ATTITUDE AND HEADING REFERENCE SYSTEM WITH GPS AIDING

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Abstract. *In several applications, such as in UAV navigation and virtual reality, attitude estimation is the paramount problem. For movements with small accelerations, attitude estimation can be obtained by means of cheap hardware, by using what is called MARG sensors (magnetometers, accelerometers and rate gyros). The authors have already developed recent works in this field, with experimental results reported elsewhere. However, for accelerated movements the output measurement necessary to drive the Kalman Filter is actually unknown, since the accelerometer reads only the specific force. Hence, alternative approaches must be devised. Estimating the acceleration by augmenting the state vector is attractive, but flawed by observation problems. A more reliable approach consists in bringing in an additional sensor, such as GPS, from which acceleration can be derived. In this paper the low-cost system developed recently by the authors has its application envelope enlarged by incorporating GPS measurements, enabling it to tackle, for instance, UAV accelerated phases and maneuvers in general. Details about the implementation are provided, with the presentation of experimental results, by comparing the attitude estimation thus obtained with the ground truth given by a high precision/high cost commercial INS/GPS navigator. As far as the authors are aware, this is the first reported experimental result of its kind in Brazil.*

Keywords*: AHRS, GPS, Kalman Filter, MEMS, UAV.*

1. INTRODUCTION

Real time control, navigation and guidance of UAVs (Unmanned Air Vehicles) and MAVs (Micro Air Vehicles) demands a good enough attitude estimate. For reasons of cost and payload, this estimate ought to be obtained by means of cheap and light sensors. This has been made possible more recently by the appearance of MARG sensors, based on the MEMS technology, which satisfies these criteria. This has motivated the appearance of several AHRS (Attitude Heading Reference System) of the low cost type. Since the MARG sensors are noisy, almost all the solutions are based on the Kalman Filter, such as in (Marins et al., 2001), (Gebre-Egziabher et al., 2004), and (Hemerly et al., 2007; Hemerly et al., 2008). These solutions differ basically due to the error model employed and the number of states which is considered in the Kalman Filter.

The basic approach for the Kalman Filter based AHRS is: given the present attitude estimate, by integrating the gyros readings a propagated attitude is obtained. Next, by using the accelerometers and magnetometers readings a Kalman Filter measurement is obtained, thereby enabling attitude correction. This approach works well if the vehicle undergoes a non accelerated trajectory. Otherwise, under what is called dynamic environments, the measurement is incorrect, since the accelerometers actually sense specific forces.

There are 2 main ways for tackling dynamic environments: a) to estimate the acceleration in real time, by augmenting the state vector, and b) to employ an independent sensor for measuring speed, from which the acceleration can be derived. The first approach, although elegant, is flawed by observation problems, since sufficient excitation is necessary for discerning the acceleration from other unknowns, such as gyros and accelerometers biases. Hence, the second one has been favored in applications, and two main sensors have been used: GPS and ADS (Air Data System). The ADS gets velocity readings from pressure sensors, and requires adequate calibration and usually has some range limitation. For general applications, GPS is then recommended.

The development of a typical GPS aided AHRS can be found in (Li et al., 2006). The GPS velocity is filtered, thereby providing the vehicle acceleration, which is employed to compensate the accelerometers readings. The attitude is then estimated by solving the associated Wahba problem, see (Gebre-Egziabher et al., 2004). Emphasis lies on how the acceleration is obtained from GPS readings, hence several relevant issues, such as recursive estimation of the attitude via a Kalman Filter and estimation of the gyros and accelerometers biases, are not considered.

This work develops a GPS aided AHRS, which differs from other approaches, including (Li et al., 2006), in the following ways: a) if there is no GPS aiding, then the norm of the acceleration is estimated and the output noise covariance is adjusted accordingly, in order to provide some robustness to acceleration; b) the error model is developed by using the quaternion formulation, which simplifies the calculations; c) a Kalman Filter is employed to estimate recursively both the attitude and the gyros biases; d) the GPS derived acceleration is employed to correct the accelerometers readings and e) performance is evaluated by using as ground truth the attitude estimates obtained by a high cost/high precision commercial navigator.

2. QUATERNIONS AND GYRO BIAS ERROR MODEL

The gyro reading is modeled as

$$\boldsymbol{w}(t) = \boldsymbol{w}_{true}(t) + \boldsymbol{b}(t) + \boldsymbol{\eta}(t) \quad \boldsymbol{w}(t) \in R^3 \tag{1}$$

where $\boldsymbol{b}(t)$ is the bias and $\boldsymbol{\eta}(t)$ stands for wideband noise. Then, as in (Titterton and Weston, 2004), the true quaternion dynamics is given by

$$\dot{\boldsymbol{q}}(t) = \frac{1}{2}\boldsymbol{q}(t) \otimes \begin{bmatrix} 0 \\ \boldsymbol{w}_{true}(t) \end{bmatrix} \tag{2}$$

where $\otimes$ stands for quaternion product.

If, at time t, the estimate $\hat{\boldsymbol{q}}(t)$ of the true quaternion $\boldsymbol{q}(t)$ is available, then the quaternion error $\boldsymbol{q}_e(t) = [1; \boldsymbol{Q}_e(t)]$ is defined by

$$\boldsymbol{q}(t) \otimes \hat{\boldsymbol{q}}^*(t) = \boldsymbol{q}_e(t) \tag{3}$$

where * stands for complementary rotation, and a dynamic equation for $\boldsymbol{Q}_e(t)$ is required by the Kalman Filter. Several procedures for obtaining the quaternion error dynamics are presented in (Creamer, 1996), and the derivation proposed by (Sherry et al., 2003) is the basic reference for almost all works concerning AHRS. The different possibilities depend upon how the quaternion error is defined and how the linearization is performed. For instance, in (Sherry et al., 2003) the linearization is carried out in the body frame and as a consequence the quaternion error dynamics does not depend on the present attitude, but only upon the gyro readings.

Here a linearization in the navigation frame was sought for by directly differentiation of Eq. (3). In order to obtain the quaternion error, by defining

$$\boldsymbol{q}_G(t) = \begin{bmatrix} 0 \\ \boldsymbol{w}(t) \end{bmatrix}, \ \boldsymbol{q}_b(t) = \begin{bmatrix} 0 \\ \boldsymbol{b}(t) \end{bmatrix}, \ \boldsymbol{q}_\eta(t) = \begin{bmatrix} 0 \\ \boldsymbol{\eta}(t) \end{bmatrix} \tag{4}$$

from Eq. (2) it follows that

$$\dot{\boldsymbol{q}}(t) = \frac{1}{2}\boldsymbol{q}(t) \otimes \left(\boldsymbol{q}_G(t) - \boldsymbol{q}_b(t) - \boldsymbol{q}_\eta(t)\right) \tag{5}$$

and

$$\dot{\hat{\boldsymbol{q}}}(t) = \frac{1}{2}\hat{\boldsymbol{q}}(t) \otimes \left(\boldsymbol{q}_G(t) - \hat{\boldsymbol{q}}_b(t)\right) \tag{6}$$

where $\boldsymbol{q}_G(t)$ is formed from the gyro readings and $\hat{\boldsymbol{q}}_b(t)$ from the available gyro bias estimates $\hat{\boldsymbol{b}}(t)$.

By using differentiation rules for quaternion products, from (3) it follows that

$$\dot{\boldsymbol{q}}_e(t) = \dot{\boldsymbol{q}}(t) \otimes \hat{\boldsymbol{q}}^*(t) + \boldsymbol{q}(t) \otimes \dot{\hat{\boldsymbol{q}}}^*(t) \tag{7}$$

where, from $\hat{\boldsymbol{q}}(t) \otimes \hat{\boldsymbol{q}}^*(t) = [1\,0\,0\,0]^T$ and Eq. (6),

$$\dot{\hat{\boldsymbol{q}}}^*(t) = -\tfrac{1}{2}\left(\boldsymbol{q}_G(t) - \hat{\boldsymbol{q}}_b(t)\right) \otimes \hat{\boldsymbol{q}}^*(t) \tag{8}$$

The quaternion error dynamics is then obtained from Eq. (7) and Eq. (8),

$$\dot{\boldsymbol{q}}_e(\boldsymbol{t}) = \frac{1}{2}\boldsymbol{q}(t) \otimes \left(\hat{\boldsymbol{q}}_b(t) - \boldsymbol{q}_b(t) - \boldsymbol{q}_\eta(t)\right) \otimes \hat{\boldsymbol{q}}^*(t) \tag{9}$$

which from Eq. (3) implies

$$\dot{\boldsymbol{q}}_e(t) = \frac{1}{2}\boldsymbol{q}_e(t) \otimes \hat{\boldsymbol{q}}(t) \otimes \left(\hat{\boldsymbol{q}}_b(t) - \boldsymbol{q}_b(t) - \boldsymbol{q}_\eta(t)\right) \otimes \hat{\boldsymbol{q}}^*(t) \tag{10}$$

By defining now the vector in R^4

$$\boldsymbol{v}_b(t) = [0\ \boldsymbol{V}_b(t)]^T = \hat{\boldsymbol{q}}_b(t) - \boldsymbol{q}_b(t) - \boldsymbol{q}_\eta(t),\ \boldsymbol{V}_b(t) \in R^3 \tag{11}$$

the quaternion multiplications by $\hat{\boldsymbol{q}}(t)$ and $\hat{\boldsymbol{q}}^*(t)$ in Eq. (10) actually produces

$$\hat{\boldsymbol{q}}(t) \otimes \left(\hat{\boldsymbol{q}}_b(t) - \boldsymbol{q}_b(t) - \boldsymbol{q}_\eta(t)\right) \otimes \hat{\boldsymbol{q}}^*(t) = \begin{bmatrix} 0 \\ \hat{\boldsymbol{C}}_{nb}(t)\boldsymbol{V}_b(t) \end{bmatrix} \tag{12}$$

where $\hat{\boldsymbol{C}}_{nb}(t)$ is the DCM associated with the quaternion estimate $\hat{\boldsymbol{q}}(t)$. Hence, by defining

$$\boldsymbol{V}_n(t) = \hat{\boldsymbol{C}}_{nb}(t)\boldsymbol{V}_b(t) \tag{13}$$

the quaternion error can be rewritten as

$$\dot{\boldsymbol{q}}_e(t) = \frac{1}{2}\boldsymbol{q}_e(t) \otimes \begin{bmatrix} 0 \\ \boldsymbol{V}_n(t) \end{bmatrix} \tag{14}$$

By using now the definitions of $\boldsymbol{q_e}(t)$, namely $\boldsymbol{q}_e(t) = [1; \boldsymbol{Q}_e(t)]$, the dynamics associated with $\boldsymbol{Q}_e(t)$ is obtained from Eq. (14) as

$$\dot{\boldsymbol{Q}}_e(t) = \frac{1}{2}\boldsymbol{V}_n(t) - \frac{1}{2}skew\left(\boldsymbol{V}_n(t)\right)\boldsymbol{Q}_e(t) \tag{15}$$

with $skew(.)$ standing for skew matrix, where, from (12) and (13). Then, by defining the gyro bias estimation error as

$$\boldsymbol{B}_e(t) = \begin{bmatrix} \hat{b}_p(t) \\ \hat{b}_q(t) \\ \hat{b}_r(t) \end{bmatrix} - \begin{bmatrix} b_p(t) \\ b_q(t) \\ b_r(t) \end{bmatrix} \tag{16}$$

and recalling that by hypothesis $\boldsymbol{Q}_e(t)$ is small, only the first term in the right hand side of Eq. (15) contributes with state noise, thereby resulting

$$\dot{\boldsymbol{Q}}_e(t) = \frac{1}{2}\hat{\boldsymbol{C}}_{nb}(t)\boldsymbol{B}_e(t) - \frac{1}{2}skew\left(\hat{\boldsymbol{C}}_{nb}(t)\boldsymbol{B}_e(t)\right)\boldsymbol{Q}_e(t) + \frac{1}{2}\hat{\boldsymbol{C}}_{nb}(t)\boldsymbol{\eta}(t) \tag{17}$$

which indicates that the state noise covariance varies with the present attitude estimate.

Regarding the dynamics for the gyro bias error estimate, the usual hypothesis is, as in (Gebre-Egziabher et al., 2004), to consider it as being a first order Gauss-Markov process, see (Grewal and Andrews, 1993). This and (17) define the dynamic error model in state space form, necessary for the Kalman Filter application. The output equation is now required, and it is formed from the magnetometer and accelerometer readings $\boldsymbol{m}_n(k)$ and $\boldsymbol{a}_n(k)$, respectively, in the navigation frame, by using charted models. As in (Gebre-Egziabher et al., 2004), it can be shown that

$$y(k) = \begin{bmatrix} -2skew(\boldsymbol{m}_n(k)) \\ -2skew(\boldsymbol{a}_n(k)) \end{bmatrix} . \boldsymbol{Q}_e(k) + \boldsymbol{v}(k) \tag{18}$$

which is linear in the quaternion error. The state space model, complemented by the sensors statistics obtained from data sheets for gyros, accelerometers and magnetometers, enable the Kalman Filter implementation. Details are omitted and can be found, for instance, in (Grewal and Andrews, 1993). For added numeric robustness, this work implements the Kalman Filter in the UD form.

3. QUATERNION PROPAGATION AND CORRECTION

Let $\hat{\boldsymbol{q}}(t)$ be the quaternion associated with the estimated DCM matrix $\hat{\boldsymbol{C}}_{nb}(t)$, which transfer from the body to the navigation frame. Then, as in (Titterton and Weston, 2004) the quaternion evolution is governed by

$$\dot{\hat{\boldsymbol{q}}}(t) = \frac{1}{2}\hat{\boldsymbol{W}}\hat{\boldsymbol{q}}(t) \tag{19}$$

where

$$\hat{\boldsymbol{W}} = \begin{bmatrix} 0 & -\hat{p} & -\hat{q} & -\hat{r} \\ \hat{p} & 0 & \hat{r} & -\hat{q} \\ \hat{q} & -\hat{r} & 0 & \hat{p} \\ \hat{r} & \hat{q} & -\hat{p} & 0 \end{bmatrix} \tag{20}$$

and $\hat{p}$, $\hat{q}$ and $\hat{r}$ are the roll, pitch and yaw estimated rates, obtained by subtracting the estimated gyro bias from their measurements. By supposing the angular rates remain constant in the integration interval T, as in (Titterton and Weston, 2004) the exact quaternion propagation is given by

$$\hat{\boldsymbol{q}}(k+1\,|\,k) = \hat{\boldsymbol{q}}(k) \otimes \boldsymbol{r}(k) \tag{21}$$

where $\otimes$ stands for quaternion multiplication and

$$\boldsymbol{r}(k) = \begin{bmatrix} a_c(k) \\ a_s(k)\sigma_p(k) \\ a_s(k)\sigma_q(k) \\ a_s(k)\sigma_r(k) \end{bmatrix}, \ \sigma_p = T\hat{p}, \ \sigma_q = T\hat{q}, \ \sigma_r = T\hat{r} \tag{22}$$

with

$$\sigma(k) = \begin{bmatrix} \sigma_p & \sigma_q & \sigma_r \end{bmatrix}^T \quad , \ a_c(k) = \cos\left(\frac{\|\sigma(k)\|}{2}\right), \ a_s(k) = \frac{\sin\left(\|\sigma(k)/2\|\right)}{\|\sigma(k)\|} \tag{23}$$

Then, at time step k the quaternion estimate is propagated as in Eq. (21). Once the Kalman filter estimate $\hat{\boldsymbol{Q}}_e(k+1)$ is available, the quaternion error $\hat{\boldsymbol{q}}_e(k+1)$ is formed and the propagated quaternion is corrected by using Eq. (3).

4. GPS AIDED AHRS

The basic block diagram for the GPS aided Attitude and Heading Reference System implemented in this work is shown in Fig. 1.

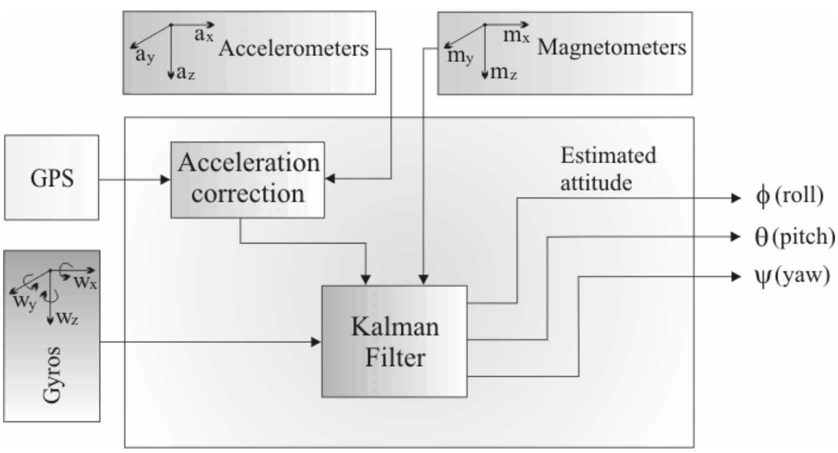


Figure 1. Block diagram of the implemented GPS aided AHRS

The AHRS in Fig. 1 incorporates main improvements with respect to (Hemerly et. al., 2008):

- if there is GPS outages, then the acceleration norm is estimated by using the present attitude estimate. If this norm indicates that the movement is accelerated, then the output noise covariance is increased proportionally. This indicates to the Kalman Filter that the accelerometer readings are not too reliable, and that more importance should be given to the propagated attitude.
- When the GPS readings are available, the vehicle acceleration is calculate in the NED frame by using a bandpass filter and this acceleration is employed to correct the readings which are used as output measurement for the Kalman Filter.

It should be noted that if the aforementioned features are disabled, then the AHRS shown in Fig. 1 behaves exactly as in (Hemerly et. al., 2008). There it was concluded that its performance is comparable to commercial systems, under the hypothesis of low dynamic movements. Therefore, the improvements introduced here aims exactly at extending the scope for AHRS usage, at a small cost incurred by the necessity of GPS. This tradeoff is considered advantageous, given the low price of basic GPS systems.

5. EXPERIMENTAL RESULTS

The GPS assisted AHRS was embarked in a van and road tests were conducted. A high cost/high precision navigator is also embarked, with the aim of providing the ground truth. A typical experiment is displayed in Fig. 2, where the movement is accelerated and both the true (obtained by the navigator) and the GPS derived acceleration norm are displayed. The difference between these accelerations is not too large, hence the compensation by using GPS must be effective. It should be noted that for low vehicle speed the GPS velocity readings can be naturally noisier than for larger speeds. Therefore, below a given GPS velocity it may be advisable to ignore the acceleration compensation scheme and proceed as if the AHRS had actually no GPS assistance.

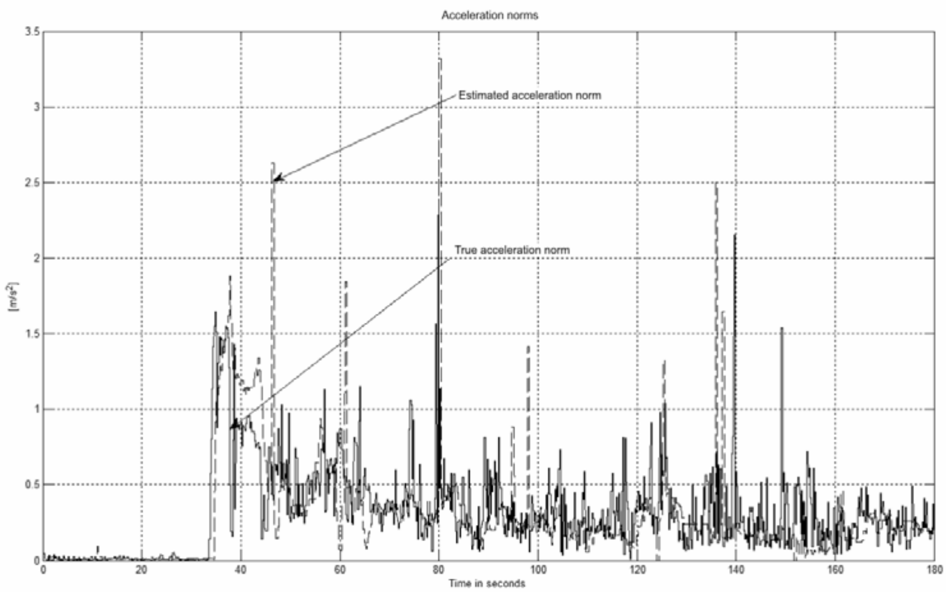
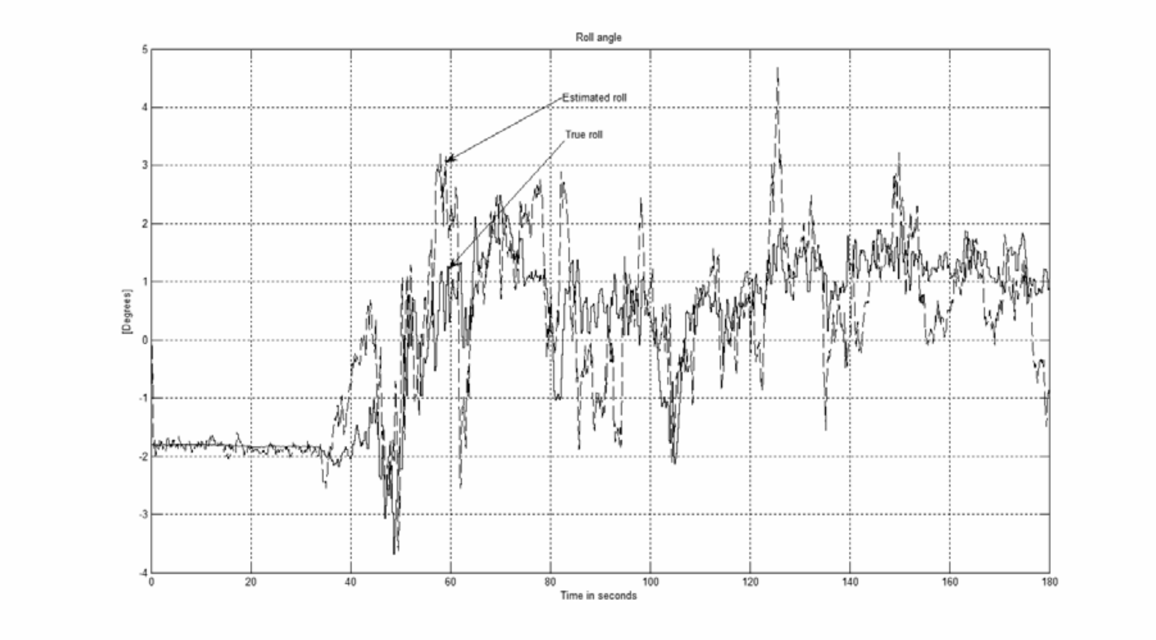
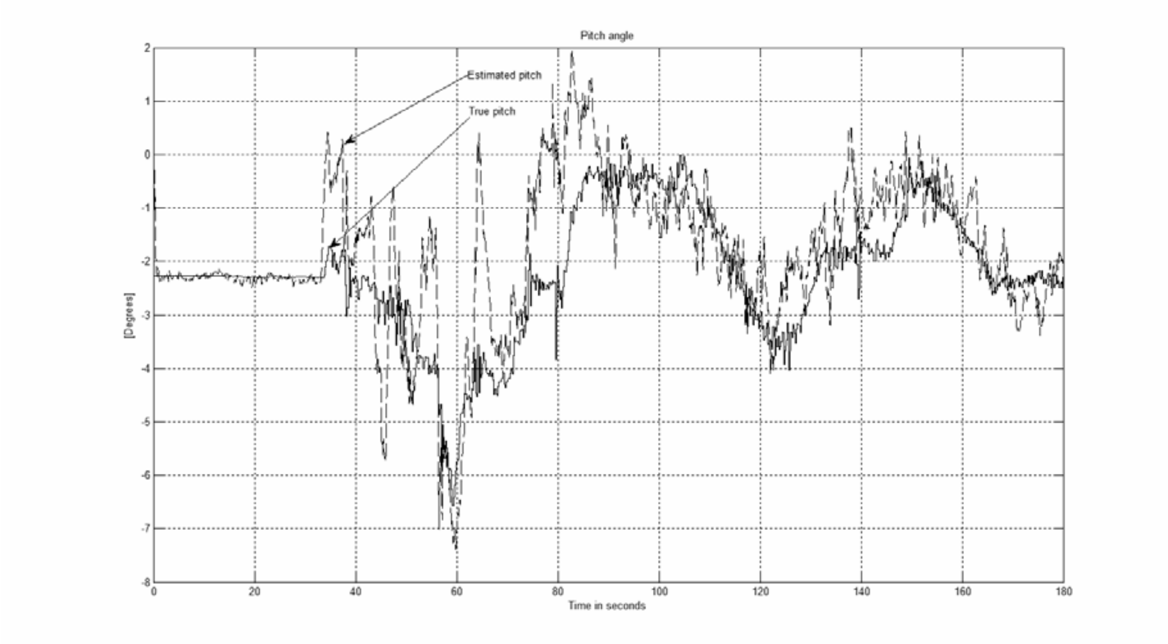


Figure 2. True and estimated acceleration norms.

The attitude angles corresponding to the experiment represented by the acceleration is Fig. 2 are shown in Fig. 3. In this figure, the true angles are those obtained by the reference system, namely the high precision navigator.


Roll angle
Estimated roll
True roll
[Degrees]
Time in seconds


Pitch angle
Estimated pitch
True pitch
[Degrees]
Time in seconds

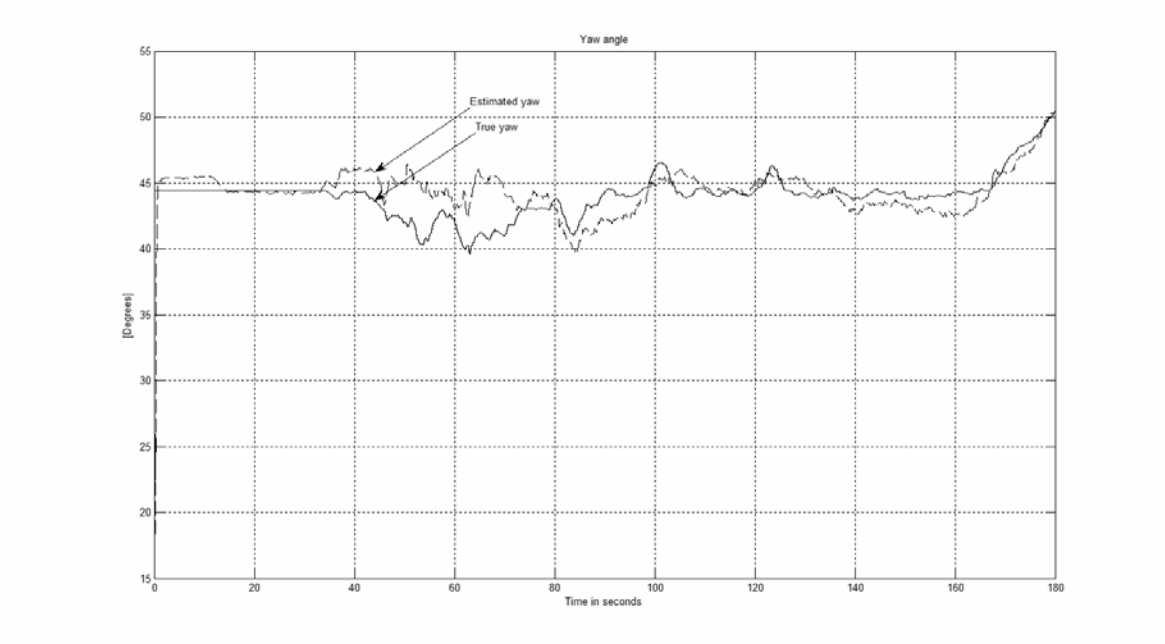


Figure 3. Euler angles (roll, pitch, yaw): true and estimated values, in degrees.

From Fig. 3 it can be concluded that the attitude estimate does not depart too much from the true values. An additional step for concluding that the acceleration correction indeed being efficient is by comparing the estimated angles in Fig. 3 with those which would be obtained with this correction. The case for the pitch angle, where the difference is more noticeable, is displayed in Fig. 4

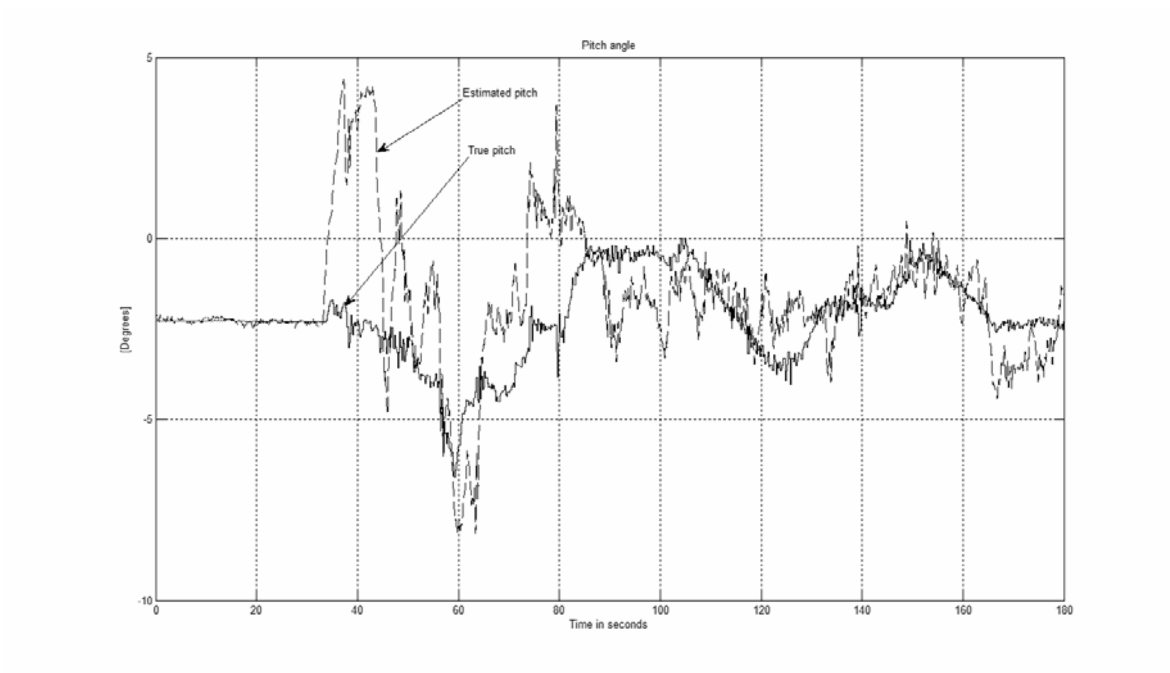


Figure 4.Pitch angle: true and estimated without acceleration correction.

By comparing the second graphic in Fig. 3 with that in Fig. 4, it is concluded that without the acceleration correction the pitch angle estimate would display a large initial error, which is exactly the trajectory phase with larger acceleration. As a matter of fact, the large estimate error shown in Fig. 4 is bound to occur in all AHRS which do not employ some sort of acceleration correction by means of an additional sensor, such as the GPS employed in this work.

6. CONCLUSIONS

A GPS assisted AHRS has been modeled and implemented in this work, aiming at extending the range of AHRS usage to dynamic scenarios where the vehicle trajectory is accelerated. Without acceleration correction, all the AHRS's are bound to provide erroneous attitude estimation, because the accelerometers measure specific forces.

An acceleration signal for correcting the accelerometers reading is derived from the GPS velocity readings, and this information is more reliable as the vehicle velocity increases. For very low speeds, it is advisable to proceed without GPS assistance, since the GPS velocity readings can be quite noisy in this case.

Experimental results were reported, and a high cost/high performance navigator was employed for providing the ground truth. It is concluded that the GPS assisted AHRS implemented in this work presents smaller attitude error estimates that those provided by an standard AHRS, i.e., which does not possess acceleration correction.

7. ACKNOWLEDGEMENTS

The authors thanks CNPq and Finep for the financial support.

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