

## CONCEPT AND KINEMATICS MODEL OF A MECHANICAL DRIVE SYSTEM BASED ON FACE AND BEVEL GEARS

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**Abstract.** *The great concern about environment issues has stimulated the development of new mechanical drives concepts in machine and automotive systems, in search of better performance, functionality and efficiency. However, the identification of the applicability of a new concept involves the analysis and tests over physical models and prototypes. This work presents the concept of a new mechanical drive system, composed of bevel and face gears of involute profile teeth, with multi-step angular velocities ratios, for industrial and automotive purposes. There is an especial interest in the use of face gears in mechanical drives because of recent innovations in manufacture and computerized design processes, witch allowed to obtain high quality parts, with high mechanical strength and considerably lower costs as compared with bevel gears. To proceed with the synthesis and analysis process over this new mechanical drive concept, it was necessary to implement two mathematical models, converted into algorithms: i) kinematics of gear meshing, with the final purpose of obtain the gear tooth profile and an accurate solid model in a CAD software (it is important to emphasize that the tooth profile of a face gear, represented by the envelope generated by a succession of involute curves, does not presents the involute profile) and ii) teeth dimensioning, based on standards and in solid mechanics tools, to obtain the gears magnitude. This paper presents in details the first mathematical model, based on linear algebra and differential geometry. The coordinate transformation matrices are shown, as well as the meshing equation for the face and bevel bands of the proposed transmission. The implemented algorithm allowed to build an accurate solid model, in a CAD software, of the bevel and face gears adopted in the system. The mathematical model also permits to define in which cases is interesting to adopt a work band (corresponding to one of the angular velocities ratio) composed by conical teeth (contrary to face gear teeth). The solid models of the gears were used to CNC code generation and manufacturing process on a high speed machining center. In succession, all the subsystems of a prototype were designed and constructed. This prototype was built to validate the functional attribute of the drive system. The concept validation was based on the verification of the movements necessary to shift the ratio of velocities simultaneously to the gears rotation, without teeth collision. Some aspects related to the system operation are discussed, and positive points and constraints observed are explained.*

**Keywords:** *mechanical transmission, face gears, bevel gears.*

### 1. INTRODUCTION

Face gear drives are composed by a cylindrical pinion, with straight or helical teeth of involute profile, and a conjugated facial gear (Dudley, 1994; Litvin e Fuentes 2004). Figure 1 shows the gears that compose this kind of mechanical transmission system.

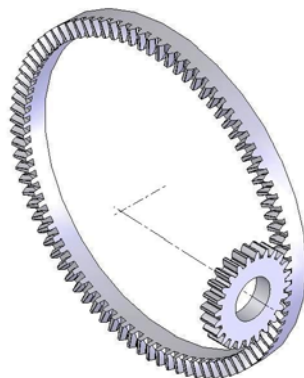


Figure 1. Face gear and cylindrical pinion.

Face gears can be considered a special case of bevel gears, being used in offset axis assembling, as well as in intercepting axis and, more frequently, in crossed axis drives. Figure 2 presents two equivalent transmission systems: a) with bevel gears – represented only by the primitive cones – and b) with a face gear and a cylindrical pinion. In both cases, the angle between axis is the same.

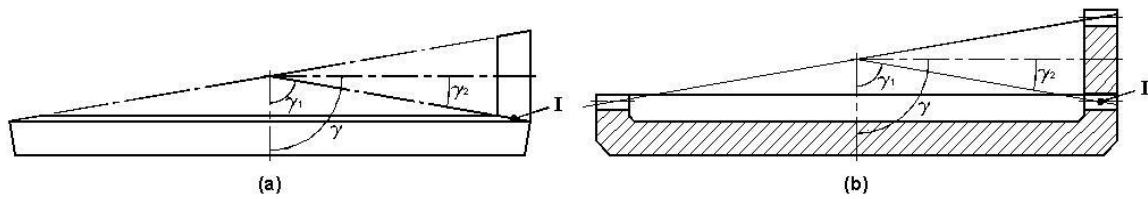


Figure 2. Equivalent transmissions with: a) bevel gears, and b) face gear and cylindrical pinion.

It is possible to see on Fig. 2 that the primitive point I, in the bevel system, is placed over the surface of both gear and pinion primitive cones. However, in a face system, the cylindrical primitive surface of pinion is not aligned with the (virtual) conical primitive surface of gear. As a result, the gear teeth present a relative complex shape. Moreover, the teeth manufacturing process in a face gear, by means of a cylindrical shaper cutter or a helical hob, is considerably simpler in relation to the cutting process of generation of bevel gears.

Until recently, face gears drawbacks were:

- i) high sensibility to angular misalignment of pinion and gear axis: a relatively small angular error moves the contact region to the teeth edge, increasing the contact pressure level and causing decrease in lifetime.
- ii) It was not possible to manufacture, by shaper generation, face gears of hardened materials.

Recent developments and mathematical models proposed by Litvin et. al. (1994, 2000, 2002 and 2005), Guingand et. al. (2005), Zanzi and Pedrero (2005), Barone et. al. (2004), Zhang and Wu (1997) and Mao (2007) allowed an amplification of face gears range of application. Nowadays, face gear drives present components of hardened materials, with high manufacturing quality, obtained by a grinding process with a worm tool. So, face gears have been used in high load and velocity applications, as happens in aeronautics industry.

Despite the advantages in costs and performance of face gear drives, these transmission systems have fixed ratio of velocities, which does not allow its application on vehicles. Bianchi and Batista (2008) presented the concept of a stepped transmission for agricultural purposes, based on face and bevel gears, with shifting of ratio of velocities. Figure 3 shows the proposed system, composed by two identical and opposed input and output gears, with a set of intermediate pinions. Both gear and pinion are divided in toothed bands. The gears presented on Fig. 3 have three bands, which means that the transmission has three ratio of velocities. However, the number of bands could be greater, since that teeth could be relatively small.

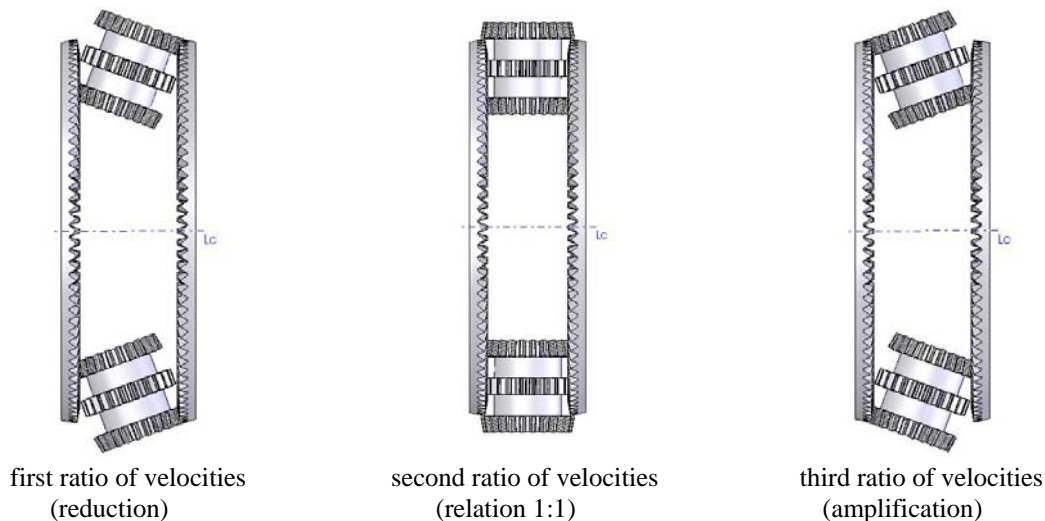


Figure 3 – Principle of operation of the proposed system (three ratio of velocities).

To validate the concept it was necessary to design and build a prototype, as well as to proceed with operational tests. The first step in the design process was the generation of a face gear teeth model, based on the procedure developed by Litvin (1994 and 2002). Such procedure was extended for bevel gears, making possible to generate all the bands and the complete gear solid model in a Computer Aided Design software (in this case, SolidWorks®). The next step comprised manufacturing and assembling of the prototype, followed by operational tests. These tests enabled to visualize if all the necessary movements for drive operation occurred effectively. The positive results obtained enable to: *i*) validate the mathematical model implemented and *ii*) validate the transmission concept in relation to functional and operational

requirements. An important aspect about the operation of the system is the shift of ratio of velocities, which shall occur without pinion and gears teeth collision.

## 2. MATERIAL AND METHODS

In a resume, for a construction of a prototype of a new mechanical gear drive system, it is necessary to make the solid model and, after that, to manufacture the gears. Another important task is the elaboration of a dimensioning process for these gears, based on standards. So, with some input data, it is possible to obtain the teeth and gear dimensional proportions and, after that, proceed to solid model generation and gears manufacturing. The steps that compose this procedure are described here, in sequence.

### 2.1. Gear teeth mathematical model

An algorithm was implemented, which perform the following sequence of tasks:

- 1) input data:
  - module (or diametral pitch)
  - teeth number for pinion and gear
  - pressure angle
  - angle between axes
- 2) preliminary calculations
  - base and primitive circles radii
  - ratio of velocities
- 3) matrices and vector assembling
  - position vector  $r_s$

$$r_s = \begin{bmatrix} x_s \\ y_s \\ z_s \end{bmatrix} = r_b \cdot \begin{bmatrix} \cos \theta_s + \theta_s \cdot \text{sen} \theta_s \\ \text{sen} \theta_s - \theta_s \cdot \cos \theta_s \\ z_s \end{bmatrix} \quad (1)$$

This vector represents the Cartesian coordinates of each point over the involute surface  $\Sigma_s$  of a pinion tooth in mesh. In this equation,  $r_b$  is the base circle radius. Observe that  $x_s$  and  $y_s$  Cartesian coordinates are parametric on the involute curvilinear coordinates  $\theta_s$  e  $z_s$ .

Figure 4 shows an involute curve, with  $x_s$  and  $y_s$  coordinate axes and the angular curvilinear displacement  $\theta_s$  (the  $z_s$  axis points out of the sheet plane).

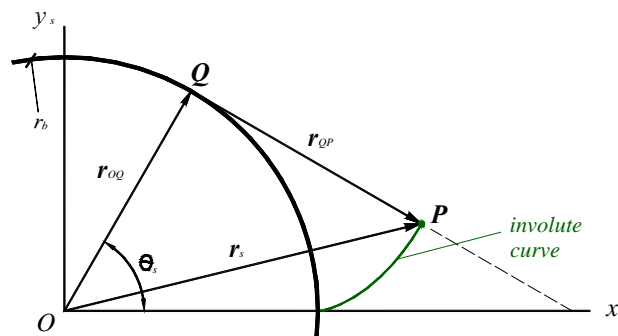


Figure 4 – Involute curve

- normal vector  $n$  to the surface  $\Sigma_s$ , in the point defined by  $r_s$ :

$$n = \begin{bmatrix} \text{sen} \theta_s \\ -\cos \theta_s \\ z_s \end{bmatrix} \quad (2)$$

- coordinate transformation matrices between  $S_2$  and  $S_s$  (gear and pinion coordinate systems, respectively):

Figure 5 shows all the coordinate systems involved. Both gear and pinion have two coordinate systems: one fixed on the rotational axis and another fixed and rotating with the part (around the respective fixed coordinate system).  $P$  is a instantaneous contact point over  $\Sigma_s$  and  $\Sigma_2$  surfaces of pinion and gear teeth in mesh.

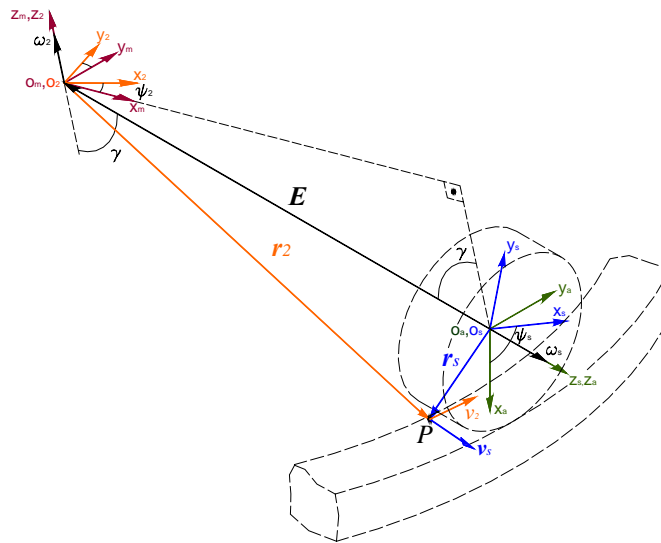


Figure 5. Coordinate systems of spur pinion and face gear in mesh.

The variable  $\psi_s$ , known as the general parameter of motion, represents the angular displacement of tool (shaper) in the gear cutting process or generation, or the angular displacement of pinion during the meshing movement. The parameter  $\gamma$  represents the angle between pinion and gear axes.

The transformation matrices are:

$$M_{as} = \begin{bmatrix} \cos \psi_s & -\sin \psi_s & 0 & 0 \\ \sin \psi_s & \cos \psi_s & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

$$M_{sa} = M_{as}^{-1} = \begin{bmatrix} \cos \psi_s & \sin \psi_s & 0 & 0 \\ -\sin \psi_s & \cos \psi_s & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

$$M_{ma} = \begin{bmatrix} -\cos \gamma & 0 & \sin \gamma & E \sin \gamma \\ 0 & 1 & 0 & 0 \\ -\sin \gamma & 0 & -\cos \gamma & -E \cos \gamma \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (5)$$

$$M_{am} = \begin{bmatrix} -\cos \gamma & 0 & -\sin \gamma & 0 \\ 0 & 1 & 0 & 0 \\ \sin \gamma & 0 & -\cos \gamma & -E \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (6)$$

$$M_{2m} = \begin{bmatrix} \cos \psi_2 & \sin \psi_2 & 0 & 0 \\ -\sin \psi_2 & \cos \psi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

$$M_{m2} = \begin{bmatrix} \cos \psi_2 & -\sin \psi_2 & 0 & 0 \\ \sin \psi_2 & \cos \psi_2 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (8)$$

– relative velocity vector

$$\mathbf{v}_{S2} = \mathbf{v}_S - \mathbf{v}_2 = \boldsymbol{\omega}_S \times \mathbf{r}_S - \boldsymbol{\omega}_2 \times (\mathbf{r}_S - \mathbf{E}) \quad (9)$$

This vector is very important because, if a point  $P$ , over  $\Sigma_S$ , is an instantaneous contact point inside the meshing range of motion, this relative velocity vector is placed over the common tangent plane to  $\Sigma_S$  and  $\Sigma_2$ . In other words, if  $P$  is an instantaneous contact point, the relative velocity vector  $\mathbf{v}_{S2}$  is normal to  $\mathbf{n}$  vector.

4) meshing equation

$$\mathbf{v}_{S2} \cdot \mathbf{n} = 0 \quad (10)$$

By applying the meshing equation, a function of parameters  $(\theta_s, \psi_s, z_s)$  is obtained. So, the problem can be stated as: for a fixed value of pinion (or shapper) angular displacement  $\psi_s$  and varying the  $z_s$  parameter in a stated range, what are the corresponding values of curvilinear angular coordinate  $\theta_s$  that satisfy the meshing equation.

All the points obtained for a given value of  $\psi_s$ , form a contact line between faces  $\Sigma_S$  and  $\Sigma_2$ . Sequentially, the value of  $\psi_s$  is increased in a iterative loop and, as a result, a family of contact lines, represented on gear coordinate system  $S_2$ , is obtained.

A MATLAB function was implemented, with two loops in  $\psi_s$  and  $z_s$ , which enabled to obtain lists of Cartesian coordinates of points that form the contact lines during the meshing motion. The family of lines forms the desired surface  $\Sigma_2$ .

A similar procedure was adopted to obtain the bevel teeth that compose the internal and external bands of input and output gears. However, a set of equations related to the cone angle  $\gamma_c$  had to be added.

## 2.2. Gear and pinion solid model elaboration in a CAD software

The lists of Cartesian coordinates of points were exported to SolidWorks®, where they have been converted in lines, by a spline-curve feature. Sequentially, a surface-loft feature was used to transform the family of contact lines in surfaces. Finally, by applying mirror and trim surface features, followed by a final knit operation, it was possible to form a face gear tooth solid model. After that, simple pattern and join features permitted to obtain a gear disc with all the face teeth of the central band of operation of input and output gears of the proposed transmission. The same procedure was adopted to form and join the bevel teeth that compose the external and internal bands of input and output gears of the proposed drive system.

The generation of pinion solid model is considerably simpler. A Matlab function based on the involute curve generated the points over the surface  $\Sigma_S$  for both spur and bevel bands. Sequentially, this points were exported to the CAD software and the same features adopted for teeth generation in face and bevel bands of the gears were used.

Figure 6 shows the final pinion and gear solid models. All the bands in the pinion have 24 teeth, while the gears have bands with 80, 88 and 96 teeth. Teeth module has a value of 3 mm.



Figure 6. Pinion and gear solid models of the proposed transmission system.

## 2.3 Design and manufacturing of a prototype to validate the drive concept

The first step to validate a new product concept is submit a prototype to tests to verify the functional attribute. So, this was the objective and focus of prototype development and not the verification of commercial applicability.

For correct operation of the proposed drive system, four movements are necessary: *i*) input gear rotation, by means of a motor; *ii*) nutation of the pinions, to shift the ratio of velocities, *iii*) input and output gears approximation or separation, depending on the desired ratio of velocities and *iv*) axial displacement of the pinions (half the value of the relative axial displacement of the gears). An electrical motor was adopted for input gear actuation, while the other movements were performed by pneumatic actuators, because of simplicity of implementation. After actuators definition, it was possible to sketch the system prototype drawing, shown on Fig.7.

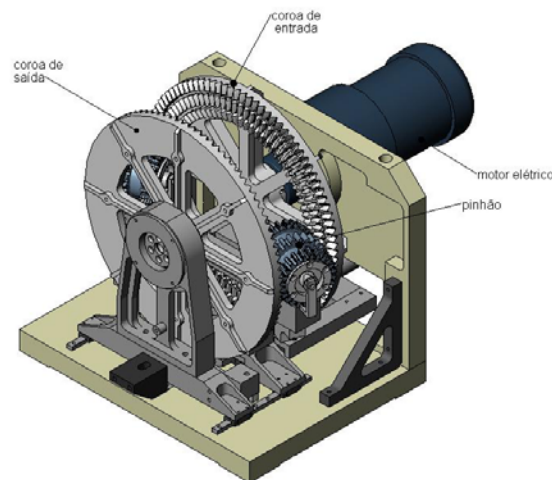


Figure 7. Assembling drawing of the prototype

The more relevant stage on prototype construction was the gears manufacturing. Because of the impossibility of generation of these gears, a milling process of cutting was adopted, with a small tool on a CNC high speed machine. The material used for pinions and gears is the cast polyamide with a small quantity of Molybdenum disulfide. The main reason for choosing this material is the silent operation, without necessity of lubrication.

Figure 8 shows a view of the constructed prototype.

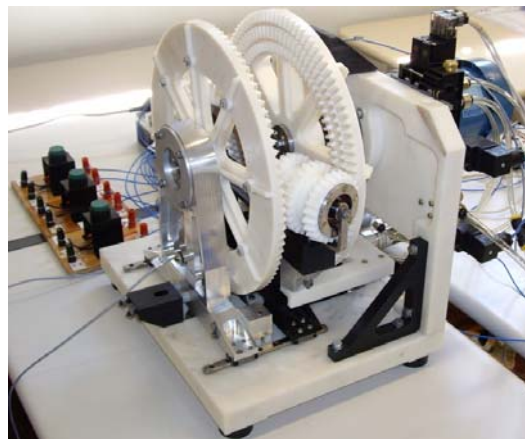


Figure 8. View of the prototype constructed for functional evaluation.

### 3. FUNCTIONALITY TESTS OVER THE PROTOTYPE

It was observed, during the first tests, that the choice for pneumatic actuation in the gears axial displacement and in the nutation movement of pinions was appropriate, due to the almost instantaneous action. However, the impact in the actuators end of course caused excessive noise and vibration. In the system design for real field applications, this problem can be solved by an accurate sizing of dampers at the end of actuators course.

However, the most important result observed is the functionality verification of the proposed gear drive system. The ratio of velocities shift occurred in an instantaneous form, without pinion and gears teeth collisions. However, certain mismatch in movements was observed in the direct shift from 1<sup>a</sup> to 3<sup>a</sup> ratio of velocities (and vice versa), because the nutation course is very large. So, it permits to state that the control system should, in the case of a drive for vehicles application, to perform the ratio of velocities shift in a sequential form.

The command of movements in the prototype was made by a PLC (Programmable Logical Controller) and the signals for ratio of velocities shift were generated by means of electrical buttons. The use of a manual lever in place of these buttons could provide the desired sequential actuation. At this moment, it is not defined if this manual lever would have an operation similar to the sequential shifting of some automobiles and motorcycles (up and down shifting) or if it would have fixed positions for each ratio of velocities.

A relatively silent operation could be observed during prototype tests, which could be naturally associated to the facial gearing as well as to the use of polyamide in pinions and gears. However, because of the rough surface finish

obtained in the cutting process adopted for gear and pinion manufacture, some noise was expected. This silent motion, as well the low heat generation observed in the gears, showed the effectiveness of the mathematical model adopted for teeth surface generation in the three work bands of gears and pinions.

Because all the main aspects were satisfied, it was observed that the proposed system is viable in respect to the functional attributes.

However, very important aspects for product validation were not still verified; as examples, the investigation of gears and pinions surface wear and the observation of shifting mechanisms behavior after grate number of cycles. To achieve these important results, a preliminary design of a small tractor mechanical gear drive is being implemented, based on high quality face gears. The steps of manufacturing and tests over product prototype could give important information related to design and operation attributes. So, the drive implementation over a small farm tractor will expand the knowledge about the product, permitting to visualize its positive and negative points, as well as delimiting its field of application.

#### 4. CONCLUSION

A mechanical gear drive concept based on face and bevel gears was presented. For concept validation, a prototype was manufactured and submitted to preliminary tests, which showed that the mechanical drive is viable in relation to functional aspects. The system operation also validated the mathematical algorithms adopted for gears solid modeling and manufacturing.

All the preliminary results point to a silent and robust mechanical drive system with a high load capacity and a relatively high moment of inertia. The system presents multi-step and relatively near ratios of velocities, and it is suitable for automatic or semi-automatic operation. So, this mechanical drive system permits small variations in torque and velocity provided by a motor, in face to changes in working load. Association with other mechanical gear drives could permit operation flexibility and comfort to the user in vehicles and other equipments. However, additional work must be done to validate this mechanical drive system as a product, attending to all the attributes related to operation, manufacturing and market.

#### 5. ACKNOWLEDGEMENTS

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