DYNAMIC INVESTIGATION OF A CABLE-STAYED TIMBER FOOTBRIDGE

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Abstract. *Presented are results of a dynamic numerical analysis of a cable-stayed timber footbridge of approximately span form 35 m and width equal to 2 m located at São Carlos Engineering School, São Paulo State University, São Carlos, Brazil. The timber footbridge modular deck is fabricated from seven interconnected curved stress-laminated timber plates. The timber pylon is made from a single log of Eucalipto Citriodora standing alone over a spatial hinge. Pinus taeda and Eucalipto Citriodora were the chosen lumbers, because they met the sustainability requirements. The timber footbridge may be described as a curved bridge, supported by 12 steel bars working as stays. The study of the effects caused by human excitation on structures has gained a significant evolution during the last few years. The increased understanding that followed the first studies is difficult to codify and, therefore, is not yet clearly stated in regulatory guidance on dynamic design of a cable-stayed timber footbridge. Walking, running and jumping on footbridges produce dynamic forces which can activate appreciable vibrations. These vibrations can cause discomfort to pedestrian and deterioration of the footbridge's structural integrity. In the course of this study, which involved the dynamic analysis of a numerical model of the three-dimensional structure developed using the SAP2000® software, vibration modes of the deck were determined. Different loading models were developed to incorporate the dynamical effects, induced by people walking and running, in the dynamical response of the cable-stayed timber footbridge. The dynamic behavior of cable-stayed timber footbridges under walking and running dynamic loads was simulated by resonant vibration caused by synchronous excitations. Completing this study, guidelines for vibrations performance are shown, focusing on the definition of the pedestrian load, frequency ranges of interest, criteria that can evaluate dynamic behavior and human comfort on footbridges with acceptable limits of vibration. The results indicated that this footbridge can reach high vibration levels hat could compromise the user's comfort limit state. Established measurement criteria allow designers to calculate new structures, and also evaluate the need of repairs of existing ones.*

Keywords*: footbridge, cable-stayed, pedestrian, vertical and lateral vibrations, numerical analysis.*

1. INTRODUCTION

Human-induced load vibrations in footbridges have long been recognized as a design problem. Timber and timber-based materials have been successfully used in pedestrian or low-traffic short to medium span bridges. However, the low density and modulus of elasticity of these materials may result in excessive vibration caused by dynamic loading, particularly in the case of synchronized action of pedestrians. The vibration serviceability limit state (SLS) criteria stated in guidelines consists of limiting the maximum value of the acceleration in deck, so that fatigue risk in structural elements is minimized and an appropriate level of comfort may be ensured. Vibration serviceability is usually evaluated for the loading generating the largest response by exciting the structure at its fundamental natural frequency in the vertical direction. This study compares the values provided by the theoretical and measuring of frequencies and acceleration with those calculated from a detailed numerical analysis. The procedure used for this purpose is described in section following.

1.1. Human-induced vibrations

For slender footbridges, vibration at low frequency is more important than that at high frequency, as the fundamental natural frequencies in vertical and lateral direction are always low. During walking on a structure, pedestrians induce dynamic time varying forces on the surface of the structure. Several studies have been performed in order to quantify pedestrian walking forces. These forces have components in all three directions, vertical, lateral and longitudinal and they depend on parameters such as pacing frequency, walking speed and pacing length, according to Wheeler (1982).

The typical pacing frequency for walking is around 2 steps per second, which gives a vertical forcing frequency of 2 Hz. This has been confirmed with several experiments, for example by Matsumoto et al. ((1972) and (1978)) who investigated a sample of 505 persons. He concluded that the pacing frequencies followed a normal distribution with a mean of 2.0 Hz and a standard deviation of 0.173 Hz. Slow walking is in the region of 1.4-1.7 Hz and fast walking in the range of 2.2-2.4 Hz. This means that the total range of vertical forcing frequency is 1.4-2.4 Hz with a rough mean of 2.0 Hz. Since the lateral component of the force is applied at half the footfall frequency, the lateral forcing frequencies are in the region of 0.7-1.2 Hz. Many footbridges have vertical and lateral natural frequencies within the limits mentioned above (1.4-2.4 Hz vertical and 0.7-1.2 Hz lateral), Bachmann et al. (1995).

According to Fujino et al. (1993), when walking, running or jumping, above a structure human-induced vertical and lateral dynamic loading at the surface of the structure. Although it is widely known that people walk with a frequency of about 2 Hz, it is not commonly known but about 10% of the vertical loading works laterally when people walk. These forces are a consequence of a lateral oscillation of the gravity center of the body and the lateral oscillations are a consequence of body movements when persons step with their right and left foot in turn. The amplitudes of these lateral oscillations are, in general, of about 1-2 cm. It should be noted that the lateral loading parameters are not well quantified. Few measurements of the magnitude of lateral loading due to walking have been made and, in addition, they have almost all been made on unmoving surfaces. The longitudinal component is not important for the vertical and lateral vibrations and is neglected.

1.2. Human-induced dinamic loading

Dynamic loading due to pedestrians movement may be considered to be a periodic excitation and can be modeled as a Fourier series, see Eq. (1) and Eq. (2), in which the fundamental harmonic has a frequency equal to the pacing rate. This is represented by the summation term, which is a Fourier series with coefficients at the discrete frequencies. The higher harmonics have frequencies that are integer multiples of the fundamental frequency Rainer and Pernica (1988). When footbridges have natural frequencies within the range of either the fundamental or the higher harmonics of the excitation source, resonance may occur resulting in large vibration-induced displacements, acceleration and discomfort to users. Human-induced dynamic loading may be due to walking, running, or jumping, the so-called vandalism loading. Each of these types of loading produces a different loading curve over time as well as frequencies in which the oscillations can occur. The vertical loading component is larger than the lateral loading component, but the lateral and longitudinal loading components can also cause vibrations problems of slender bridges especially if a pedestrian-bridge-interaction develops. The forcing function due to moving pedestrians can be mathematical described by the Eq. (1) and Eq. (2) at vertical and lateral directions, respectively.

$$F_{vert}(t) = F_0 \begin{bmatrix} 1 + \alpha_{1,vert} \cdot \sin(2 \cdot \pi \cdot f_p \cdot t) + \alpha_{2,vert} \cdot \sin(4 \cdot \pi \cdot f_p \cdot t - \varphi_2) + \alpha_{3,vert} \cdot \sin(6 \cdot \pi \cdot f_p \cdot t - \varphi_3) + \\ \alpha_{4,vert} \cdot \sin(8 \cdot \pi \cdot f_p \cdot t - \varphi_4) + \alpha_{5,vert} \cdot \sin(10 \cdot \pi \cdot f_p \cdot t - \varphi_5) \end{bmatrix} \tag{1}$$

$$F_{lat}(t) = F_0 \begin{bmatrix} \alpha_{1,lat} \cdot \sin\left(2 \cdot \pi \cdot \frac{f_p}{2} \cdot t\right) + \alpha_{2,lat} \cdot \sin\left(4 \cdot \pi \cdot \frac{f_p}{2} \cdot t - \varphi_2\right) + \alpha_{3,lat} \cdot \sin\left(6 \cdot \pi \cdot \frac{f_p}{2} \cdot t - \varphi_3\right) + \\ \alpha_{4,lat} \cdot \sin\left(8 \cdot \pi \cdot \frac{f_p}{2} \cdot t - \varphi_4\right) + \alpha_{5,lat} \cdot \sin\left(10 \cdot \pi \cdot \frac{f_p}{2} \cdot t - \varphi_5\right) \end{bmatrix} \tag{2}$$

Where F_0 = dead load of the pedestrian equal to 0.70 kN, α_i = Fourier coefficient of the i*th* harmonic, f_p = activity rate (Hz), t = time (s), φ_i = phase angle of the i*th* harmonic, i = number of the i*th* harmonic, F_{vert} = vertical force (kN) and F_{lat} = lateral force (kN).

According to Bachmann and Ammann (1987) and Bachmann et al. (1995), in Tab. 1 and Tab. 2 pertinent values for the Fourier coefficients and phase angle proposed to the representative type of activity. Bachmann and Ammann (1987) pointed out that dynamic pavement load is dominated by the pacing frequency, see Tab. 3. The dynamic load functions time records for normal walking at vertical and lateral directions is showed in Fig. 1-a and Fig. 1-b, respectively. These load functions were generated with the aid of Eq. (1) and Eq. (2).

Table 1. Fourier coefficient and phase lag at vertical direction

Fourier coefficient		$\alpha_{1,vert}$	$\alpha_{2,vert}$	$\alpha_{3,vert}$	$\alpha_{4,vert}$	$\alpha_{5,vert}$
Normal walking Bachmann and Ammann (1987)	f_p = 2.0 Hz	0.37	0.10	0.12	0.04	0.08
	phase lag - φ_i	-	$\pi/2$	$\pi/2$	$\pi/2$	$\pi/2$
	pacing rate (Hz)	1.4-2.4	2.8-4.8	4.2-7.2	5.6-9.6	7.0-12.0
Normal running Bachmann et al. (1995)	f_p = 2.5 Hz	1.60	0.70	0.20	-	-
	phase lag - φ_i	-	-	-	-	-
	pacing rate (Hz)	2.0-3.0	4.0-6.0	6.0-9.0	-	-

Table 2. Fourier coefficient and phase lag at lateral direction

Fourier coefficient		$\alpha_{1,lat}$	$\alpha_{2,lat}$	$\alpha_{3,lat}$	$\alpha_{4,lat}$	$\alpha_{5,lat}$
Normal walking Bachmann and Ammann (1987)	$f_p/2 = 1.0$ Hz	0.039	0.010	0.042	0.012	0.015
	phase lag - φ_i	-	$\pi/2$	$\pi/2$	$\pi/2$	$\pi/2$
	pacing rate (Hz)	0.6-1.1	1.2-2.2	1.8-3.3	2.4-4.4	3.0-5.5

Table 3. Typical values for pacing frequency, velocity and length

activity type	f_p (Hz)	v_p (m/s)	l_p (m)
slow walking	1.7	1.0	0.60
normal walking	2.0	1.5	0.75
fast walking	2.3	2.3	1.00
normal running	2.5	3.1	1.25
fast running	> 3.2	5.5	1.75

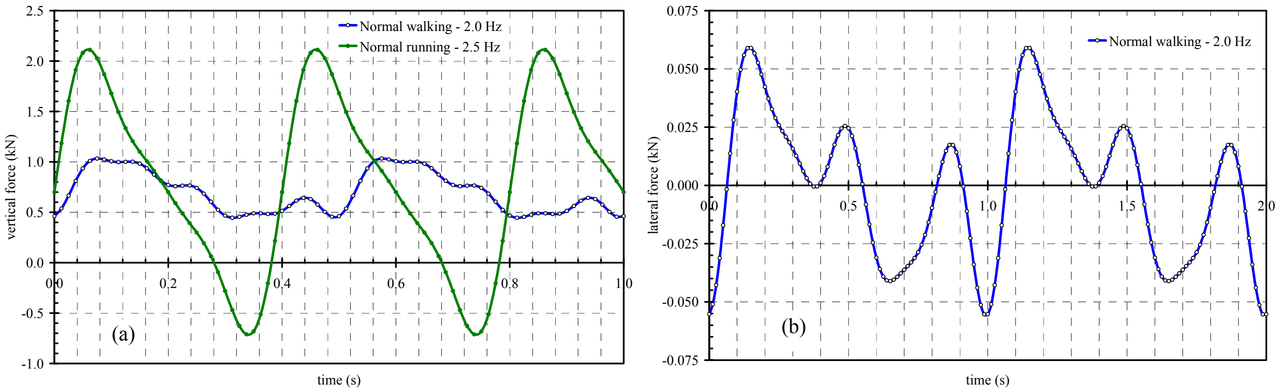


Figure 1. Dynamic load function at (a) vertical and (b) lateral directions due to a single pedestrian normal walking and normal running along the longitudinal axis of the cable-stayed timber footbridge

2. FOOTBRIDGE STRUCTURAL DETAILS

According to Pletz (2003), a cable-stayed timber footbridge at São Carlos Engineering School is fabricated from seven interconnected curved stress-laminated timber plates, that is, timber deck. The tower is made from a single log of Eucalyptus citriodora (Young's modulus 14232 MPa and density 999 kg/m^3), standing alone over a spatial hinge. Pinus taeda (Young's modulus 6448 MPa and density 476 kg/m^3) and Eucalyptus citriodora were the chosen lumbers, because they met the sustain ability requirements. The footbridge may be described as a curved bridge, supported by 12 steel bars (Young's modulus 210000 MPa and density 8030 kg/m^3) working as stays, with an overall length of 35 m and a walkway with of 2 m, Fig. 2 up to Fig. 5.

A set of 37 treated 5 x 20 x 520 cm Pinus taeda laminations were used in the stress laminated plates. Because of the laminations length, no butt joints occurred in the plates. At the edges of the plates two extra Eucalyptus citriodora laminations were adopted, according to Ritter and Lee (1996) recommendation. A thirteen meters long treated log of Eucalyptus citriodora, with a 45 cm top diameter and a 55 cm base diameter was used as tower, see Fig. 2 up to Fig 5. The twelve stays are the same type of the steel bar used to stress the SLT decks, Dywidag (ST 85-105) bars of 15 mm diameter. The timber tower two back-stays are the same steel bar, with 32 mm diameter. Connections and plates for stress distribution are of SAE 1020 steel. All metallic devices are hot dipped galvanized. According to showed in Fig. 2 up to Fig 5, the abutment 1 is anchored at the beam of the laboratory end the abutment 2 is anchored at the top ravine, respectively.

The Brazilian code loading requirements for pedestrian bridges is 5 kPa. Although cable-stayed footbridges are usually flexible structures, this particular footbridge is not sensitive to wind action because of its arched shape in plan. Effects of temperature variation were also not considered. On site measurements of these actions proved those effects to be negligible. Cable fatigue was not investigated because it does not govern the design of footbridges like these one, used by small groups of students, similar to the footbridge designed by Carter and Fayers (1994).

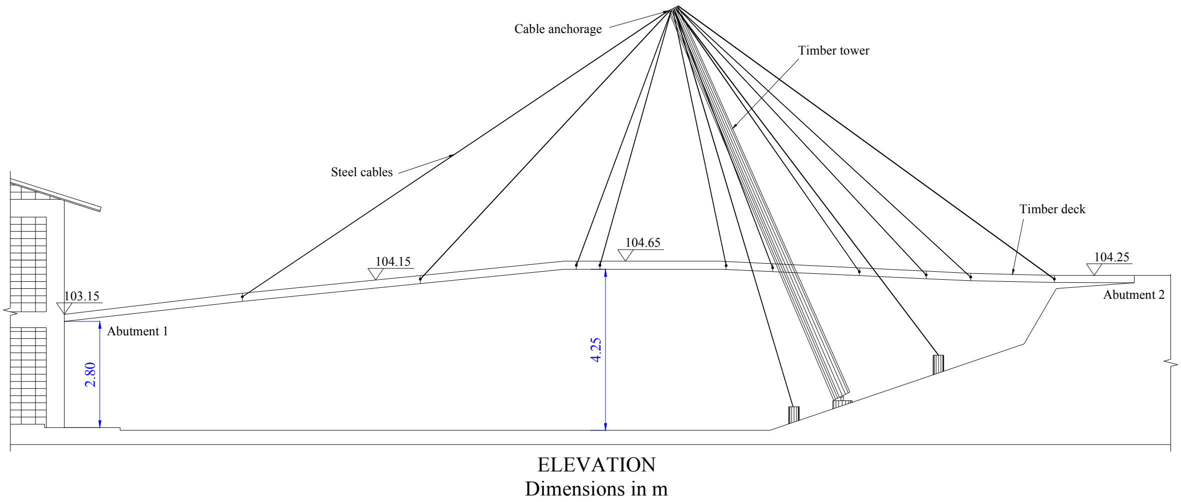


Figure 2. Elevation lateral of the cable-stayed timber footbridge, Pletz (2003)

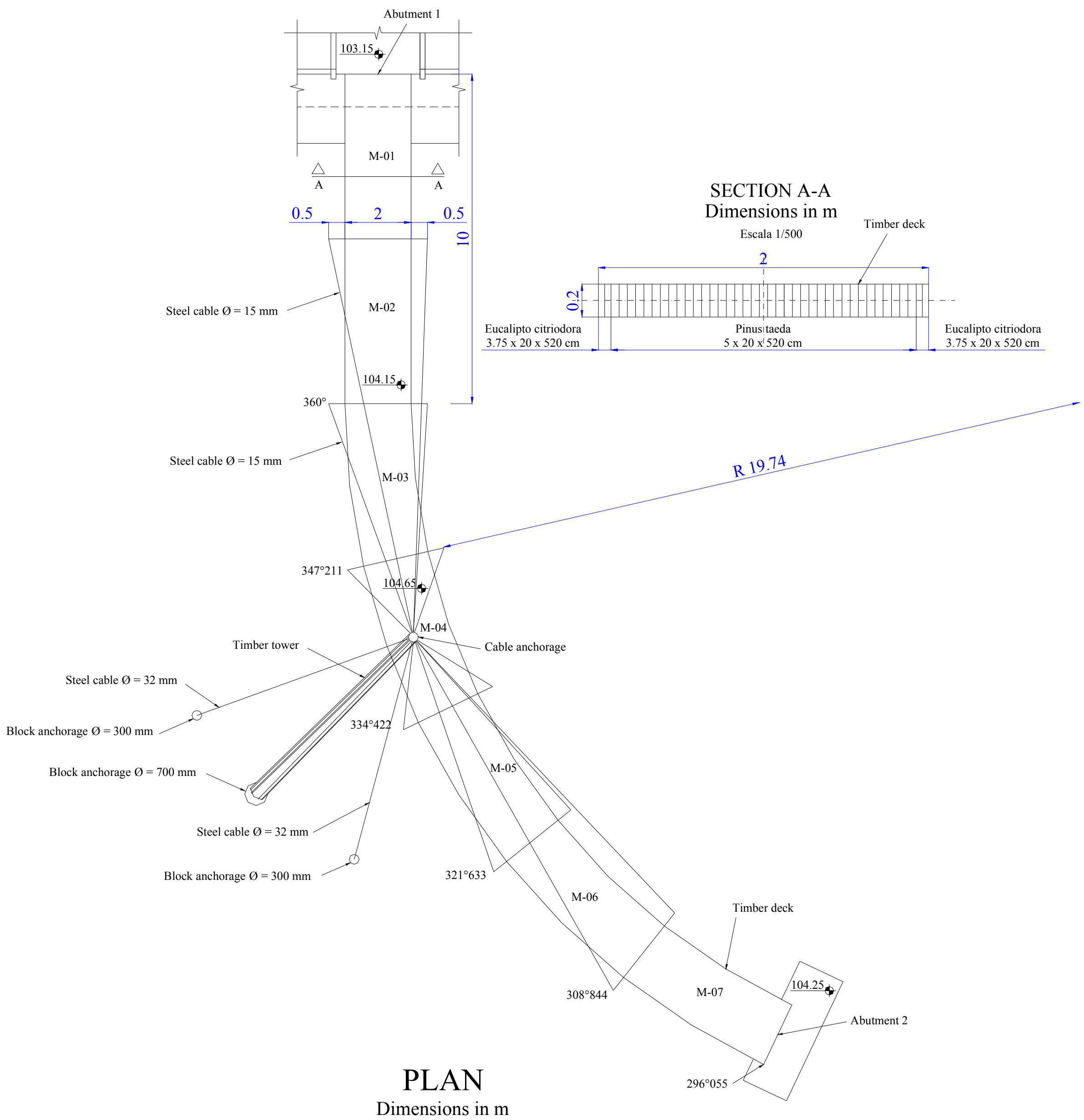


Figure 3. Plan view and section A-A of the cable-stayed timber footbridge, Pletz (2003)

Figure 4. Cable-stayed timber footbridge (a) lateral and (b) top view

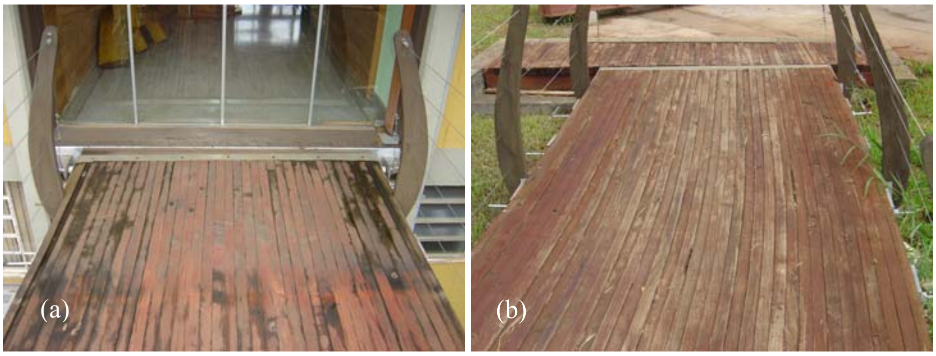
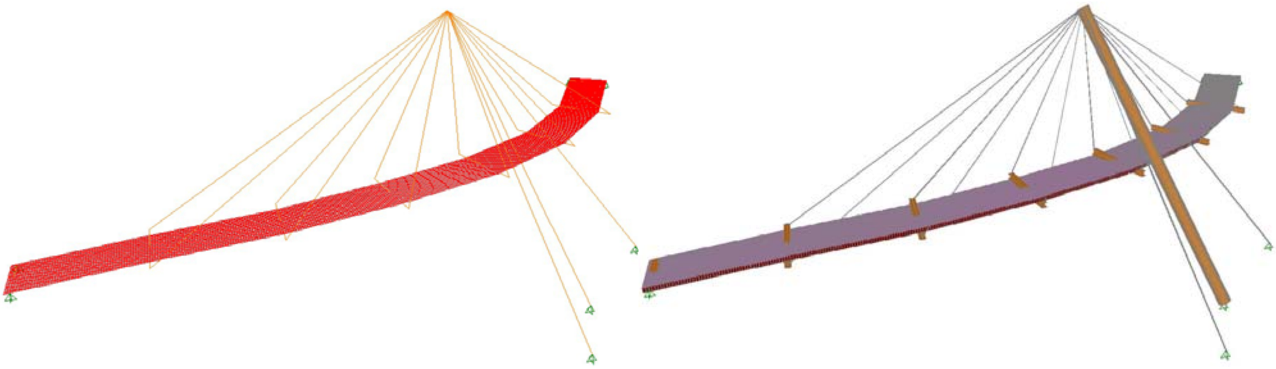


Figure 5. Cable-stayed timber footbridge (a) abutment 1 and (b) abutment 2

3. FINITE ELEMENT MODELLING OF THE CABLE-STAYED TIMBER FOOTBRIDGE

At item is describes the dynamic analysis numerical model of the first cable-stayed timber footbridge built in Brazil. A complete 3D Finite Element (FE) numerical model at São Carlos Engineering School timber footbridge was developed using the structural analysis package SAP2000® (2006) and adopted to carry out numerical analysis, see Fig. 6. With advances in numerical modeling it is often expected that FE models based on technical design data and best engineering judgment can reliably simulate both the static and dynamic behavior of the bridge. The aim was to construct a detailed model which would be able to simulate the dynamic behavior of the structure as well as possible. This was based on the limited technical data available and best engineering judgment.



3D Finite Element Model 3D Finite Element Model Extrude

Figure 6. Finite element model of the cable-stayed footbridge

In the developed 3D Finite Element Model, steel cables are represented by three-dimensional beam elements, where flexibility and torsion effects are considered. The timber modular deck is represented by finite shell elements. The key finite element modeling assumptions were as follows: The timber modular deck is assumed to be fixed and is modeled as shell elements with end releases. The cable anchorages and steel cables were modeled using three-dimensional beam elements by suitably restrained with properly computed characteristics. The timber tower was modeled using three-dimensional beam as fully fixed considering solid rock foundations. The handrails were not modeled at design process.

4. LIMIT VALUES FOR ACCELERATION ACCORDING TO INTERNATIONAL STANDARDS

Limit values for acceleration at international codes are directly linked to human comfort. Due to the plethora of studies on such a subjective matter as pedestrian comfort, there are many different acceleration limits in the international codes. An overview of the peak acceleration limits in codes and relevant literature are provided in Fig. 7 and Fig. 8. Some of these limits accelerations are dependence of the fundamental frequency while others are constant for the whole range of pedestrian induced loading frequencies.

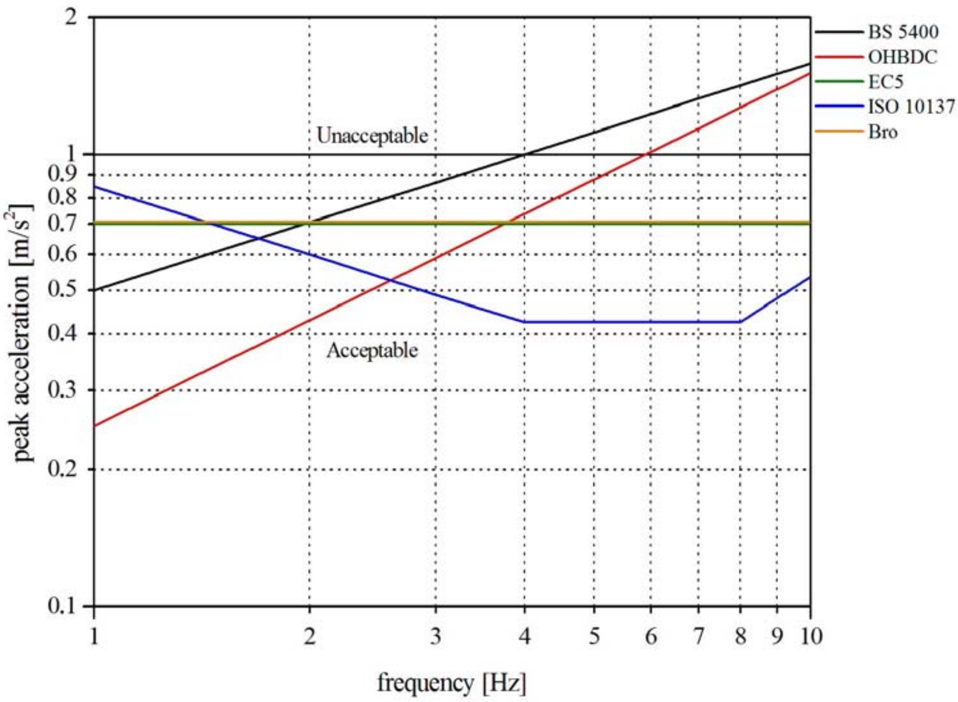


Figure 7. Acceptability of vertical vibration at footbridges

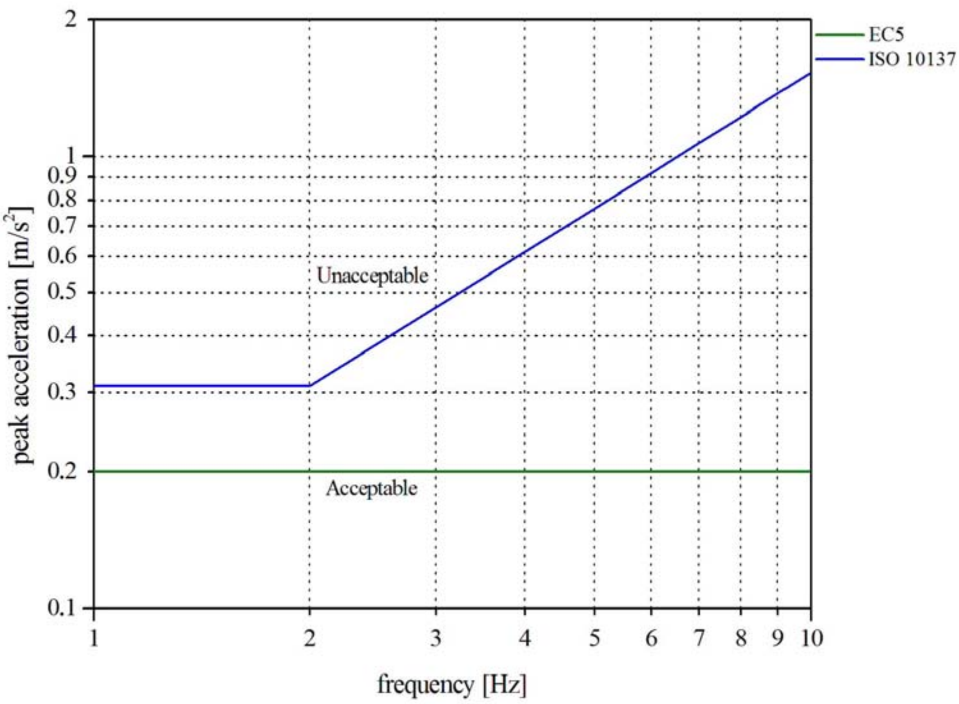


Figure 8. Acceptability of lateral vibration at footbridges

5. RESULTS AND DISCUSSION

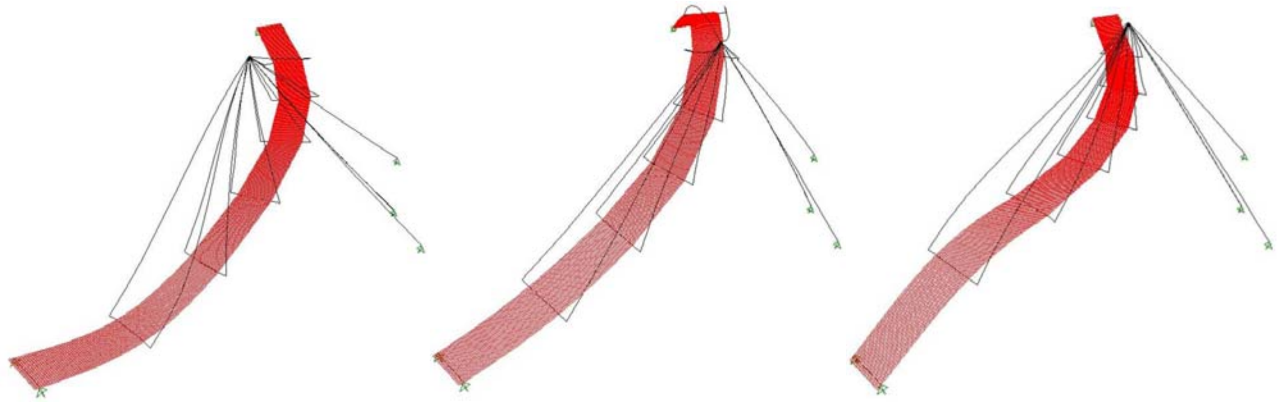
At stage the comparison is made between the maximum acceleration values obtained from the load numeric model presented before and those show at the Eurocode (1993), BS 5400 (1978), OHBDC (1991), ISO 10137 (1992) and Bro

(2004). In Tab. 4 contains a compilation of case studies found in the literature describing cable-stayed footbridge at São Carlos Engineering School. In Tab. 4, it can be seen that at vertical direction the footbridge have a fundamental vertical natural frequencies of less than 5 Hz and that this is possibly a troublesome frequency range, as acknowledged in standards. A more careful look in Tab. 4 shows that the case reports are mostly concerned with footbridge having the vertical modes of vibration in a narrower frequency range of 1.70-3.20 Hz. As previously mentioned, this corresponds to the usual pedestrian pacing rate. The compilation presented in Tab. 4 favors the frequency tuning approach, since all lively structures reported had frequencies at vertical direction within the critical frequency ranges.

Table 4. Case reports of fundamental frequency at cable-stayed timber footbridge at São Carlos Engineering School

Reference	Fundamental frequencies
Pletz (2003)	First mode - Numerical (ANSYS) fundamental vertical frequency of 1.79 Hz
	First mode - Experimental (accelerometer) fundamental vertical frequency of 1.83 Hz
	Second mode - Numerical (ANSYS) fundamental vertical frequency of 2.67 Hz
	Second mode - Experimental (accelerometer) fundamental vertical frequency of 2.74 Hz
	Theoretical fundamental vertical frequency of the footbridge equal to 3.20 Hz
Larocca (2006)	Third mode - Experimental (GPS) fundamental vertical frequency of 3.11 Hz
	Third mode - Experimental (displacement transducer) fundamental vertical frequency of 3.06 Hz

Following the rules commended by recent standards the load model can be applied in three different loading situations concerning pedestrian activity, namely one single pedestrian, one group of 10 pedestrians and finally a constant stream of pedestrians crossing the bridge with a pre-established density over the deck. But, this paper was applied loading situations for one single pedestrian. The results from the one-span deck loaded with one pedestrian crossing the bridge with a pace rate can be coincident with the eigenfrequency, gives a substantially lower maximum acceleration than the one computed. However, the value obtained from this code for the effect of a continuous stream of pedestrians acting in resonance with the structure compares well with that one obtained from the load model. The mode shapes with lower frequencies (mode 1-3) involve mainly vertical acceleration, highlighting a greater tendency of the structure to vertical swaying rather than lateral vibration. The numerical fundamental modes shapes of vibration were found at vertical direction and are show in Fig. 9-a up to Fig. 9-c.



(a) f = 1.454 Hz (b) f = 2.570 Hz (c) f = 2.975 Hz

Figure 9. Numerical modes of vibrations at vertical direction are (a) mode 1, (b) mode 2 and (c) mode 3

The dynamic analysis was therefore useful to take account of the effects caused by the passage of either a single pedestrian above the structure. The response to a moving pedestrian was simulated using normal walking and normal running forcing function model as showed in Eq. (1) and Eq. (2). But, it was not necessary to simulate the pedestrians load at lateral direction because appeared modes of vibration with frequencies out of the pacing rate, as shown in Fig. 9-a up to Fig 9-b. The following peak acceleration values due to the passage of a pedestrian normal walking and normal running were showed in Fig. 10 and Fig. 11.

The values shown in Fig. 10 and Fig. 11 refer to maximum acceleration situations, so that the resonance of one harmonic of the load, the first or the second, with the first eigenfrequency is the main loading situation. However, footbridges are mostly used by normal walking people at pace rates of about 2.0 Hz.

The diagrams in Fig. 10 and Fig. 11 illustrate the time history of the acceleration at deck level, with the pedestrian on the footbridge at different moments in time. In diagrams Fig. 11, continuous amplification of the vibration occurs when the pedestrian normal running across; whereas diagrams in Fig. 10, the response history do not shows amplification of the vibration due to normal walking across.

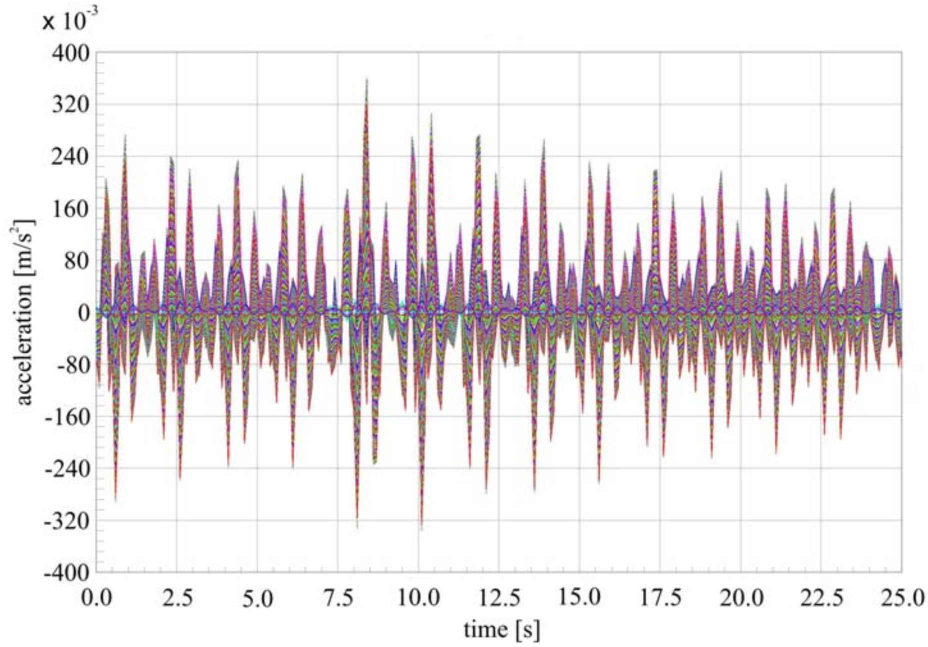


Figure 10. Acceleration response due to the passage of a pedestrian normal walking (f_p = 2.0 Hz) at direction vertical

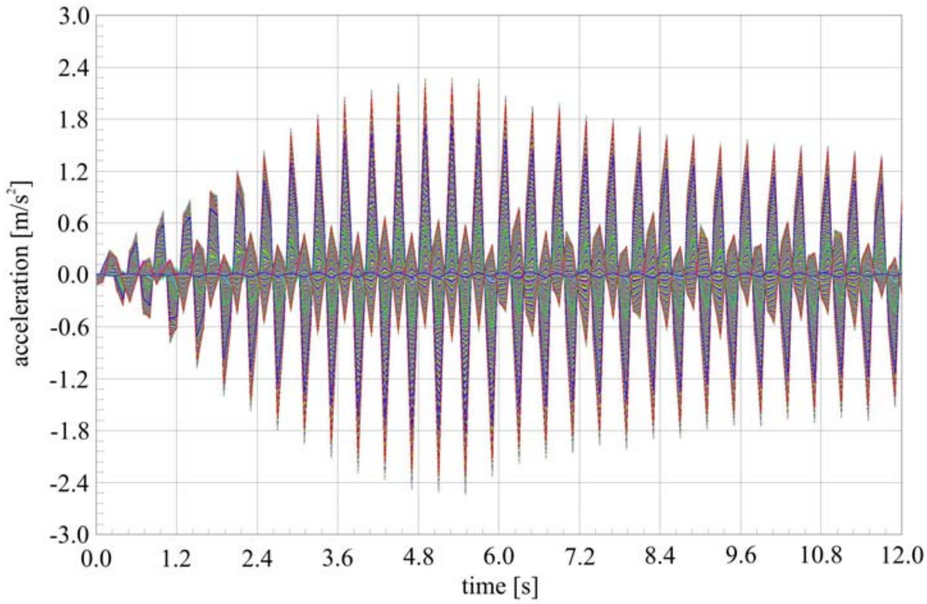


Figure 11. Acceleration response due to the passage of a pedestrian normal running (f_p = 2.5 Hz) at direction vertical

6. CONCLUSIONS

Natural frequencies which are in range coinciding with frequencies typical for human-induced dynamic loading can be avoided by increasing structural stiffness. Increasing stiffness can be an expensive measurement and will almost always have negative effects on the aesthetics of the structure. This, together with the coincidence of the structure's main natural frequencies and the range of frequencies for maximum load from footfalls, makes it susceptible to vibrations induced by pedestrians. This is good for testing but undesirable for other users (and in extreme cases structural integrity). Human-induced vibrations are a subject of serviceability. It was therefore assumed that structures respond linearly to applied loads and that dynamic response can be found by solving the numerical models. The loading models proposed by the above mentioned standards are all based on the assumptions that pedestrian loads can be approximated as periodic loading.

A good agreement with the numerical results showed in Fig. 9 and experimental results showed in Table 4 have been found when comparison. The footbridge dynamic response in terms of acceleration was obtained and compared to the limit proposed by standards as showed in Fig. 7. The dynamic analysis indicated that the footbridge is sensitive to large vertical vibrations due to pedestrian normal walking and normal running, and this resulted in accelerations that exceeded the criteria proposed by the standards. However, it was unable to detect the structures vulnerability to horizontal vibrations. The results of the analysis providing evidence that the maximum acceleration can be cause discomfort to users and that there is risk of limit conditions being reached.

7. ACKNOWLEDGMENTS

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