A COMPARATIVE STUDY OF THE CONSTITUTIVE EQUATIONS TO PREDICT WORK HARDENING CHARACTERISTICS OF STAINLESS STEELS 304 AND ACE P439A.

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Abstract *The process of sheet metal forming using numerical methods requires good knowledge of the mathematical model that represents the plastic region of the material used. The modeling of the elastic-plastic behavior of the stainless steels in numerical codes requires laws that describe the materials' yield surface and work hardening. The growing demand for precision in the results obtained with numerical simulation programs, motivated by economic issues or related to quality aspects, also implicates in the necessity of new constitutive models or even the adaptation of known models. This study's purpose was to evaluate constitutive equations proposed by several researchers, which best represented the plastic behavior of austenitic (304) and ferritic (439A) stainless steels. In this work, using stress-strain curves obtained from traction tests it was possible to calibrate, identify parameters and evaluate different work hardening laws, such as: Hollomon, Swift, Voce, Ludwick, Misiolek, and Ramberg-Osgood. The optimization module of the Matlab® software was used in order to find the best value for the laws parameters. It was observed that the laws that got better adjusted to the experimental data for stainless steels 304 and 439A were Ludwick and Swift, respectively. Therefore, the use of a methodology based on the minimization of the global error decreased the error of the description of the plastic behavior of the stainless steels, thus becoming a robust and accurate tool to determine the coefficients of the work hardening laws from traction tests.*

Keywords*: work hardening, stainless steel, optimization.*

1. INTRODUCTION

One of the most important elements of computer simulation of mechanical forming processes is the type of material. The simulation of phenomena such as reduction of thickness and springback are highly dependent on the description of plastic behavior during strain, (Oliveira et. al., 2007). The mathematical model used to describe the plastic behavior should take into account the physical phenomenon to which the material is submitted, depending on the type of material, the mechanical forming conditions and the history of strain, (Gronostajski, 2000).

The introduction and use of new materials is a challenge to numerical simulation, because so far there are no constitutive models of work hardening to adequately describe all aspects of plastic behavior (Chaparro et. al., 2005). The growing need for precision in the results obtained by numerical simulations, motivated whether by economic or quality issues, requires a detailed study of existing models.

There are different laws for work hardening; however, researchers still disagree as to which best describe the plastic behavior of stainless steel.

This work presents a methodology for assessment and determination of parameters, work hardening laws, having as objects of study, austenitic (304) and ferritic (439A) stainless steel. Using stress versus strain curves, obtained by tension tests, it was possible to calibrate, identify parameters and evaluate different work hardening laws as: Hollomon, Swift, Voce, Ludwick, Misiolek and Ramberg-Osgood. In order to do so, optimization techniques based on minimizing a functional were used. It was found that the laws that best fit the experimental data for stainless steel 304 and 439A were respectively Swift and Ludwick. Therefore, a study involving experimental characterization and numerical simulations creates the possibility of establishing a methodology that allows the knowledge of the process parameters and also contributes to the development and improvement of commercial software for finite elements.

2. STAINLESS STEEL

Stainless steels are iron/carbon and chromium alloys, with or without additional elements, containing around 11% of chromium. This is the minimum quantity of chromium necessary for stability.

The stainless steel has five (05) basic classifications: austenitic (300 series), ferritic (400 series), martensitic (400 series), duplex and hardened by precipitation.

The austenitic stainless steel, have excellent corrosion resistance, excellent ductility and weldability. The 304 stainless steel is the most popular among the austenitic stainless steel. One of the problems faced by the 304 (the same happens with other stainless steel) is the corrosive action caused by the chloride anion. Depending on the concentration of chlorides in the environment, temperature and pH, three types of corrosion can occur: by pit, in cracks and under stress. From these three forms of corrosion, the ferritic are also prone to the first two and it may be said that, in general, the austenitic have better resistance than the ferritic to corrosion by pit and in cracks (due to the action of nickel, which favors material's repassivation in regions where the passive film was broken by these forms of corrosion). Even if austenitic steels are not magnetic, after a process of printing, or in a cold mechanical forming, as in rolling, in the parts that have suffered greater strain, a certain magnetic character can be observed. This is a consequence of partial transformation of austenite into martensite, which happens by cold strain. Reductions in the amount of nickel, (as compared with the 304), decrease the stability of austenite, allowing a greater formation of martensite in the cold rolling, (Carbó, 2001).

The ferritic stainless steels have Cr content higher than the martensitic steels, i.e. 16-30%, and a relatively low percentage of carbon (C), (0.08-2.2%). They resist corrosion better than the martensitic. When the content of chromium is high, they resist will to the oxidation at high temperatures. They are generally of a low mechanical strength and relatively fragile. The addition of Mo to these steels improves the corrosion resistance. Its applications are numerous due to its resistance to corrosion and ease of processing (mechanical forming). They are much used in home appliances, exhaust, etc.

In general, the characteristics of hardening and elongation of ferritic stainless steels are comparable to those of carbon steel of high resistance.

The composition of the ferritic stainless steel P439A is equivalent to the UNS S43932 material of the ASTM A240 standard. The ferritic stainless steel ACE P439A is a material with superior corrosion resistance than the ferritic AISI 430. Like other stabilized ferritic stainless steels, it has excellent weldability and stamping, in addition to full immunity to corrosion under stress and absence of nickel, which makes the steel more competitive in the market. Because it is ferritic steel, the ACE P439A shows a very high elongation in tension test. It is a material with excellent performance in the operations of mechanical forming, bending and stamping.

3. WORK HARDENING

Many metals when plastically strained show work hardening, so their strength increases due to plastic strains. In the simulation of mechanical forming of metals, it is assumed that the yielding surface of the material evolves, expanding in an isotropic way (isotropic hardening) due to plastic work, overlapping to this effect, sometimes, a shift in the yielding surface area of tension (kinematic hardening) (Alves, 2003).

To describe the work hardening phenomenon, it is necessary to use hardening laws such as Hollomon, Swift, Voce, Ludwick, Misiolek and Ramberg-Osgood.

According to Gronostajski (2000), for numerical simulation of the forming process of metals at room temperature and with low strain rates, constitutive equations are employed in this case the effect of temperature caused by plastic work in the stress flow can be neglected. It is very common to use the Law of Hollomon, Eq (1) to describe the behavior of plastic materials, where K is the strength coefficient and *n* the strain hardening exponent. The values of the exponent *n*, range from 0.1 to 0.6 (Kocks, 1982).

$$\sigma = K\varepsilon^n \tag{1}$$

A modification of the law of Hollomon, Eq (1), was introduced by Ludwick, adding the yield stress σ_y, according to Eq (2):

$$\sigma = \sigma_y + K\varepsilon^n \tag{2}$$

Another modification of the Hollomon law was proposed by Swift (1952), adding a strain constant, as shown in Eq (3).

$$\sigma = K\left(\varepsilon + \varepsilon_0\right)^n \tag{3}$$

Where K is the strength coefficient; ε_0 is a strain constant, *n* is the strain hardening exponent, experimentally determined. For Gosh (1997) the strain hardening exponent, *n*, is the most important factor in the distribution of strain.

Another model with a saturation level was suggested by Voce. As shown in Eq. (4) the flow stress approaches an asymptotic value, σ_0, at high strains. This model is applied to a large number of aluminum alloys, (Hosford, 2005).

$$\sigma = \sigma_S - (\sigma_S - \sigma_0)exp(-n\varepsilon) \tag{4}$$

Where σ_0 is the yield stress, σ_S is the saturation stress, n is the strain hardening exponent.

Different work hardening laws have been proposed by several authors, according to Alves (2003), among the models described, one can also cite the Laws of Misiolek, Eq (5) and Ramberg-Osgood (1943), Eq (6):

$$\sigma = K\varepsilon^n \, exp(n_1\varepsilon) \tag{5}$$

$$\varepsilon = \frac{\sigma}{E} + K(\sigma/E)^n \tag{6}$$

Where n, n_1 = strain hardening exponent, E is the Young's modulus, K is the strength coefficient.

4. MATERIAL CHARACTERIZATION

To describe the plastic behavior of materials the work hardening laws are used, being necessary to determine the parameters that can be adjusted to the behavior of the material. The adjustment is performed using tensile tests. For description of the isotropic component of work hardening of materials used in printing the phenomenological laws proposed by, for example, Hollomon, Swift, Voce, Ludwick, Misiolek and Ramberg-Osgood were used.

4.1. Characterization of Mechanical Properties

For characterization of stainless steel 304 and 439A tensile tests were performed in different orientations on the rolling direction (0°, 45º and 90º), as Fig. 1. It should be emphasized that these tests were performed by ArcelorMittal Inox Inox Brasil (Timóteo, Minas Gerais, Brazil).

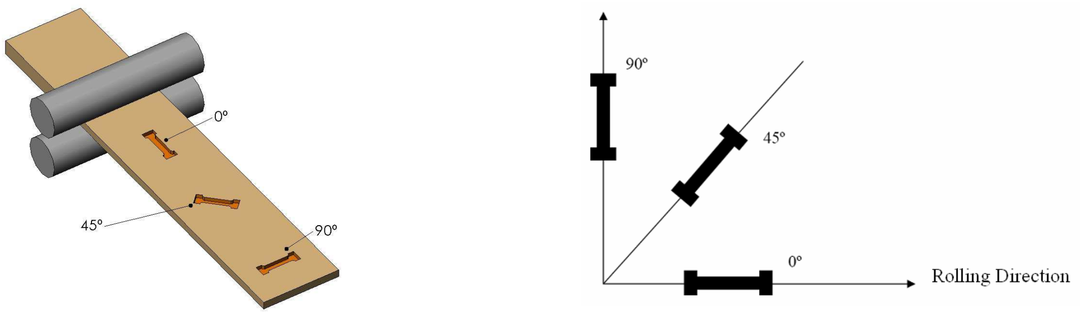

Figure 1: Removal of specimens from the plate, (Source: Magalhães, 2005).

By means of tensile tests, the stress curve versus the conventional strain is obtained, in different directions with reference to the rolling direction.

For the analysis of the material's mechanical properties a stress versus true strain curve was outlined, using the Matlab® program. The achievement of the Young's modulus was carried out in the linear elastic region of the stress versus strain diagram. The yield stress was obtained by means of an offset to 0.2% of strain, related to the straight of the elastic stage. The coefficients of anisotropy or Lankford coefficients (Stampack®, 2003) were obtained according to Hosford (2005), for a 15% strain. Tables 1 and 2 show the values found, and as it may be seen, the coefficients of variation of anisotropy in the plane of the plate.

4.2. Identification of Parameters of Work Hardening Laws

Through stress versus true strain curves, it was possible to perform the calibration of the laws of work hardening by minimizing the error function given by Eq (7), using the optimization module of the Matlab® program.

For the optimization of the experimental curves according to the work hardening laws, the elastic component of the real curve of tension versus strain was withdrawn, so the plastic region did not show flat portions or other more complex phenomena.

$$error = \sqrt{(\sigma_{analyt} - \sigma_{exp})^2} \tag{7}$$

Where σ_{exp} is the stress obtained by the tensile tests and σ_{analyt} is the stress analytically obtained by the work hardening laws.

Table 1: Characterization of the Austenitic Stainless Steel 304

Angle with the Rolling Direction [º]	Anisotropy Coefficient	Yield Stress [MPa]	Young's modulus [MPa]
0	1.22	282.40	181.03
45	0.82	259.01	217.30
90	1.45	265.73	252.32

Table 2: Characterization of the Ferritic Stainless Steel 439A

Angle with the Rolling Direction [º]	Anisotropy Coefficient	Yield Stress [MPa]	Young's modulus [MPa]
0	1.86	298.3732	215.53
45	1.69	301.9764	187.61
90	1.85	303.5942	256.37

In minimizing the error function in the optimization module of the Matlab® program, we have resorted to the Nelder-Mead simplex algorithm.

The Simplex method (Nelder-Mead) is a heuristic method of finding the minimum of any function of size N. The method searches for values in the parametric space that minimize the objective function. It is based on geometric grounds. From an initial guess, and a known the number of parameters to be optimized (N), the algorithm constructs a polyhedron in the parametric space, the polyhedron has N +1 vertices where the objective function is assessed and it is decided which new values of the parameters best fit the desired goal. For example, for adjustment of parameters, the algorithm constructs a triangle, in whose vertices the objective function is assessed, and according to their values the polyhedron moves and strains itself in search of the great.

The work hardening laws of used in the study were: Hollomon, Swift, Voce, Ludwick, Misiolek and Ramberg-Osgood. In Tables 3 and 4 values of the parameters found for each of the laws studied in the rolling direction may be found.

Table 3: Parameters obtained for the work hardening laws, in the characterization of the austenitic stainless steel, in the rolling direction.

Work hardening laws	K [MPa]	n	ε_0	σ_S [MPa]	n_1
Hollomon	1658.9	0.5	-	-	-
Swift	1684.3	0.5	0.0035	-	-
Voce	-	1.1	-	2668	-
Ludwick	1934.9	0.9	-	-	-
Misiolek	786.3	0.22	-	-	1.50
Ramberg-Osgood	7330	1.9	-	-	-

The predicted values of stresses for each equation were compared to the real values of stresses obtained from the experiments. The results are shown in Figures 2 and 3. To evaluate the adaptation of the work hardening laws to the experimental data, the concept of normalized mean square error was used, which, according to Bendat and Piersol (1986) is given by Eq (8).

$$\text{Normalized mean square error} = \frac{\sqrt{E[(\hat{\theta} - \theta)^2]}}{\theta} \tag{8}$$

Where $\hat{\theta}$ is an estimate of θ; θ is an arbitrary statistical parameter related to the stress and the strain $E[\ ]$ is the expected value of [ ].

Table 4: Parameters obtained for the work hardening laws, in the characterization of the ferritic stainless steel, in the rolling direction.

Work hardening laws	K [MPa]	n	ε_0	σ_S [MPa]	n_1
Hollomon	856.4387	0.2423	-	-	-
Swift	872.1767	0.2548	0.0034	-	-
Voce	-	7.4361	-	661.2792	-
Ludwick	859.3459	0.6710	-	-	-
Misiolek	840.7655	0.2372	-	-	0.0572
Ramberg-Osgood	1.35×10^{10}	4.2163	-	-	-

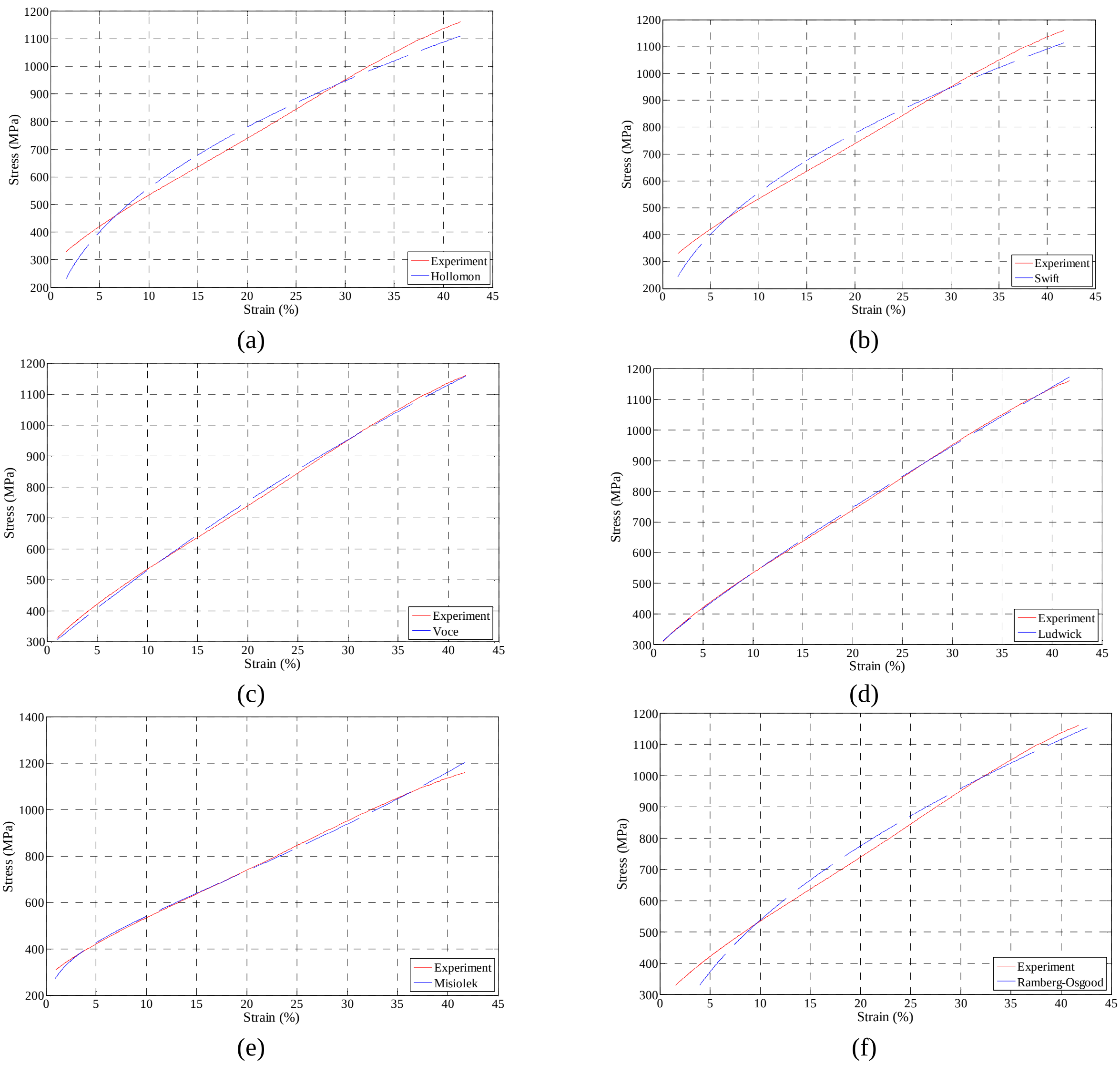

Figure 2: Optimization of different work hardening laws for the austenitic stainless steel 304, in the rolling direction. (a) Hollomon (b) Swift (c) Voce (d) Ludwick (e) Misiolek (f) Ramberg-Osgood.

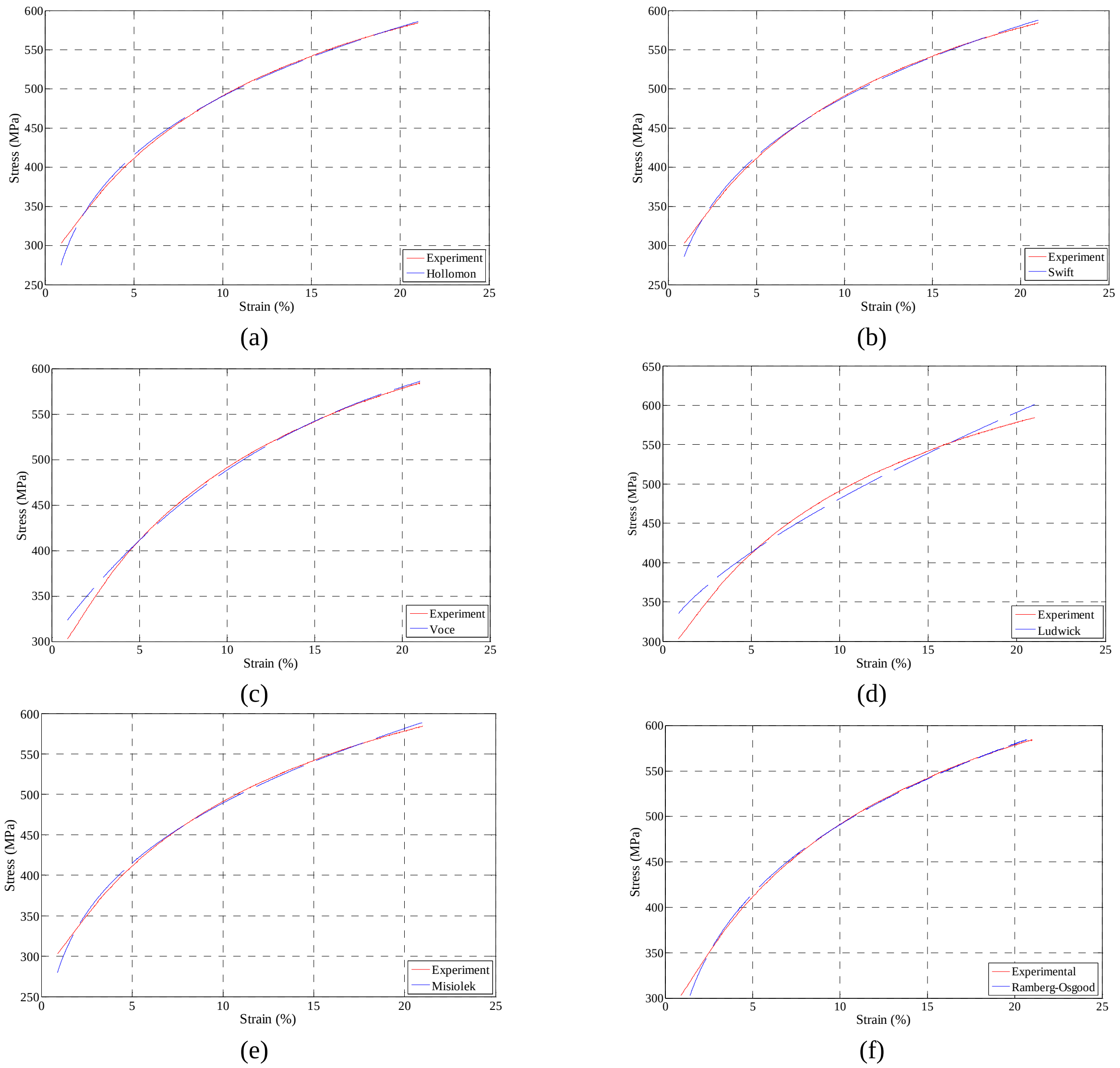

Figure 3: Optimization of different work hardening laws for the ACE P439A ferritic stainless steel in the rolling direction. (a) Hollomon (b) Swift (c) Voce (d) Ludwick (e) Misiolek (f) Ramberg-Osgood.

5. DISCUSSION

Through the study, using optimization procedures, it was possible to identify parameters of the work hardening laws and also evaluate what laws that better describe the plastic region of austenitic and ferritic stainless steels. To this end, we used the standardized mean square error. It can be observed by Figures 4 and 5 that the errors are considerably lower for all the work hardening laws analyzed.

Observing Tables 3 and 4, it is found that the strain hardening exponent, n, is high for austenitic stainless steels and for low ferritic ones. As n is related to the work hardening rate, high values of n imply low work hardening rate and therefore, large associated strains.

When analyzing the graphs stress versus strain of the materials studied, Figures 2 and 3, it appears that the ferritic stainless steel has maximum strains in the plastic region around 20% and the austenitic around 40%. Therefore, n is an important factor in the distribution of strain, that is, it has great influence on the ductility of the material; it should be noted that the strain hardening exponent n is an important factor for measuring formability determination.

Figures 2 and 4 indicate that the work hardening of the 304 austenitic stainless steel in the rolling direction is best described by the equation of Ludwick, Eq (2), as it has been suggested by some authors such as Singh (2004) and Antunes and Antunes (2007), presented a normalized mean square error equal to 0.0241%. For other diagonal and transversal directions related to rolling direction, the law that has been better adapted to the work hardening was the Misiolek's one, Eq (5), with normalized mean square error equal respectively to 0.0522% and 0.0555%.

In the study it was found that the stress versus strain curve of the 304 stainless steel has a linear character not obeying the work hardening laws as Hollomon, Swift, and Ramberg-Osgood in a large portion of the curve.

The ferritic stainless steel 439A had its work hardening well described by the Law of Swift in all evaluated directions, as Figure 3 and 5. Since the normalized mean square error was equal to 0.0090% for the rolling direction, 0.0118% for the diagonal and 0.0123% for the transversal.

It was found through Table 4 that the equations of Hollomon, Swift and Misiolek had approximate values of the parameters K and n, for the ferritic stainless steels. These equations have a common feature that is to present the strain hardening exponent and they do not need the yield stress and the Young´s modulus of the material to describe the plastic region.

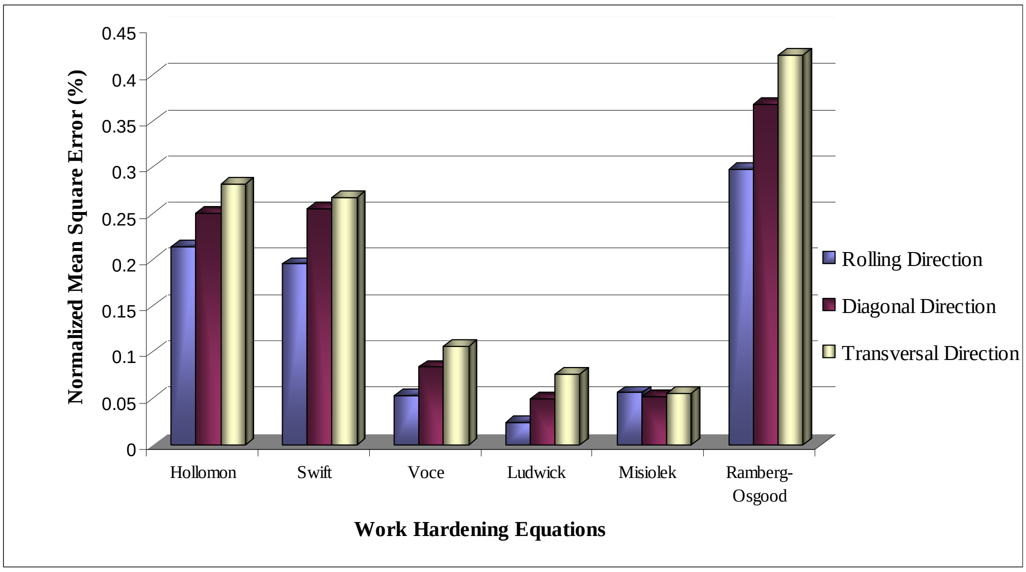

Figure 4: Mean Square Error showed by the different laws analyzed fot the austenitic stainless steel 304.

In Figure 3, it may be noticed that the initial and final regions of the curve of plasticity of ferritic stainless steel does not show a perfect fit, even for the equation of Swift, which showed better adaptation to the experimental curve. This is mainly due to the highly linear nature of the beginning of the curve of plasticity, being adjusted by power laws. For the austenitic stainless steel, the law with linear characteristic, equation of Ludwick had a better adaptation to the experimental data, because of the linear characteristic of the material's plastic region.

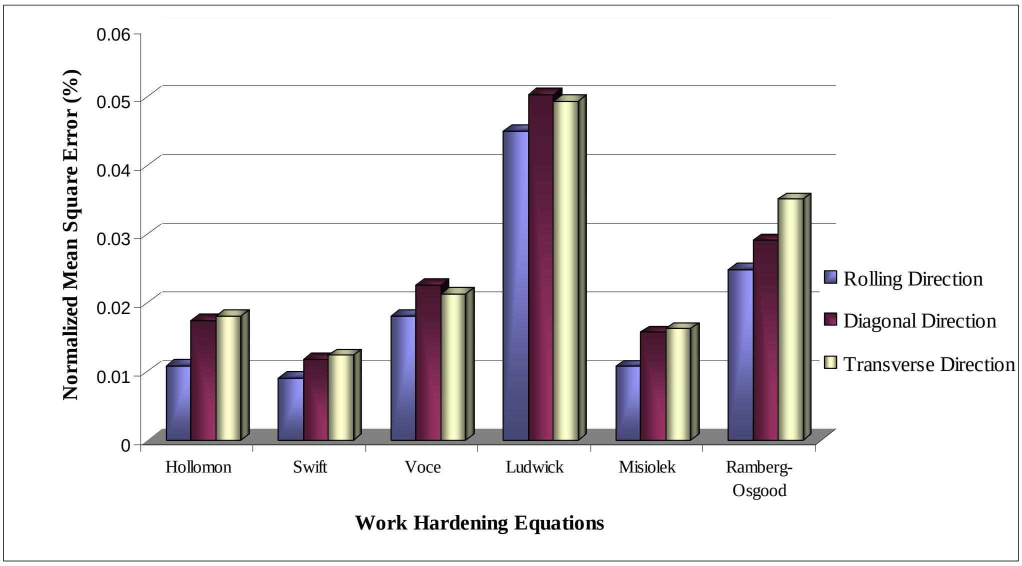

Figure 5: Mean square error presented by the different laws analyzed, for the ferritic stainless steel 439A.

The Hollomon's equation, Eq (1), commonly used to describe the work hardening of metal, represented in a reasonable manner the plastic behavior of ferritic stainless steels, as shown in Fig. 3-a. However, this equation does not show a good fit for austenitic stainless steels, Fig 2-a, due mainly to the instability of the phases present in the material, which alter the strain hardening exponent (n) with the strain.

It should be noted that K and n, although considered constants in the material, depend on its thermomechanical history, therefore, K and n, are highly dependent on the microstructure of the material analyzed.

The equations considered on the paper are adequate for a large family of metals and alloys, but not necessarily to model austenitic steels. The inelastic behavior of austenitic stainless steels is strain rate dependent, even at room temperature. The effect of strain rate is very important on micro-structural change and mechanical behavior. The equations considered in this paper do not account for the rate dependency.

6. CONCLUSIONS

The work shows a methodology to determine parameters and evaluate work hardening laws, commonly used in scientific studies involving plasticity.

The use of a methodology based on error minimization, reduces problems in the description of the plastic behavior of stainless steel, making it a robust and efficient tool to determine the parameters of work hardening laws.

For the evaluation of the different work hardening laws, it was observed that the use should be accompanied by a meticulous evaluative study, noting the ones that best fit to the experimental data, i.e., which adequately describe the plastic behavior of the analyzed material.

7. ACKNOWLEDGEMENTS

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