EFFECTS OF TIME DELAYS AND DATA INTERPOLATION METHODS ON LONGITUDINAL AERODYNAMIC DERIVATIVES ESTIMATES

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Abstract. *Aircraft parameter estimation from flight data has reached maturity as standard procedure for experimental modeling in the aircraft industries and related research institutes, as can be seen from the large number of recent books on the subject. One of the central issues here is the existence of time delays and different data sampling rates in the flight data measurements that can, if neglected, cause accuracy loss: the aerodynamic parameter estimates will change in order to explain the delays. This report is concerned with the effect of a correction algorithm used to eliminate the time delay inherent to the Pulse Code Modulation data acquisition system, and the performance of two data interpolation scheme used to tackle the differences in sampling rates between measured data. More precisely, first part of this report analyses the influence of delays introduced by the data acquisition system, and the second part consists of a comparison between two different interpolation methods: a linear interpolation and a spline interpolation, used to deal with sub-sampled flight data, using, in both cases, the standard deviation of the estimated parameters as a comparison standard. The analysis was carried out using parameter identification by output-error with a Levenberg-Marquardt method, with corrected standard deviation for the hypothesis of colored noise. The data used herein are from real flight of a regional type aircraft.*

Keywords*: Time delays, aircraft parameter estimation, flight data, experimental modeling.*

1. INTRODUCTION

Recent works have addressed the problem of aircraft identification from experimental data, see Mendonça et al. (2006-2007) and Hemerly et al. (2006-2007). The goal in these referenced works was to obtain accurate aerodynamic derivatives from flight data. More details about aircraft identification can be found in recent books by Jategaonkar (2006) or Klein and Morelli (2006).

The present report is concerned with how the accuracy of the identified model is affected by the quality of the data acquisition system, more specifically time delays and acquisition rate.

A particularly relevant approach in the aforementioned references is the output error method based on Levenberg-Marquardt technique; it employs dynamic equations used to model the data generating system, which are integrated during the estimation process.

The performance of the output error is typically addressed by appropriate input selection, such as in Brasil Neto et al. (2006), so as to provide adequate excitation. There are however, other facts impacting upon parameter estimation accuracy. For instance, the existence of time delays in the flight data measurements can, if neglected, cause accuracy loss: the aerodynamic parameter estimates will change in order to explain the delays.

In the first part of this work, the influence of real case delays introduced by the PCM (Pulse Code Modulation) architecture of data acquisition system is analyzed. In this architecture the measured data is collected in sequence instead of simultaneously, introducing a constant (usually known) time delay on each measured data but the first one.

In the second part of this work a comparative study is made between two different methods used to deal with sub-sampled flight test data – data with slower dynamic and greater sampling time. The most usual method is to repeat the last measured value while there is no new measurement; others methods use offline interpolation between measurements to resample the data. Two interpolation methods were subject of this study: a linear interpolation method and a spline interpolation method.

The output error parameter identification, based on the Levenberg-Marquardt optimization technique, is used in this analysis. The standard deviation of the identified parameters are corrected for the fact that the residuals are not white, but colored. The data came from real flight tests of a regional jet aircraft.

2. LONGITUDINAL RIGID-BODY EQUATION OF MOTION

Aircraft parameter identification is concerned with obtaining a mathematical description of the aerodynamic forces and moments acting on the aircraft, in terms of measured quantities such as control surfaces deflections, airspeed, aircraft orientation relative to wind and angular rates. A general formulation would include several degrees of freedom, such as elastic effects, time varying mass properties, moving components (rotors), etc. The motion of aircraft in free flight can be extremely complicated. Common simplifying assumptions mostly used in the study of aircraft motion are:

1) The aircraft is a rigid body with constant mass and time interval too small for any significant variation of mass in most cases (except load ejection).
2) The air is at rest relative to the Earth
3) Flight is close enough to the Earth's surface so that the surface can be considered as being flat.
4) Gravity is uniform and does not chance with aircraft position.

Given these assumptions, the motion of the aircraft can be described by the Newton's second law of motion in translational and rotational forms. In aircraft motion studies, one must always be sure that the assumptions made are appropriate for the problems at hand. In this study shall be restricted, without loss of generality, to the equations representing the longitudinal motion and an aircraft.

The longitudinal equations of motion are simple, three degree of freedom, ordinary differential equations with constant coefficients. The coefficients in the equations are made up of aerodynamic stability derivatives, mass and inertia characteristics of the airplane. These equations are represented bellow.

$$
\begin{aligned}
\dot{u} &= r \cdot v - q \cdot w + g \cdot \sin(\theta) + \frac{\bar{q} S_W}{m} \cdot CX + \frac{T_X}{m} \\
\dot{w} &= q \cdot u - p \cdot v + g \cdot \cos(\theta) \cdot \cos(\phi) + \frac{\bar{q} S_W}{m} \cdot CZ + \frac{T_Z}{m} \\
\dot{q} &= \frac{1}{I_{YY}} \left[(I_{ZZ} - I_{XX}) \cdot p \cdot r - I_{XZ} \cdot \left(p^2 - r^2\right) + \bar{q} S_W \cdot \bar{c} \cdot Cm_{CG} + M_T \right] \\
\dot{\theta} &= q \cdot \cos(\phi) + r \cdot \sin(\phi)
\end{aligned}
\tag{1}
$$

where $\bar{q}$ is the dynamic pressure, S_W is the wing reference area, m is the aircraft mass, T_X is the longitudinal axis thrust force, T_Z is the vertical axis thrust force, $\bar{c}$ is the mean aerodynamic cord and M_T is the pitching moment due to engine thrust, for more details see Klein and Morelli (2006).

The aerodynamic forces and moment acting on the aircraft are more conveniently expressed in the form of non-dimensional coefficients. This removes the known dependence on the airspeed and air density, and normalizes forces, moments and their respective derivatives. The aerodynamic model adopted in this investigation is given bellow; Jategaonkar (1990).

$$
\begin{aligned}
CX &= CL_{WB} \cdot \sin(\alpha) - CD \cdot \cos(\alpha) + \frac{S_{HT}}{S_W} \cdot CL_{HT} \cdot \sin(\alpha_{HT} - I_H) \\
CZ &= -CL_{WB} \cdot \cos(\alpha) - CD \cdot \sin(\alpha) - \frac{S_{HT}}{S_W} \cdot CL_{HT} \cdot \cos(\alpha_{HT} - I_H) \\
Cm_{CG} &= Cm_{REF} + CX \cdot (Z_{REF} - Z_{CG})/\bar{c} - CZ \cdot (X_{REF} - X_{CG})/\bar{c} \\
CL_{WB} &= CL_0 + CL_{\alpha_{WB}} \cdot \alpha \\
CL_{HT} &= CL_{\alpha_{HT}} \cdot \alpha_{HT} + CL_{\delta_E} \cdot \delta_E + CL_{CT} \cdot T/T_{REF} \\
Cm_{REF} &= Cm_0 - \frac{S_{HT} \cdot CL_{HT}}{S_W \cdot \bar{c}} \left[X_{HT} \cdot \cos(\alpha_{HT} - I_H) + Z_{HT} \cdot \sin(\alpha_{HT} - I_H) \right] + Cm_q \cdot \frac{q\bar{c}}{2V} \\
CD &= CD_0 + \frac{CL^2}{\pi \cdot \left(b^2/S_W\right) \cdot e_f} \\
CL &= CL_{WB} + \frac{S_{HT}}{S_W} \cdot CL_{HT} \cdot \cos(\alpha_D - \varepsilon)
\end{aligned}
\tag{2}
$$

where α is the angle of attack, S_{HT} is the horizontal tail reference area, I_H is the horizontal stabilizer deflection, X_{REF} and Z_{REF} are the positions, in the X and Z body axis, of the reference point, X_{CG} and X_{CG} the position in the same axis of the center of gravity, T is the thrust, T_{REF} is any reference thrust, V is the true airspeed, b is the wing span, e is the Oswald Efficiency Factor, e_f is the increment in horizontal tail angle of attack due to pitch rate and ε is the downwash.

3. OUTPUT ERROR METHOD BASED ON LEVENBERG-MARQUARDT

Here, the time delay estimation is performed through the Levenberg-Marquardt method. Basically, the functional to be minimized considers the prediction error

$$e(k) = z(k) - \hat{y}(k) \tag{3}$$

where $\hat{y}(k)$ is the predicted output by using the present parameter estimate vector $\hat{\theta}$ of the unknown parameter vector θ, $z(k)$ is the measurement and k is the discrete time index. More precisely, the cost to be minimized is

$$J_{LM}(\theta,R) = \frac{1}{2}\sum_{k=0}^{N-1} e^T(k+1)\cdot R^{-1}\cdot e(k+1) + \frac{N}{2}\cdot \ln\big(\det(R)\big) \tag{4}$$

where N is the available number of data points. The minimization is a two steps procedure, where:

1) for the available $\hat{\theta}$, an estimate of the prediction error covariance matrix is obtained from

$$R = \frac{1}{N}\cdot\sum_{k=0}^{N-1} e^T(k+1)\cdot e(k+1) \tag{5}$$

2) then Levenberg-Marquardt method is employed to solve the minimization problem and then update the estimate, by calculating $\Delta\hat{\theta}$ according to

$$\left[\sum_{k=0}^{N-1} S^T(k)\cdot R^{-1}\cdot S(k) + \lambda I\right]\cdot\Delta\hat{\theta} = -\sum_{k=0}^{N-1} S^T(k)\cdot R^{-1}\cdot e(k) \tag{6}$$

where S is the sensitivity matrix, Klein and Morelli (2006).

These steps are repeated till convergence is attained. It is worth to mention that the Levenberg-Marquardt method makes a tradeoff between minimization and size of the parameter update step. This procedure makes it less susceptible to numerical errors than the Gauss-Newton method. For further details, see Press et al. (1990).

4. COLORED RESIDUAL CORRECTION

A technique for the determination of the confidence intervals based on higher-order sensitivity analysis of an estimator for a general dynamic system is used herein. Applied to a maximum likelihood estimator this technique leads to the following form for the covariance matrix,

$$Cov(\hat{\theta}) = P = \left[\sum_{i=1}^{N} S^T(i)\cdot R^{-1}\cdot S(i)\right]^{-1} \tag{7}$$

where $S(i)$ is the sensitivity matrix of the outputs with respect to the parameters, and R is the noise covariance matrix. The standard deviation is calculated as the square root of each diagonal element of the P matrix.

The covariance matrix expressed by Eq. (7) assumes that the residuals are white, which is not always the case, noise is generally colored. A correction is necessary in order to take into account the spectral distribution of the residuals in different frequencies, otherwise the estimated parameters will display unrealistic confidence intervals. Further improvement can be achieved by replacing the white noise assumption by assuming colored residual Eq. (7). The new estimate of the covariance matrix is then given by:

$$Cov(\hat{\theta}) = P \cdot \left[\sum_{i=1}^{N} S^T(i) \cdot R^{-1} \cdot \sum_{j=1}^{N} R_C(i-j) \cdot R^{-1} \cdot S(j) \right]^{-1} \cdot P$$
$$R_C(k) = R_C(-k) = \frac{1}{N} \sum_{i=1}^{N-k} \upsilon(i) \upsilon^T(i+k) \tag{8}$$
$$\upsilon(i) = z(i) - \hat{y}(i)$$

where $z(i)$ is the measurement vector, and $\hat{y}(i)$ is the simulation result with the estimated parameter vector; both at instant $t(i)$. The standard deviations are the square root of the diagonal terms of the covariance matrix given by Eq. (8) and for 95% of confidence the uncertainty is the double of the standard deviation. For more details see Klein and Morelli (2006).

5. TIME DELAY CORRECTION

The PCM data acquisition architecture introduces time delays on the measurements. It uses a single AD converter and a multiplexer to connect the converter to the numerous measuring channels, each one connected to a sensor or an analog measurement system. As one channel is read at time the measurements are not simultaneous but the delays are known due to its constant switching frequency and channels mapping matrix.

For time delay correction, the measurements have a new individual time stamp that takes account the initial time stamp of the cycle, its position in the reading map and the switching frequency. As there are new time stamps the measurements are merged according to their own time stamps on a new database.

All sampling times are integer multiples of the fundamental switching time. A new database is created with the highest sampling rate of all the measurements and the measurement value on a certain time is considered to be the nearest value.

6. DATA INTERPOLATION METHODS

The interpolation methods used on this work are the linear and spline interpolation methods. The results given by each method is used in the comparative study of this report. The goal is to access the benefits, if any, of using an elaborated method like the spline interpolation in place of a more simple method like a linear interpolation.

The linear method consists of just finding a bias and a gain in the interval defined by the time interval t_j and t_{j+1}. For any given time interval the values of the interpolated data are given by

$$x = \frac{x_{j+1} - x_j}{t_{j+1} - t_j} \cdot (t - t_j) + x_j \tag{9}$$

The spline interpolation method is based on a 3rd order polynomial, making use of the interval's two previous points and one after. Using five consecutives samples (t_j, t_{j+1}, t_{j+2}, t_{j+3}, t_{j+4}) the coefficients of the 3rd order interpolation function are determined for the time interval [t_{j+2}, t_{j+3}] and the spline interpolation function is called $S_3(t)$ and is defined by

$$S_3(t) = \frac{(t_3 - t)^3 \cdot \varphi_3 + (t - t_3)^3 \cdot \varphi_4}{6 \cdot (t_4 - t_3)} + \frac{(t_4 - t) \cdot x_3 + (t - t_3) \cdot x_4}{(t_4 - t_3)} - \frac{(t_4 - t_3)}{6} \left[(t_4 - t) \cdot \varphi_3 + (t - t_3) \cdot \varphi_4 \right] \tag{10}$$

where t is the time in which the interpolated value is computed, and φ_3 and φ_4 are the interpolation function coefficients. These coefficients are calculated by solving the following linear system given by the general equation

$$\frac{(t_i - t_{i-1})}{6} \varphi_{i-1} + 2 \cdot (t_{i+1} - t_{i-1}) \cdot \varphi_i + (t_{i+1} - t_i) \cdot \varphi_{i+1} = 6 \cdot \left(\frac{x_{i+1} - x_i}{t_{i+1} - t_i} - \frac{x_i - x_{i-1}}{t_i - t_{i-1}} \right) \tag{11}$$

for i = 2, 3, 4. The polynomial $S_3(t)$ at each interval is used to calculated the interpolated values that belong to these time intervals.

7. EXPERIMENTAL RESULTS

Two influences were analyzed in this report: one due to time delay correction, introduced by the PCM architecture, on the measurements and the one due to the use of two different interpolation methods, used to minimize the effect of parameters with low sampling rate.

Before identifications the data has been checked for consistency by use of a Flight Path Reconstruction (FPR) technique, more details can be found on Jategaonkar (2006). All uncertainties are for 95% confidence interval and have been corrected for colored residuals.

Flight data were first corrected for the lag effect on each individual measurement, then the identifications were made using the original data (no time correction) and time corrected data, both with repeated values of low sample rated parameters. The data used for those analyses were sampled at 20 Hz. The confidence interval for each identified parameter is presented on Tab. 1. The last column represents the percent reduction on the confidence interval due to the time correction.

Table 1. Results for time correction

Parameter		Confidence Interval		Reduction
Name	Dimension	Original Data	Corrected Data	
CLAWB	rad^{-1}	0.114	0.122	-6.81%
CLAHT	rad^{-1}	0.0731	0.0707	3.21%
CLDEHT	rad^{-1}	0.0262	0.0259	1.01%
CLCTHT	[]	0.000600	0.000566	5.69%
DEDA	[]	0.0170	0.0161	5.27%
CMQWB	rad^{-1}	0.00292	0.00301	-3.29%

The parameter CLAWB represents the variation of wing-body lift due to variation on the wing-body angle of attack, CLAHT the variation of horizontal tail lift due to variation on local angle of attack, CLDEHT the variation of horizontal tail lift due to elevator deflection, CLCTHT the variation of horizontal tail lift due to thrust effects, DEDA the variation of downwash on the horizontal tail caused by the variation of wing-body angle of attack and CMQWB the wing-body pitch damping derivative due to pitch rate.

The confidence interval of most parameters is smaller with time lag correction on the measured data. This is expected because the identification process assumes that all data in a time instant are taken simultaneously while in the PCM acquisition system the measurements are taken one at a time. However, some parameters have smaller confidence intervals on original data because they, in a certain way, explain the error introduced by the acquiring delay.

The second analysis deals with two different interpolation methods: linear interpolation and spline interpolation. The time corrected data, from the previous analysis, had the repeated parameters interpolated by either one of the two interpolation method. Then the parameter identification was executed in the same way as before. The comparative results are presented in Tab. 2. As in the first analysis the last columns represent the percent reduction of the confidence interval compared due to data interpolation.

Table 2. Results for Interpolation Methods

Parameter		Confidence Interval			Linear Reduction	Spline Reduction
Name	Dimension	Repeated	Linear	Spline		
CLAWB	rad^{-1}	0.1218	0.0961	0.0973	21.08%	20.13%
CLAHT	rad^{-1}	0.0707	0.0699	0.0702	1.13%	0.77%
CLDEHT	rad^{-1}	0.0259	0.0260	0.0259	-0.47%	0.13%
CLCTHT	[]	0.000566	0.000568	0.000578	-0.30%	-2.04%
DEDA	[]	0.0161	0.0156	0.0158	2.92%	1.95%
CMQWB	rad^{-1}	0.00301	0.00255	0.00262	15.48%	12.92%

Taking the confidence interval as a measurement of good identification results it can be seen that all the interpolation methods give us overall better results. Some parameters are not affected too much by interpolation because they have a high sampling rate like, for example, the elevator position. This is equivalent to say that the elevator influence on the simulation characteristics is well determined even without any interpolation.

It was expected that the spline interpolation, a more sophisticated method that uses more information to interpolate the data, would yield narrower confidence intervals than the linear interpolation. However, the uncertainties of the spline interpolation are bigger because it filters some high frequency components that should be important to some parameter estimation.

Another relevant aspect is the Mean Square Error (MSE) between the flight data and simulation results with the parameters identified by the estimation method. It can be seen a significant reduction in the MSE with the use of any interpolation method. The results are presented in Tab. 3 and an example of visual comparison of the pitch angle on a maneuver with generic elevator input is presented on Fig. 1.

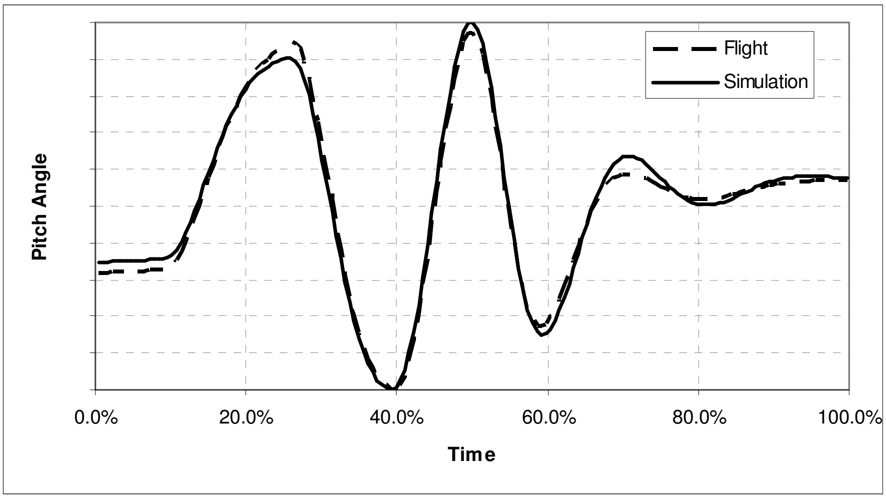

Figure 1. Time plot of pitch angle of short period generic excitation of flight (———) and simulated (- - - - - -)

Table 3. MSE for Different Interpolation Methods

Interpolation	MSE
Without	554.17
Linear	457.96
Spline	443.88

It is expected that any interpolation method of mainly the output variables give a smaller MSE because the sample rate of the acquired data (20 Hz in this case) is much higher than the highest aircraft mode frequency (something about 1 Hz) and the low sample rated measurements are not constant during the measurement intervals.. So any attempt to vary them on these time intervals is benefic. As the linear interpolation gives a discontinuity of the first derivative of the signal, the MSE for the spline interpolation is expected to be smaller than the MSE for the linear interpolation, which is confirmed by the results.

8. CONCLUSIONS

Two analyses for real time data correction for aircraft identification were carried out in this work with real flight test data of a regional jet. The identification used the output error method with Levenberg-Marquardt optimization and the uncertainties were corrected for colored noise.

The first analysis was the impact on the uncertainties of time lag correction introduced by the PCM data acquiring system architecture. The second one was the comparison of the effect of two different interpolation methods to adjust the measurements with low sample rate: linear and spline interpolation.

The identifications for these conditions showed us that the time lag correction decreases the uncertainties of most parameters hence it is an advisable first step. With this correction the spline interpolation give us smaller Mean Square Errors (MSE), but the linear interpolation reduced all parameters uncertainties except for the control derivatives' one. As the differences of uncertainties are proportionally greater than MSE reduction and the uncertainties are better indicators of good estimation according to Jategaonkar (2006) and Klein and Morelli (2006) the linear interpolation revealed to be the best choice.

A more detailed statistic analysis should be performed as continuation, by using, for instance, others models, maneuvers and estimation methods, like equation error methods and frequency domain methods.

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