

# MODEL PREDICTIVE CONTROL WITH CONSTRAINTS ON ACCUMULATED DEGRADATION OF ACTUATORS

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**Abstract.** *Process industry has experienced important changes over the last two decades. The development of new technologies has made automated systems more complex and, eventually, more vulnerable to faults. Fault-tolerant operation in the presence of actuator faults requires some form of redundancy, especially if it is necessary to keep the system running until the next scheduled maintenance. This period of time is defined in the present work as the maintenance horizon. In some cases the control effort can be redistributed among the available actuators to alleviate the work load and the stress factors on the equipments with worst conditions to avoid their break down. For this purpose, an appropriate policy should be developed to redistribute this effort until maintenance actions can be taken. In this paper, a solution to this problem is proposed by using a Model Predictive Control (MPC) approach. An important feature of MPC is the possibility of explicitly including input and state constraints in the control law. In the proposed approach, the plant model is augmented with additional states associated to the accumulated degradation of the actuators. It is assumed that the degradation rate is proportional to the control effort demanded from each actuator. Constraints are imposed on the additional states to ensure that the accumulated degradation will be acceptable at the end of the maintenance horizon. The onset of an actuator fault is modelled as either an increase in the degradation rate or a step increment in the accumulated degradation. A simulated tank level control system is used as a case study to illustrate the efficiency of the proposed approach. This system is composed by a set of redundant pumps that supply fluid to the tank. The results show that the predictive controller distributes the control effort in a suitable manner to relieve the pumps with larger accumulated damage. Moreover, this distribution is promptly changed upon the occurrence of a fault.*

**Keywords:** *Model Predictive Control, Fault-tolerant Control, Maintenance Horizon, Redundant Actuators.*

## 1. INTRODUCTION

Industrial systems are subject to failures that may compromise the operation of the system. In many cases, failures result from a degradation (or deterioration) process [3]. In critical systems, the degradation should not be allowed to exceed a fault threshold before the next scheduled maintenance. These classes of systems often requires some form of sensors and actuator redundancy to implement a fault-tolerant operation [2, 7].

The present work is concerned with systems that suffer actuator wear. The objective is to keep the system running along a period of time defined here as maintenance horizon without exceed a degradation threshold on actuators and avoiding a fault. One way to implement this capability is to provide the controller with an ability to redistribute the control effort among the available actuators. In this way, it is possible to alleviate the work load and the stress factors on the equipments with worst conditions to avoid their early break down. For this purpose, an appropriate policy is required to redistribute this effort until maintenance actions can be taken.

In this paper a Model Predictive Control (MPC) is used to implement a fault tolerant-operation for systems with actuator redundancy. In the proposed approach, the plant model is augmented with additional states associated to the accumulated degradation of the actuators. It is assumed that the degradation rate is proportional to the control effort demanded from each actuator. Constraints are imposed on the additional states to ensure that the accumulated degradation will be acceptable at the end of a given maintenance horizon.

A simulated tank level control system is used as a case study to illustrate the efficiency of the proposed approach. The results show that the predictive controller distributes the control effort in a suitable manner to relieve the pumps with larger accumulated damage.

The paper is organized as follow. Section 2 reviews the basic concepts of MPC. In section 3 describes the proposed MPC formulation with constraints on accumulated degradation. Results and conclusions are presented in sections 4 and 5, respectively.

### 1.1 Notation

The Table 1 presents a list of symbols that will be used throughout the text.

Table 1. List of symbols

Notation	Description
$I_\beta$	Identity matrix of size $\beta$ -by- $\beta$ .
$T_\beta^\Lambda$	Block lower triangular matrix composed by $\beta$ -by- $\beta$ blocks of $\Lambda$ matrices.
$[\Lambda]_\beta$	Block column matrix composed by $\beta$ -by-1 blocks of $\Lambda$ matrices.
$[\Lambda]_\beta^T$	Block row matrix composed by 1-by- $\beta$ blocks of $\Lambda$ matrices.
$[\Lambda]_{\alpha \times \beta}$	Block matrix composed by $\alpha$ -by- $\beta$ blocks of $\Lambda$ matrices.

## 2. MODEL PREDICTIVE CONTROL

MPC was developed in the 1970s to address the needs of the petroleum industry [5]. Nowadays, many applications can be found in other areas such as Flight Controls [4], Energy-saving Systems [1], and others. MPC employs a mathematical model of the system to predict the future evolution of the controlled variables over a moving window, which is called Prediction Horizon. The objective is to optimize a sequence of control actions along Control Horizon in order to minimize a cost function defined over the Prediction Horizon. The resulting problem is solved and the first value of the optimal control sequence is applied to the system. This process is repeated at each sampling time [5]. Figure 2 shows the basic elements of an MPC scheme employing state feedback.

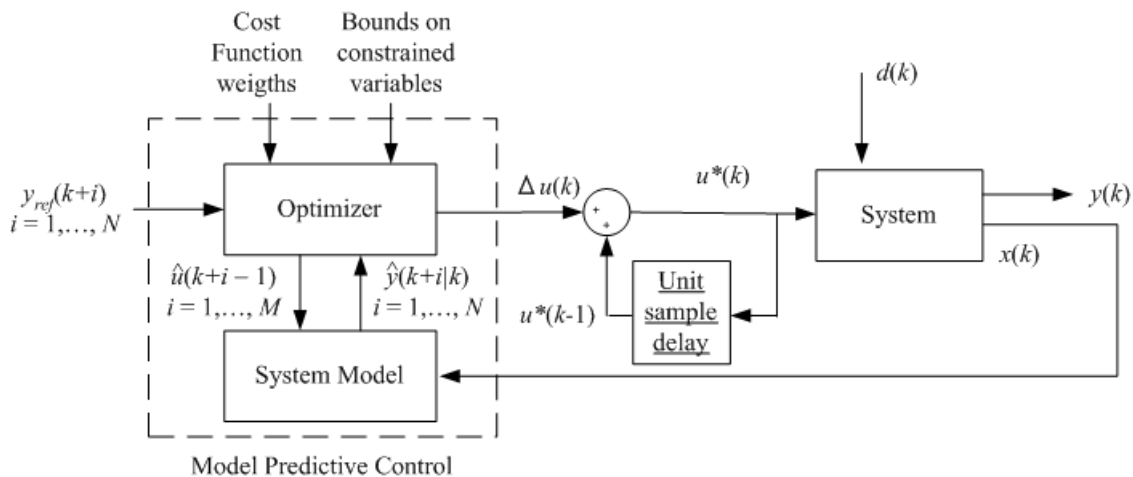


Figure 1. MPC scheme employing state feedback.

In the Figure 2  $\hat{u}(k+i|k), i = 1, \dots, M$  are the values of the control sequence,  $u^*(k)$  is the optimal value of  $\hat{u}(k|k)$ ,  $\Delta u(k)$  is the increment of control value,  $\hat{y}(k+i|k)$  is the values of output sequence,  $y_{ref}(k)$  is the values of reference output,  $x(k)$  is the values of states,  $M$  is the number of samples in the Control Horizon and  $N$  is the number of samples in the Prediction Horizon. It is considered that the value of sequence control in the interval between  $M$  and  $N$  is constant and equal to the value of  $\hat{u}(k+M|k)$ .

## 3. PROPOSED METHOD

### 3.1 MPC with actuator degradation

The work is concerned with a class of system that can be represented by discrete-time linear state-space models of the form

$$\begin{cases} x(k+1) = Ax(k) + Bu(k) + Ed(k) \\ y(k) = Cx(k), \end{cases} \quad (1)$$

where, for each  $k \in Z^+$ ,  $x(k) \in R^n$  is the state,  $u(k) \in \mathfrak{R}^p$  is the control input with  $u(k) \geq 0$  for all  $k$ ,  $y(k) \in \mathfrak{R}^q$  is the output,  $d(k) \in \mathfrak{R}^m$  is the disturbance,  $A \in \mathfrak{R}^{n \times n}$  is the state matrix,  $B \in \mathfrak{R}^{q \times p}$  is the input matrix,  $E \in \mathfrak{R}^{n \times m}$  is the disturbance matrix and  $C \in \mathfrak{R}^{q \times n}$  is the output matrix.

The fault considered here is assumed to be related to a degradation process with rate associated to a measurable variable [3]. Assuming that the wear of the actuators is proportional to the exerted control effort  $u$  the normalized accumulated degradation  $z(k)$  can be written

$$z(k+1) = z(k) + \frac{\Gamma u(k)}{z_{k_M}}. \quad (2)$$

where  $z_{k_M}$  is the *maximum* acceptable degradation up to the Maintenance Horizon and  $\Gamma$  is a diagonal matrix of deterioration coefficients  $\gamma_1, \gamma_2, \dots, \gamma_p$ , associated with the  $p$  element of input vector  $u(k)$ . These coefficients are assumed to be known.

The first step in MPC formulation is to obtain a prediction model to be used on optimization along the Prediction Horizon. To facilitate the MPC formulation, lets consider the disturbance  $d(k)$  as an extended state and the system presented in (1) became:

$$\begin{cases} \hat{x}_e(k+1) = A_e \hat{x}_e(k) + B_e u(k) \\ y(k) = C_e x_e(k), \end{cases} \quad (3)$$

where

$$A_e = \begin{bmatrix} A & E & 0_{n \times p} \\ 0_{m \times n} & E'_{m \times m} & 0_{m \times p} \\ 0_{p \times n} & 0_{p \times m} & I_p \end{bmatrix}, \quad (4)$$

$E' \in \mathbb{R}^{m \times m}$  is state matrix for the disturbance,

$$B_e = \begin{bmatrix} B \\ 0_{n \times p} \\ \Gamma \end{bmatrix}, \quad (5)$$

$$C_e = [ C \quad 0_{q \times n} \quad 0_{q \times p} ], \quad (6)$$

and

$$\hat{x}_e(k) = \begin{bmatrix} x(k) \\ \hat{d}(k) \\ z(k) \end{bmatrix}. \quad (7)$$

The notation  $\hat{d}(k)$  is used instead of  $d(k)$  because it considered in this work that the value of disturbance is unknown and it is estimated by using of a Luenberger State Estimator [6] given by

$$\begin{cases} \hat{x}_s(k+1) = A_s \hat{x}_s(k) + B_s u(k) + L(y(k) - \hat{y}(k)) \\ \hat{y}(k) = C_s \hat{x}_s(k), \end{cases} \quad (8)$$

where

$$A_s = \begin{bmatrix} A & E \\ 0_{m \times n} & E'_{m \times m} \end{bmatrix}, \quad (9)$$

$$B_s = \begin{bmatrix} B \\ 0_{n \times p} \end{bmatrix}, \quad (10)$$

$$C_s = [ C \quad 0_{q \times n} ], \quad (11)$$

$L$  is the gain matrix of the state estimator and

$$\hat{x}_s(k) = \begin{bmatrix} \hat{x}(k) \\ \hat{d}(k) \end{bmatrix}. \quad (12)$$

The predictive equation to the states on the Prediction Horizon and Control Horizon is:

$$\begin{bmatrix} \hat{y}(k+1|k) \\ \hat{y}(k+2|k) \\ \vdots \\ \hat{y}(k+N|k) \end{bmatrix} = \begin{bmatrix} C_e B_e & 0_{q \times p} & \dots & 0_{q \times p} \\ C_e A_e B_e & C_e B_e & \dots & 0_{q \times p} \\ \vdots & \vdots & \ddots & \vdots \\ C_e A_e^{N-1} B_e & C_e A_e^{N-2} & \dots & C_e B_e \end{bmatrix} \begin{bmatrix} \hat{u}(k|k) \\ \hat{u}(k+1|k) \\ \vdots \\ \hat{u}(k+N-1|k) \end{bmatrix} + \begin{bmatrix} C_e A_e \\ C_e A_e^2 \\ \vdots \\ C_e A_e^N \end{bmatrix} x_e(k) \quad (13)$$

If  $M < N$  we have

$$\underbrace{\begin{bmatrix} \hat{y}(k+1|k) \\ \hat{y}(k+2|k) \\ \vdots \\ \hat{y}(k+N|k) \end{bmatrix}}_{\hat{Y}} = \underbrace{\begin{bmatrix} C_e B_e & 0_{q \times p} & \cdots & 0_{q \times p} \\ C_e A_e B_e & C_e B_e & \cdots & 0_{q \times p} \\ \vdots & \vdots & \ddots & \vdots \\ C_e A_e^{N-1} B_e & C_e A_e^{N-2} & \cdots & \sum_{i=0}^{N-M} C_e A_e^i \end{bmatrix}}_H \underbrace{\begin{bmatrix} \hat{u}(k|k) \\ \hat{u}(k+1|k) \\ \vdots \\ \hat{u}(k+M-1|k) \end{bmatrix}}_{\hat{U}} + \underbrace{\begin{bmatrix} C_e A_e \\ C_e A_e^2 \\ \vdots \\ C_e A_e^N \end{bmatrix}}_F x_e(k) \quad (14)$$

or,

$$\hat{Y} = H\hat{U} + Fx_e. \quad (15)$$

In this work the incremental input ( $\Delta u(k)$ ) formulation is used.

$$\underbrace{\begin{bmatrix} \hat{u}(k|k) \\ \hat{u}(k+1|k) \\ \vdots \\ \hat{u}(k+M-1|k) \end{bmatrix}}_{\hat{U}} = \underbrace{\begin{bmatrix} I_p & 0_{p \times p} & \cdots & 0_{p \times p} \\ I_p & I_p & \cdots & 0_{p \times p} \\ \vdots & \vdots & \ddots & \vdots \\ I_p & I_p & \cdots & I_p \end{bmatrix}}_{T_M^{I_p}} \underbrace{\begin{bmatrix} \Delta \hat{u}(k|k) \\ \Delta \hat{u}(k+1|k) \\ \vdots \\ \Delta \hat{u}(k+M-1|k) \end{bmatrix}}_{\Delta \hat{U}} + \underbrace{\begin{bmatrix} I_{p \times p} \\ I_{p \times p} \\ \vdots \\ I_{p \times p} \end{bmatrix}}_{[I_p]_M} \hat{u}(k-1) \quad (16)$$

or

$$\hat{U} = T_M^{I_p} \Delta \hat{U} + [I_p]_M u(k-1). \quad (17)$$

The matrix version of the prediction equation for  $\Delta u(k)$  formulation is obtained from (15) and (17) as follows

$$\hat{Y} = HT_M^{I_p} \Delta \hat{U} + H[I_p]_M u(k-1) + Fx_e, \quad (18)$$

Consider now the MPC problem which the cost function has the quadratic form given by

$$J(\Delta \hat{U}(k)) = \sum_{i=1}^N [\hat{y}(k+i|k) - y_{ref}(k+i)]^2 + \rho_1 \sum_{i=1}^M [\Delta \hat{u}_1(k+i-1|k)]^2 + \dots + \rho_q \sum_{i=1}^M [\Delta \hat{u}_q(k+i-1|k)]^2. \quad (19)$$

where  $\rho_q$  is the cost associated to control increment  $\Delta u_q(k)$ . Equation 19 can be written in matrix form as

$$J(\Delta \hat{U}(k)) = [\hat{Y} - Y_{ref}]^T [\hat{Y} - Y_{ref}] + \Delta \hat{U}^T D_M^R \Delta \hat{U} \quad (20)$$

and we have

$$D_M^R = \begin{bmatrix} R & 0_{p \times p} & \cdots & 0_{p \times p} \\ 0_{p \times p} & R & \cdots & 0_{p \times p} \\ \vdots & \vdots & \ddots & \vdots \\ 0_{p \times p} & 0_{p \times p} & \cdots & R \end{bmatrix} \quad (21)$$

and

$$R = \begin{bmatrix} \rho_1 & 0 & \cdots & 0 \\ 0 & \rho_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & \rho_p \end{bmatrix}, \quad (22)$$

$$Y_{ref} = \begin{bmatrix} y_{ref}(k+1) \\ y_{ref}(k+2) \\ \vdots \\ y_{ref}(k+N) \end{bmatrix} \quad (23)$$

Using (18) and (20), the cost function can be written as

$$J(\Delta \hat{U}(k)) = [HT_M^{I_p} \Delta \hat{U} + H[I_p]_M u(k-1) + Fx_e - Y_{ref}]^T [HT_M^{I_p} \Delta \hat{U} + H[I_p]_M u(k-1) + Fx_e - Y_{ref}] + \Delta \hat{U}^T D_M^R \Delta \hat{U} \quad (24)$$

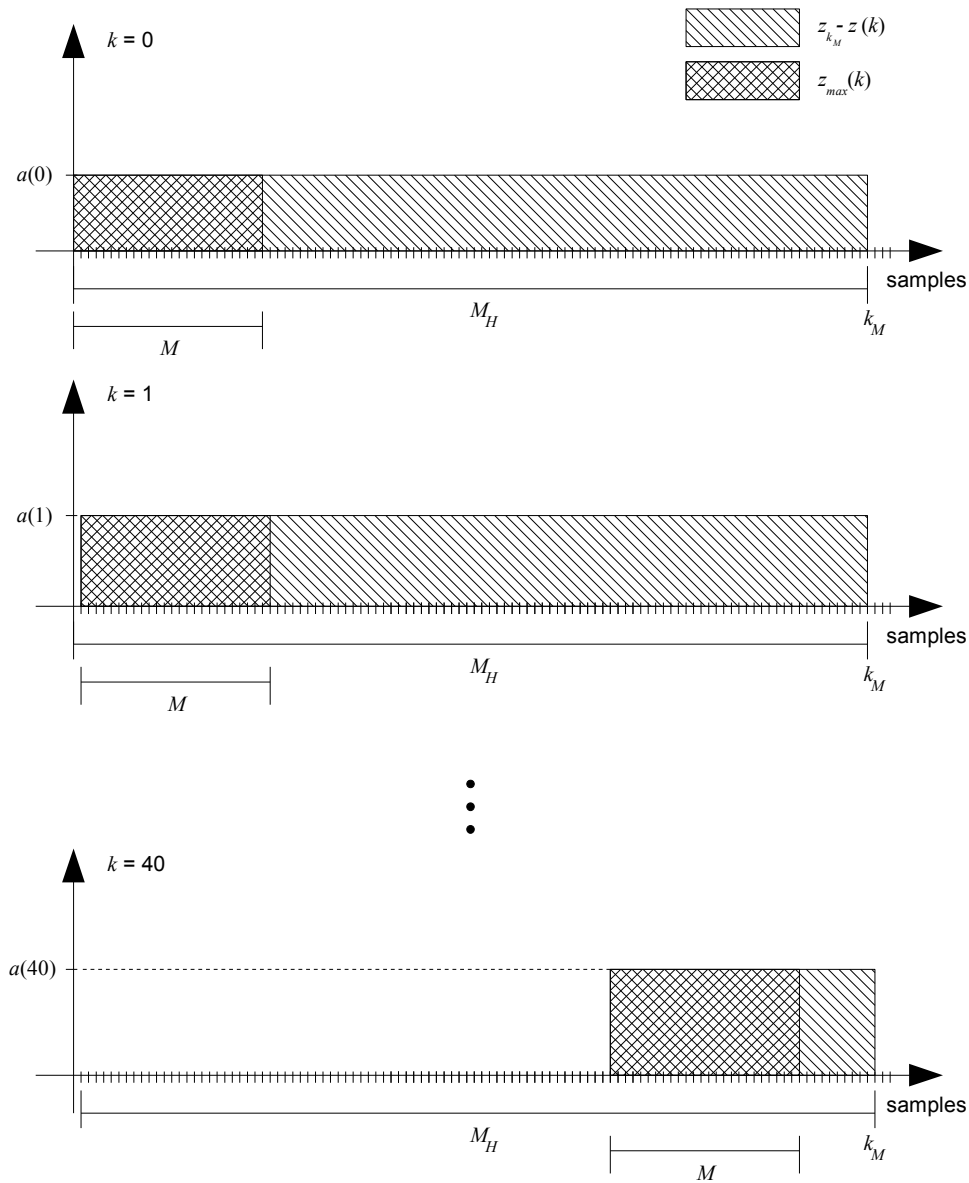


Figure 2. Evolution at time of the amount of accumulated degradation  $Na(k)$  that can be spent during a predict window.

### 3.2 MPC with restriction on actuators degradation

Consider the time  $k = k_M$  which correspond to the instant of a programmed maintenance. The period of time started at  $k = 0$  and ending at  $k = k_M$  is defined here such Maintenance Horizon ( $M_H$ ). The problem to be solved now is how to guarantee that the accumulated degradation will not exceeds a threshold  $z_{k_M}$  before  $k = k_M$ . Mathematically, this condition can be expressed by

$$z(k_M) \leq z_{k_M}. \quad (25)$$

However, the Prediction Horizon in MPC is typically shorter than Maintenance Horizon. Therefore, one needs to fix a threshold  $z_{max}(k + N)$  of the degradation that can be accumulated up to the time  $k + N$ . If one consider that degradation is uniformly distributed along the Maintenance Horizon one can propose that the accumulated degradation along the Prediction Horizon at time  $k$  should not be larger than the area of a rectangle with base  $N$  and height  $a(k)$  given

by

$$a(k) = \frac{z_{k_M} + N - z(k)}{k_M - k}. \quad (26)$$

in that

$$z_{max}(k) = Na(k), \quad (27)$$

It is worth noting that  $a(k)$  is a fiction degradation that is equivalent to the average of  $\hat{u}(k)$  in the prediction window. Intuitively, the area  $Na(k)$  is the amount of accumulated degradation that can be spent during a predict window so as to reach  $k = k_M$  without crossing the threshold. Figure 2 illustrates this idea. At time  $k + 1$  we need to calculate the new value of the height  $a(k + 1)$  subtracting  $z(k_M)$  of  $z(k + 1)$  and distributing again the remaining degradation uniformly from instant  $k + 1$  until time  $k = k_M$ . This process is repeated for all  $k$  until  $k = k_M$ . In this way it possible to write a prediction equation for the normalized accumulated degradation using (2) in a recursive way

$$\hat{z}(k + N|k) = z(k) + [\Gamma]_N^T \Xi \Delta \hat{U} + [\Gamma]_N^T [I_P]_N u(k - 1) \quad (28)$$

where

$$[\Gamma]_N = \begin{bmatrix} \Gamma \\ \Gamma \\ \vdots \\ \Gamma \end{bmatrix}, \quad (29)$$

$$\Xi = \begin{bmatrix} T_M^{I_p} \\ [I_p]_{(N-M) \times M} \end{bmatrix} = \begin{bmatrix} I_p & 0_{p \times p} & \dots & 0_{p \times p} \\ I_p & I_p & \dots & 0_{p \times p} \\ \vdots & \vdots & \ddots & \vdots \\ I_p & I_p & \dots & I_p \\ \vdots & \vdots & \ddots & \vdots \\ I_p & I_p & \dots & I_p \end{bmatrix} \quad (30)$$

The bounds in the acceptable accumulated degradation can now be stated as

$$\hat{z}(k + N|k) \leq z_{max}(k). \quad (31)$$

or

$$[\Gamma]_N^T \Xi \Delta \hat{U} \leq z_{max}(k) - z(k) - [\Gamma]_N^T [I_P]_N u(k - 1) \quad (32)$$

The constraint in equation (32) and the cost function given by (24) to defines an optimal control problem. In this work it is considered that  $u(k) \geq 0$  and so it is necessary to impose the excursion constraint to the optimal control problem as follows

$$0 \leq \underbrace{\begin{bmatrix} \hat{u}(k + 1) \\ \hat{u}(k + 2) \\ \vdots \\ \hat{u}(k + M - 1) \end{bmatrix}}_{T_M^{I_p}} \leq [I_p]_M u_{max} \quad (33)$$

Using (17) and (33), the constraint in matrix form is given by

$$\begin{bmatrix} T_M^{I_p} \\ -T_M^{I_p} \end{bmatrix} \Delta \hat{U} \leq \begin{bmatrix} [I_p]_M (u_{max} - u(k - 1)) \\ [I_p]_M (u(k - 1)) \end{bmatrix} \quad (34)$$

Finally, including the restriction on accumulated degradation given by (32) the MPC problem resumes to  $\min J(\Delta \hat{U})$  (equation 24) subject to

$$\begin{bmatrix} T_M^{I_p} \\ -T_M^{I_p} \\ [\Gamma]_N \Xi \end{bmatrix} \Delta \hat{U} \leq \begin{bmatrix} [I_p]_M (u_{max} - u(k - 1)) \\ [I_p]_M (u(k - 1)) \\ z_{max}(k) - z(k) - [\Gamma]_N [I_P]_N u(k - 1) \end{bmatrix} \quad (35)$$

This is a special type of mathematical optimization problem called Quadratic Programming (QP).

#### 4. TANK LEVEL CONTROL

A classical regulation problem of Tank Level control is used as a case study for illustrative purposes. The objective is to apply the MPC formulation in a system composed by two actuators (pumps) that supply fluid to a tank and guarantee that their degradation threshold will not be violated. Tank Level control was simulated in Simulink/Matlab environment using the block diagram as presented in Figure 3 and the Matlab function *quadprog* was used to solve the Quadratic Problem resulted from MPC optimization. The parameters used in the simulation are shown in the Table 4.

Table 2. List of parameters

parameter	value	unit	parameter	value	unit
$\rho_1$	1	-	$\rho_2$	1	-
$\gamma_1$	0.5	sample <sup>-1</sup>	$\gamma_2$	0.1	sample <sup>-1</sup>
$N$	10	samples	$M$	5	samples
$M_H$	500	samples	$\Delta T$	0.1	s
$u_{1min}$	0	l/s	$u_{2min}$	0	l/s
$u_{1max}$	100	l/s	$u_{2max}$	100	l/s
$y_{ref}$	90	%	$d(k)$	30 (for all $k$ )	l/s

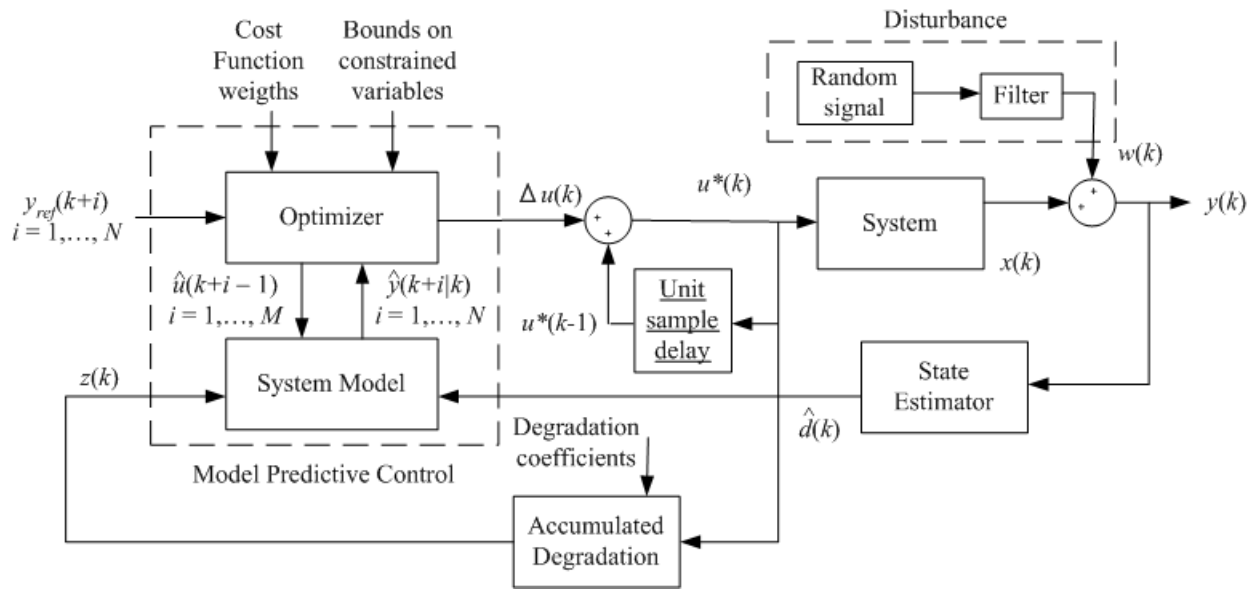


Figure 3. Block Diagram of MPC used to optimizing the degradation on actuators in a Tank Level Control.

Equation (36) is the state space representation of Tank in the simulation. The disturbance  $w(k)$  in the output of the tank are additive filtered random signals  $N(0, 5^2)$  and constant term  $d(k)$  (see Figure 3).

$$\begin{cases} x(k+1) = x(k) + \Delta T u_1(k) + \Delta T u_2(k) - \Delta T w(k) \\ y(k) = x(k), \end{cases} \quad (36)$$

The prediction model used in the MPC formulation is obtained by the extended state space form of (36) where  $x_1(k) = y(k)$ ,  $x_2(k) = \hat{d}(k)$ ,  $x_3(k) = z_1(k)$ , and  $x_4(k) = z_2(k)$ . In the prediction model it is considered that the disturbance  $w(k)$  is composed only by the constant term  $\hat{d}(k)$ .

$$\begin{bmatrix} \hat{x}_1(k+1) \\ \hat{x}_2(k+1) \\ \hat{x}_3(k+1) \\ \hat{x}_4(k+1) \end{bmatrix} = \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} \underbrace{\begin{bmatrix} 1 & -\Delta T & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}}_{A_e} + \underbrace{\begin{bmatrix} \Delta T & \Delta T \\ 0 & 0 \\ \gamma_1 & 0 \\ 0 & \gamma_2 \end{bmatrix}}_{B_e} \begin{bmatrix} u_1(k) \\ u_2(k) \end{bmatrix} \quad (37)$$

$$y(k) = \underbrace{\begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix}}_{C_c} \begin{bmatrix} x_1(k) \\ x_2(k) \\ x_3(k) \\ x_4(k) \end{bmatrix} \quad (38)$$

In this first case study, the objective was to illustrate the operation of MPC with restriction on the actuator degradation. Figure 4.a shows the time series of output  $y(t)$ . Figures 4.b shows the evolution of control signal and Figure 4.c shows the accumulated degradation along the Maintenance Horizon.

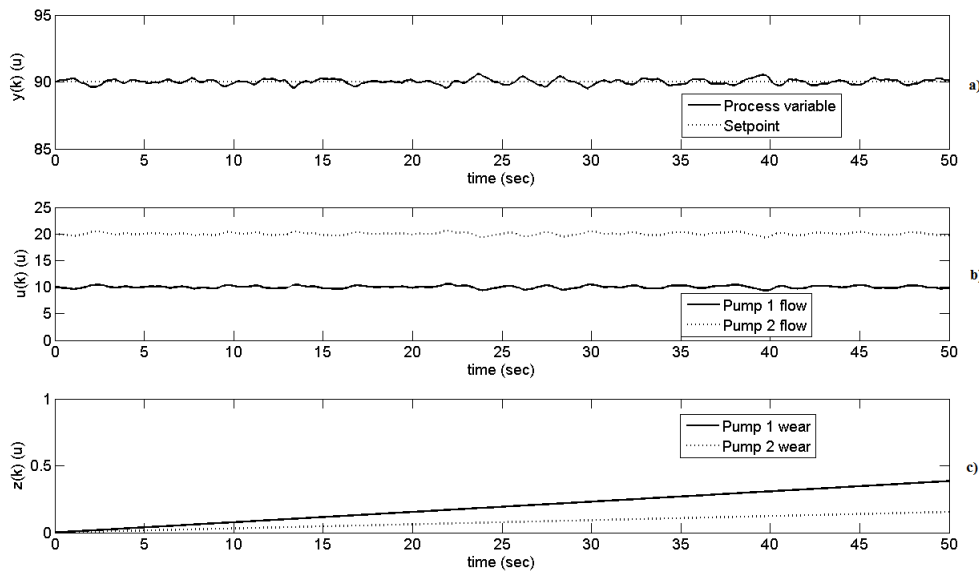


Figure 4. Optimizing the degradation among actuator in a Tank Level Control.

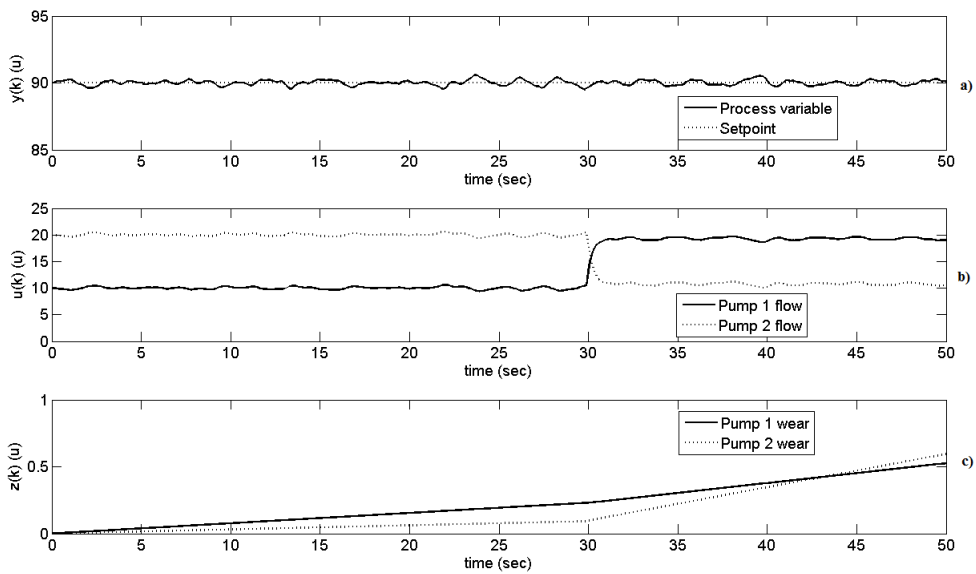


Figure 5. Optimizing the degradation among actuator in a Tank Level Control in the presence of actuator fault.

The Figure 4.b shows the control efforts along the Maintenance Horizon. It can be noted that the effort in actuator 2 is greater than that in actuator 1 as expected because the deterioration rate of actuator 1 is greater than the deterioration rate of actuator 2.



#### 4.1 Inclusion of actuators faults

Consider that at instant  $k = 30$  an abrupt fault occurs. It is considered here that this fault changes only the degradation rate of the actuator are deteriorated. It can be represented this mathematically by

$$\gamma_2 = \begin{cases} 0.1, & \text{if } k < 30 \\ 1.5, & \text{if } k \geq 30 \end{cases} \quad (39)$$

and  $\gamma_1 = 0.5$  for all  $k$ .

Figure 5.a shows the time series of output  $y(t)$ . Figures 5.b shows the evolution of control signal in the presence of a fault and Figure 5.c shows the accumulated degradation along the Maintenance Horizon. Figure 5.a shows that the controller start to redistribute the control effort among actuator in the instant that the fault was detected. In this example it was supposed that the fault made the deterioration rate of actuator 2 became greater than the deterioration rate of actuator 2.

A problem arises when the deterioration rate became so great leadily to a fault so that the optimal control problem becomes unfeasible. One possible solution for this problem is to determine a new Maintenance Horizon.

#### 5. CONCLUSIONS

This paper presented an MPC approach that is capable of redistributing the control effort among the available actuators to alleviate the work load and the stress factors on the equipments. In the proposed approach, the plant model is augmented with additional states associated to the accumulated degradation of the actuators. It is assumed that the degradation rate is proportional to the control effort demanded from each actuator. Constraints are imposed on the additional states to ensure that the accumulated degradation will be acceptable at the end of the maintenance horizon. A simulated tank level control system is used as a case study to illustrate the efficiency of the proposed approach. The results show that the predictive controller distributes the control effort in a suitable manner to relieve the pumps with larger accumulated damage.

#### 6. ACKNOWLEDGEMENTS

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