DIRECT SYNTHESIS FROM FREQUENCY RESPONSE MEASUREMENTS APPLIED TO THE CONTROLLER DESIGN OF A SERVO POSITIONING SYSTEM

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Abstract. *In classical control theory, one must be able to predict the system characteristics in order to find the proper gains of the controller. This prediction of system behavior requires reliable and precise models for the plant, actuators, and sensors, which may not always be available in some practical situations. Simplifying assumptions and unknown or difficult to measure parameter values are a few causes of non ideal behavior of closed loop systems when the believed "optimum" feedback gains are used, calculated from imprecise models. In order to overcome modeling difficulties, one can rely on frequency response measurements of the system to design the appropriated controller. In this work, the direct synthesis of first order controllers, based on system frequency response information (input/output information), is used and experimentally applied to a servo positioning system. The stabilizing gains G_1, G_2 and G_3 of the controller are obtained by the Keel-Bhattacharyya algorithm, and used to experimentally control the angular position of an electric motor subjected to a desired trajectory. The advantage of this approach lies on the fact that only input and output information is necessary (frequency response), and detailed information of plant, actuator or sensors, necessary to build a mathematical model, is not required to find the stabilizing gains of the controller. In fact, plant, sensors, and actuators can be considered as a whole integrated system (black box), whose input is the signal to the actuator and output is the signal from the sensors.*

Keywords: *first order controller, direct synthesis, frequency response, servo positioning*

1. INTRODUCTION

Control systems are widely employed in most areas of Engineering. In industry, control systems can be found in the whole production line: supply chain, means of production (machine tools), production process (industrial automation), quality control, and finally reaching stocking and distribution systems. In all these cases, one can have open loop control systems, where decision is usually taken by humans, or closed loop control systems, where decision is automatically taken based on a control law (Ogata, 2001; Dorf and Bishop, 2004)

In the case of closed loop control systems, where the decision on the moment of actuation depends on a control law, one has to determine appropriated feedback gains to be inserted in the control law. For instance, in the design of PID controllers (the controller most commonly adopted in industrial applications), the proportional, derivative, and integral gains must be known for a proper work of the control system. The problem is how to determine the gains in order to achieve the expected performance of the control system.

The most common ways of finding appropriated feedback gains of linear controllers, like the PID controller, are based on mathematical modeling of the system (root locus, state space analysis), empirical testing (Ziegler-Nichols), or trial-and-error. Many times, different techniques are used together, where the gains can be initially estimated empirically or by mathematical modeling, and lately adjusted by trial-and-error during the physical implementation of the controller. If the system to be controlled (plant) must be modeled, it must be considered as a linear system, where equations of motion are linear and it is possible to write its transfer function. Such simplifying assumption may result in feedback gains whose controller performance is only guaranteed in a narrow operating range of the system. In this case, the controller works well around the linearization point, but it is not robust enough to variations in system parameters (Marczyk, 2003).

In this context, arises the idea of finding the feedback gains of the controller through information gathered from experimentation. In this case, there is no need to linearize the system because the system is not modeled mathematically. The system is considered as a whole, like a "black box", where the input and output relationships are experimentally obtained *in loco*. Examples of this kind of approach can be found in literature, like the works on robust control (Zhou, 1998; Tantaris et al., 2003) and artificial neural network (Tang, 1996).

In this work, one presents the experimental implementation of a technique for calculating the feedback gains based on the analysis of frequency response functions (FRFs) of the plant. In this case, the FRFs are obtained experimentally, thus avoiding the adoption of simplifying assumptions and considering the system as it is. In this approach, the controller must have the following form:

$$C(s) = \frac{G_1\, s + G_2}{s + G_3} \tag{1}$$

and throught the theorems proposed by Keel and Bhattacharyya (2005), it is possible to find a volume in the space of gains G_1, G_2, G_3 inside which any set of gains stabilizes the closed loop system.

This technique is an alternative to other gain calculation techniques because the system can be considered as a whole, including sensors and actuators which, on their sides, have own dynamic characteristics (own FRFs). Hence, the experimental FRF used in the calculation of the gains may include dynamic effects of the sensors and actuators which will enhance the quality of the calculated gains (Fig. 1).

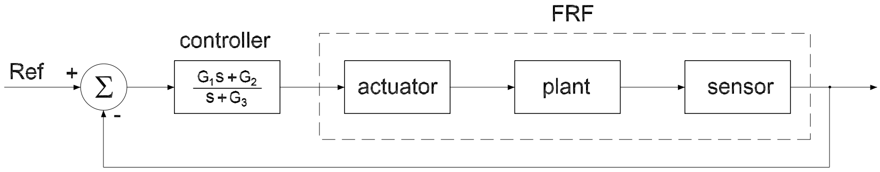


Figure 1. Block diagram of the closed loop system.

2. DIRECT SYNTHESIS FROM FREQUENCY RESPONSE FUNCTIONS

According to Theorem 1 of Keel and Bhattacharyya (2005), any set of gains G_1, G_2, G_3 bounded by the curves defined by Eqs. (2) to (4) will result in a closed loop system with an invariant number of open loop poles in the left hemi-plane.

$$G_3 + G_2 H(0) = 0 \tag{2}$$

$$\begin{cases} G_1(\omega) = \dfrac{1}{|H(\omega)|^2}\left[\dfrac{H_I(\omega)}{\omega}\,G_3 - H_R(\omega)\right] \\ \\ G_2(\omega) = \dfrac{-1}{|H(\omega)|^2}\left[H_R(\omega)G_3 + \omega H_I(\omega)\right] \end{cases} \tag{3}$$

$$1 + H(\infty)G_2 = 0 \tag{4}$$

where $H(\omega)$ is the frequency response function of the system (actuator + plant + sensor), composed of real ($H_R(\omega)$) and imaginary ($H_I(\omega)$) parts.

Equation (2) results in a straight line in G_1–G_2 plane, where G_2 assumes a constant value depending on the given value of G_3 and on the value of the frequency response function for zero frequency ($H(0)$). Equation (3) gives pairs of gains G_1 and G_2 as a function of frequency ω and of the given value of G_3, resulting in a curve in the G_1–G_2 plane. Equation (4) is usually not applied because in many dynamic systems (mechanical systems) the value of the frequency response function for infinity frequency tends to null ($H(\infty) \to 0$).

Hence, by adopting a value for G_3, one can find bounded regions in G_1–G_2 plane defined by curves whose equations are given above. By varying the values of G_3, these regions form volumes in the space of gains. According to Keel and Bhattacharyya (2005), if a set of gains, belonging to one of these volumes, stabilizes the closed loop system, then any set of gains within this volume will stabilize the system. Likewise, if a set of gains, belonging to one of the volumes, does not stabilize the closed loop system, then any set of gains within this volume will not stabilize the system. System stability can be easily verified by looking for right hemi-plane poles in the Nyquist plot of the open loop system.

The procedure for finding the gains of the controller based on the frequency response functions of the actuator + plant + sensor can be summarized as follows:

i) measure the system response in frequency domain (FRF);

ii) calculate the curves (G_1 and G_2 pairs) for given values of G_3 and frequency ω;

iii) plot the curves in the space of gains (G_1, G_2, G_3) and obtain the gain volumes;

iv) pick sets of gains in each volume and test for stability.

After that, one will find groups of gains G_1, G_2, G_3 that make the closed loop system stable. The appropriated set of gains to be used, chosen from the group of stabilizing gains, will depend on the performance specifications of the system.

3. EXPERIMENTAL SET-UP

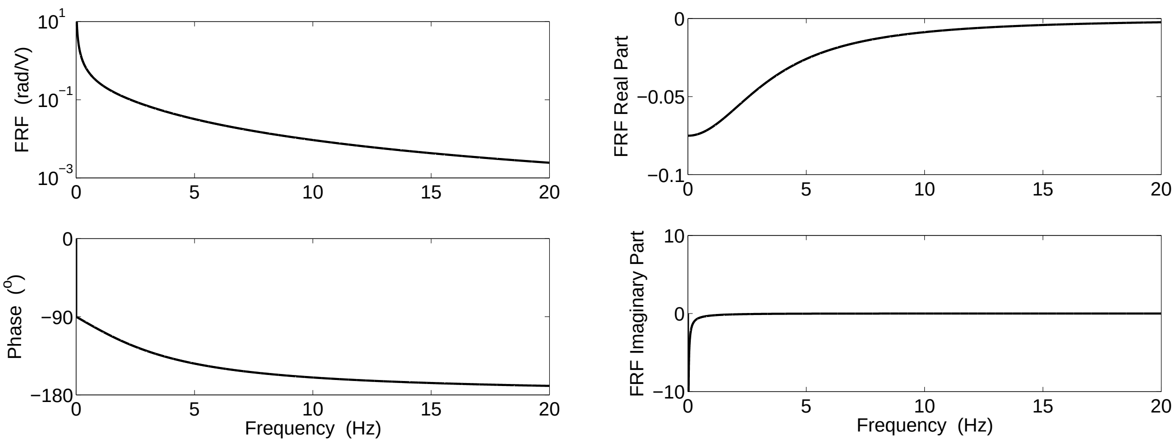
The procedure was tested in the Quanser™ servo positioning system shown in Fig. 2. The arm (1) is fixed on the main shaft (3), which is impelled by the electric motor through an encapsulated gear box and gear plates (4). An encoder is mounted in the main shaft, whereas a potentiometer is mounted in a second gear plate (2). System response is measured by the encoder or by the potentiometer and respective signals are sent to the acquisition board (6), which is connected to a LabView acquisition board in a PC. The control signal is sent through the acquisition board (6) to the power module (5), which sends the power signal to the electric motor (4) in the platform, working as the actuator.

Figure 2. Quanser servo positioning arm platform: 1) arm; 2) potentiometer; 3) main shaft + encoder; 4) electric motor + gear box; 5) power module; 6) acquisition board.

As one can see in Fig. 2, the sensor system (encoder) and the actuator system (power module + electric motor + gears) have inherent dynamic characteristics that affect the performance of the closed loop system. For this reason, the system will be considered as a whole (actuator + plant + sensor) in the FRF identification.

3.1 Identification of Experimental FRF of the Servo Positioning System

In order to obtain the system FRF, the system must be excited with a broad frequency band signal. In this work, a chirp signal is used, ranging from 0 to 20 Hz, period of 30 s and amplitude of 3 V. The chirp signal is repeated 10 times, totaling a sampling period of 300 s, with sampling frequency of 40 Hz. The chirp signal is sent to the actuation system (power module + electric motor) and the angular displacement of the arm (output response of interest) is measured through the encoder. With input and output signals in hand, one can calculate the frequency response function estimators H_1 and H_2, and associated coherence (Maia and Silva, 1997). The obtained results for the experimental set-up are shown in Fig. 3. As one can see in Fig. 3, the servo positioning system presents a strong decay in angular displacement as input signal frequency increases, which is typical for electric motors.



(a) amplitude and phase

(b) real and imaginary parts

Figure 3. FRF of the servo positioning system (experimental results).

4. IMPLEMENTATION OF THE PROCEDURE

Applying Eqs. (2) and (3) of the procedure, using the measured FRF of the servo positioning system (Fig. 3), one obtains the results shown in Fig. 4 for given values of G_3 between -42.8 and -2.8. As one can see in Fig. 4, Eqs. (2) and (3) form two surfaces in the space of gains: the darker surface, which comes from Eq. (2) and is a constant level plane at $G_2 = 0$; and the lighter surface, which comes from Eq. (3). When the frequency tends to zero ($\omega \to 0$), the lighter surface converges to the darker surface at $G_2 = 0$.

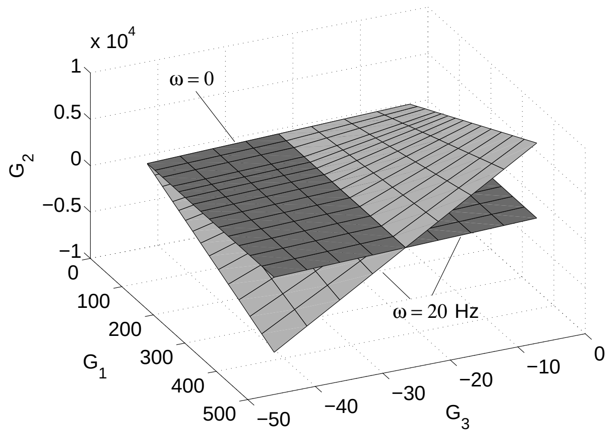
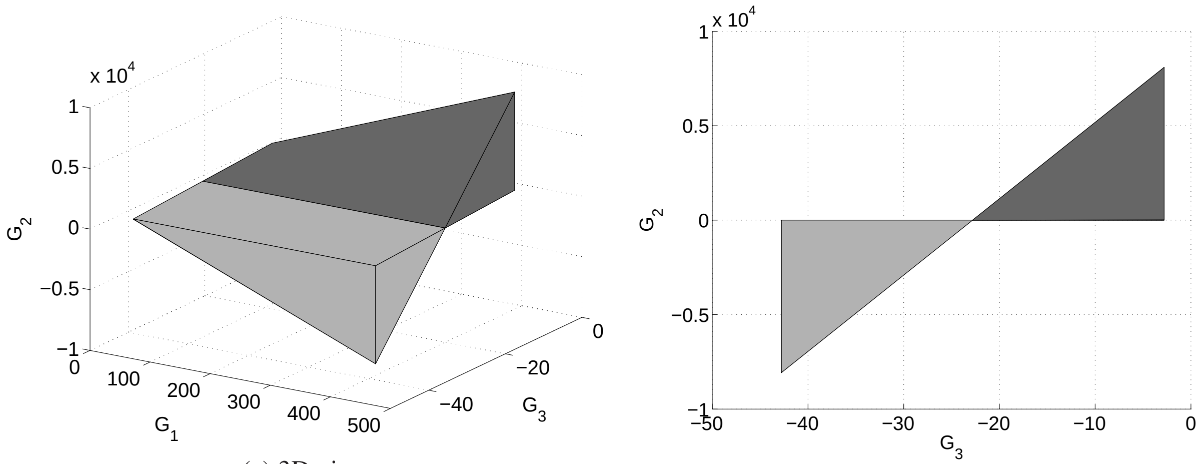


Figure 4. Surfaces in the space of gains for the servo positioning system ($0 < \omega < 20$ Hz, $-42.8 < G_3 < -2.8$).

In this case, the surfaces bound two prismatic volumes in the space of gains, with triangular shapes (Fig. 5). The darker volume in Fig. 5(a) is composed of gains G_1, G_2, G_3 with positive G_2, whereas the lighter volume is composed of gains G_1, G_2, G_3 with negative G_2. A turn over point is observed for $G_3 = -22.8$ (Fig. 5(b)), where G_2 is zero for any G_1. The turn over point can be mathematically obtained from Eq. (3), setting $G_2(\omega) = 0$:

$$G_3^* = -\omega \frac{H_I(\omega)}{H_R(\omega)} \tag{5}$$

where G_3^* is the value of G_3 at the turn over point. It is interesting to note that this value is constant whatever frequency is considered.

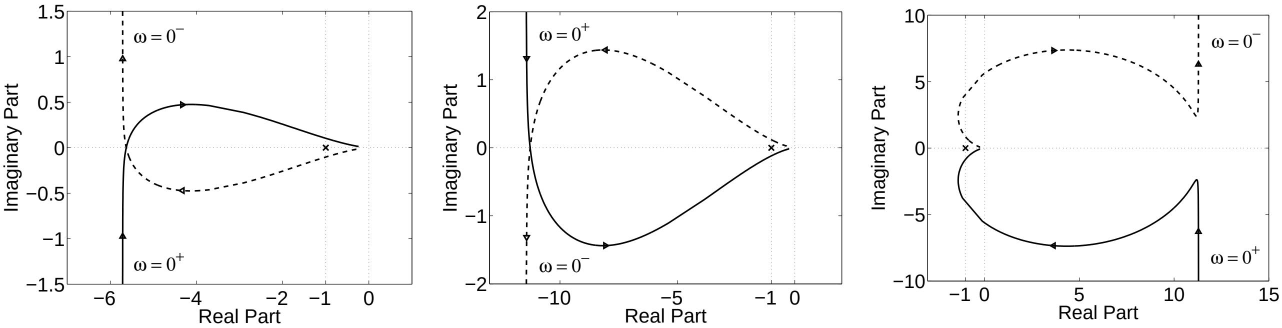


(a) 3D view

(b) G_2–G_3 plane

Figure 5. Volume of gains bounded by the surfaces ($0 < \omega < 20$ Hz, $-42.8 < G_3 < -2.8$).

The Nyquist stability criterion is used to verify the stability of the system when gains chosen from the volumes are adopted. The plant (servo-positioning system) has no open loop poles in the right hemi-plane. The controller (Eq. (1)) has one open loop pole in the right hemi-plane when $G_3 < 0$. Hence, one has three different conditions to be analyzed: gains in the darker volume of Fig. 5 with $G_3 > 0$ (no poles in the right hemi-plane); gains in the darker volume of Fig. 5 with $-22.8 < G_3 < 0$ (one pole in the right hemi-plane); and gains in the lighter volume of Fig. 5 with $G_3 < -22.8$ (one pole in the right hemi-plane). The Nyquist plots of these three conditions are shown in Fig. 6, adopting $G_1 = 100$ and $G_2 =$ +10 or -10 for the lighter or the darker volumes, respectively.



(a) $G_2 = -10$, $G_3 = -30$ ($G_3 < -22.8$, one pole in the RHP)

(b) $G_2 = -10$, $G_3 = -15$ ($-22.8 < G_3 < 0$, one pole in the RHP)

(c) $G_2 = 10$, $G_3 = 15$ ($G_3 > 0$, no pole in the RHP)

Figure 6. Nyquist plots for gains pertaining to the volumes in the gain space ($G_1 = 100$, $-20 < \omega < 20$ Hz).

In the case of $G_3 < -22.8$ (Fig. 6(a)), the Nyquist plot circumscribes (-1,0) in the clockwise direction, thus leading to an unstable system according to the Nyquist stability criterion. In the case of $-22.8 < G_3 < 0$ (Fig. 6(b)), the Nyquist plot circumscribes (-1,0) in the counter-clockwise direction. According to the Nyquist stability criterion, this is the condition for stability of systems with open loop poles in the right hemi-plane (one counter-clockwise loop for one pole). Therefore, this condition leads to a stable system. In the case of $G_3 > 0$ (Fig. 6(c)), the Nyquist plot does not circumscribe (-1,0), thus leading to a stable system according to the Nyquist criterion.

Hence, the system is stable for $G_3 > -22.8$, which represents the darker volume in the space of gains, and unstable for $G_3 < -22.8$, which represents the darker volume in the space of gains (Fig. 5(b)). According to Theorem 1 of Keel and Bhattacharyya (2005), any set of gains belonging to the darker volume will lead the system to stability, whereas any set of gains belonging to the lighter volume will lead the system to instability. Therefore, the stabilizing gains must be chosen from the darker volume.

4.1 Relationship to Lead/Lag Compensation

Linear compensators have transfer functions of the following form:

$$G_c(s) = \frac{K(s+z)}{s+p} \tag{6}$$

where if $|z| < |p|$ one has a lead compensator, whereas if $|z| > |p|$ one has a lag compensator (Dorf and Bishop, 2004). The lead compensator provides phase lead at high frequencies, thus increasing phase margin and system response which enhances responsiveness and stability of the system. The lag compensator provides phase lag at low frequencies, which reduces the steady state error but also decreases system responsiveness.

Comparing the controller in study (Eq. (1)) to the lead/lag compensator (Eq. (6)), one can see the similarity between them, where $K = G_1$, $z = G_2/G_1$ and $p = G_3$. Hence, one can estate that:

$$\text{if } \left|\frac{G_2}{G_1}\right| < |G_3| \Rightarrow \text{lead compensation} \tag{7}$$

$$\text{if } \left|\frac{G_2}{G_1}\right| > |G_3| \Rightarrow \text{lag compensation} \tag{8}$$

The relationships (7) and (8) give an idea of what to expect from a chosen set of gains G_1,G_2,G_3 of the volume in the space of gains. Actually, the volume in the space of gains can be divided into regions where the controller will behave as a lead or a lag compensator, as shown in Fig. 7. With this information, one can look for sets of gains in the volume that achieve desired performance criteria.

In order to test the controller applied to the servo positioning system, sets of gains were chosen in the volume for a given value of G_3 (Fig. 8). The parameter to be controlled is the angular displacement of the servo positioning system. System response is observed for unitary step disturbances (reference value). The experimental results are shown in Fig. 9.

By varying the value of gain G_1, keeping $G_2 = 40$ and $G_3 = 4$, one can clearly see the lead/lag effect on system response (Fig. 9(a)). At gain set #1, the controller is a lag compensator, and system response presents small steady state error with higher rising time and overshoot. As the controller changes to a lead compensator (gain set #3), the system presents higher responsiveness (smaller rising time), but settling time increases considerably.

Figure 9(b) presents the effect of varying gain G_2, keeping $G_1 = 10$ and $G_3 = 4$ at constant values. In this case, rising time is almost the same for all tested conditions, however overshoot and settling time differ. At gain set #5, the controller

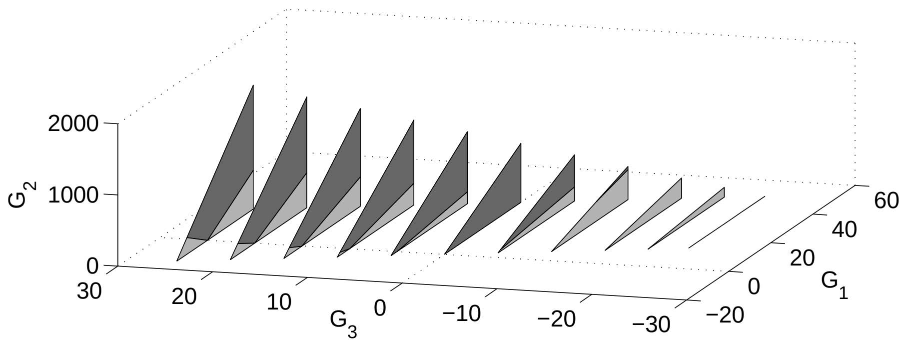


Figure 7. Lead/lag areas in the volume of stabilizing gains (light = lead, dark = lag).

set	1	2	3	4	5
G_1	5	10	20	10	10
G_2	40	40	40	20	100
G_3	4	4	4	4	4

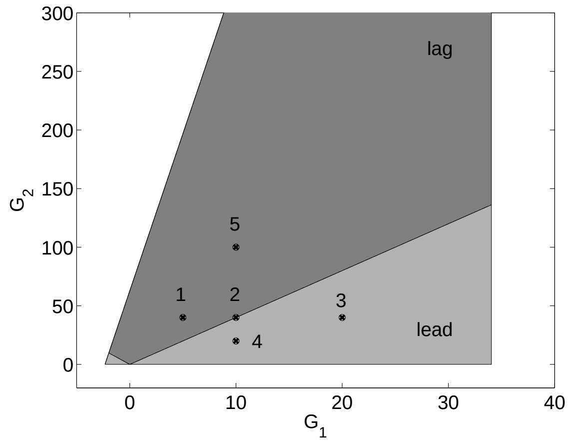
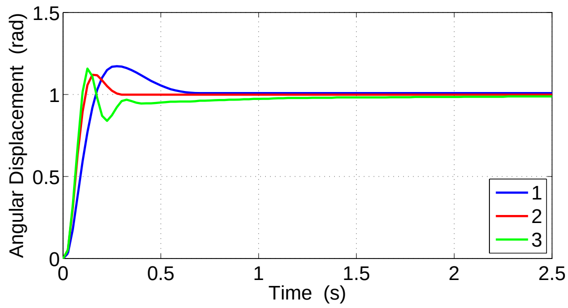
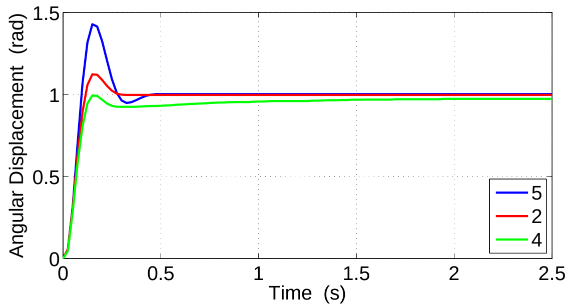


Figure 8. Sets of gains and lead/lag areas in the volume for $G_3 = 4$.



(a) variation of G_1 ($G_2 = 40$, $G_3 = 4$)



(b) variation of G_2 ($G_1 = 10$, $G_3 = 4$)

Figure 9. Unitary step response of the servo positioning system with chosen gains from the volume in the space of gains (experimental results).

is a lag compensator, presenting higher overshoot with small steady state error and settling time. As the controller changes to a lead compensator (gain set #4), there is a reduction in overshoot with an increase in settling time and steady state error.

As one can see, appropriated gain sets can be found in the volume of stabilizing gains according the desired performance of the system. However, this search for an appropriated gain set must be done in a trial and error basis, by knowing that the controller in study behaves as a lead/lag compensator. It must be noted that Theorem 1 of Keel and Bhattacharyya (2005) only defines the region (volume) in the space of gains where the closed loop system is stable. The relationship between the volume of stabilizing gains and specific performance criteria is not clear and must be further investigated. However, one must still bear in mind that the proposed procedure allows the determination of stabilizing gains for a system whose mathematical model is not known, and only input/output information is available. This fact alone is already an advance.

5. CONCLUSION

The gains of a linear controller, applied to a servo positioning system, was determined by Theorem 1 of Keel and Bhattacharyya and experimentally implemented. The following aspects of the procedure can be summarized as follows:

- Only input/output relationships must be known from the system in order to find the stabilizing gains. This relationship can be obtained experimentally, where sensors and actuators are considered part of the system, thus including dynamic effects of such components in the analysis with no need of any modeling;

- The volume of gains in the space of gains can be divided into regions where the controller behaves as a lead or a lag compensator. This information helps finding appropriated gains to the system aiming at attending performance criteria. Nevertheless, this gain search depends on a trial and error approach because the relationship between the gains in the volume and the performance criteria must still be further investigated;

- An important drawback of the procedure lies on the fact that it only works for SISO systems. In previous studies on MIMO systems performed by the authors, all the volumes found in the space of gains contained unstable gains, and no additional information could be extracted from the system.

6. ACKNOWLEDGEMENTS

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7. REFERENCES

Dorf, R.C., Bishop, R.H., 2004, "Modern Control Systems", Prentice Hall, Upper Saddle River, USA. 880p.
Keel, L.H., Bhattacharyya, S.P., 2005, "Direct Synthesis of First Order Controllers From Frequency Response Measurements", Proceedings of the 2005 American Control Conference, Portland, USA, pp.1192-1196.
Marczyk, J., 2003, "Beyond Optimization in Computer Aided Engineering", CIMNE - International Center for Numerical Methods in Engineering, Barcelona, Spain. 300p.
Ogata, K., 2001, "Modern Control Engineering", Prentice Hall, Upper Saddle River, USA. 970p.
Tang, Y., 1996, "Active Control of SDF Systems Using Artificial Neural Networks", Computers and Structures, Vol.60, No.5, pp.695-703.
Tantaris, R.N., Keel, L.H., Bhattacharyya, S.P., 2003, "H_∞ Design With First Order Controllers", Proceedings of the 42^{nd} IEEE Conference on Decision and Control, Las Vegas, USA, pp.1084-1089.
Zhou, K., 1998, "Essentials of Robust Control", Prentice-Hall, Upper Saddle River, USA. 410p.

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