PERFORMANCE COMPARISON OF ATTITUDE DETERMINATION ALGORITHMS DEVELOPED TO RUN IN A MICROPROCESSOR ENVIRONMENT

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Abstract. *This paper presents a study concerning the practical implementation and the performance analysis of attitude determination procedure for artificial satellites implemented in a microprocessor environment. Three different algorithms, their experimental execution in microcontrollers, and the obtained results are detailed and analyzed. The main objective of the project is the development of an embedded system for application in Brazilian satellites. The project is financially and technically supported by the UNIESPAÇO Program of the Brazilian Space Agency - AEB. Our purpose is to obtain a processing system able to properly calculate the attitude using an architecture based on low-cost COTS components, that will include fault tolerance techniques in future project stages. The adopted Earth magnetic field sensor is a three axis solid state magnetometer (Honeywell HMC2003), the solar sensor is an experimental arrangement using a position sensitive detector (Hamamatsu S2044), and the processor is a PIC microcontroller (Microchip PIC24FJ64GA002). The performance analysis of the algorithms considers mainly the memory demands, the computation time, the precision of obtained results, and the system reliability. The relevance of this study is a consequence of the necessity of providing an efficient onboard computation in artificial satellites to ensure its accurate attitude, i.e., the spatial orientation precision of the satellite must be adequate to its operation requirements. The three algorithms considered in this study are the TRIAD, the Q-Method and the QUEST, and all of them use two measurement vectors given by the Earth magnetic field and the sun directions. The TRIAD algorithm is based on a deterministic strategy, and the Q-Method and QUEST algorithms are based on optimization techniques to obtain the satellite attitude. The attitude representation in the projet is parameterized using the quaternions. Concluding remarks and next steps of the project close the paper.*

Keywords: *attitude determination; artificial satellites; embedded systems*

1. INTRODUCTION

The researches involving the attitude determination of artificial satellites have become relevant in consequence of the crucial functionality of attitude control for diverse satellites missions. The main objective of this study is to implement and to analyze determination algorithms running in microcontroller based systems, taking into account the computation performance, memory, precision on the calculation, and also the reliability. An algorithm running in a microcontrolled system must be robust and reliable, including fault tolerance to ensure adequate performance during the satellite flight life. The precise space orientation of a satellite is a requirement of the mission payload to accomplish tasks of remote sensing, meteorology, data collecting, communication, etc., and it is also a requirement of the satellite itself.

The attitude is determined in a known reference system, and can be expressed and parameterized in different ways. The most common adopted parameters are the Euler angles and quaternions (Shuster, 1993). The attitude is usually represented by a rotation matrix, showing the relation between the known reference system and another reference system fixed in the satellite body. This matrix is named attitude matrix or director cosines matrix. In the attitude determination procedure, observations of known directions are considered. In this study, we consider the directions of the Earth magnetic field and the Sun. They are observed using appropriate sensor devices fixed in the satellite body, and their directions in the known reference system are supposed available. With this two pair of direction vectors, observed in the body reference and available in the known reference, it is possible to calculate the matrix attitude and to obtain the attitude parameters. The use of the directions of the Earth magnetic field and the Sun is an interesting issue in terms of sensing because there are low cost and precise sensor devices in the market for these specific measurements (HONEYWELL, 2004; HAMAMATSU, 2009).

This work deals with the selection and evaluation of performance of attitude determination algorithms to be applied in satellites three axis estabilization, using tests of these algorithms running in a development platform for microcontrollers-based systems (MPLAB, 2009).

2. THE EMBEDDED SYSTEM FOR ATTITUDE DETERMINATION

This work is part of a research project concerning the development of a low cost prototype of an embedded system for attitude determination of small size scientific satellites, in the context of the UNIESPAÇO Program, of the Brazilian Space Agency (Duarte *et alii*, 2008a; Duarte *et alii*, 2008b; Martins-Filho *et alii*, 2008). This system is based on the measurement of the magnetic field and the Sun direction. The project involves the whole attitude determination task and the facilities to integrate it to the attitude control system. The general scheme of the system is showed in Fig. 1, and the main functional modules are: magnetometer, sun sensor, signal conditioners, A/D converters, CPU (processor), memory, serial ports and timers.

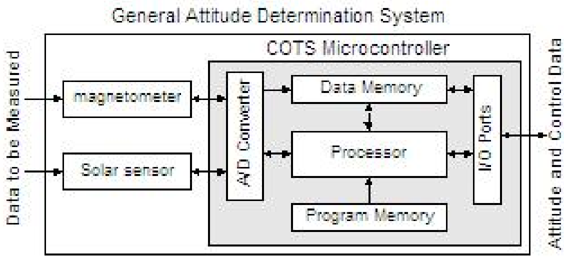

Figure 1. Scheme of the attitude determination system.

The adopted sensor for the components Earth magnetic field is the three axis magnetometer HMC2003 (Honeywell). This device can measure fields below $100\mu G$ (typical resolution bandwidth of $+/-2G$ (HONEYWELL, 2004). The HMC2003 signals represent the magnitude of the three components of the magnetic field, varying between 0 and 5 Volts, and taking the value $2.5V$ for $0G$ magnetic field. These signals are converted to digital by the microcontroller. The measurement of the sun direction will be done using an experiment set based on a photo-sensor PSD two-dimensional S2044 (HAMAMATSU, 2009). The two sensors are considered as absolute measurements sensors, taking the two directions in terms of the body reference system (Shuster and Natanson, 1993).

The data processing is done by a microcontroller 16-bits PIC, which comprises memories, serial communication interfaces, serial ports, timers, A/D converters and CPU (MICROCHIP, 2007). The microcontroller PIC can be considered adequate to the project requirements, and its programming environment facilitates the fast development of the necessary routines (MPLAB, 2009).

3. ATTITUDE REPRESENTATIONS

A rigid body attitude can be represented using different parameters, and the most popular in Space Engineering are the Euler angles and the quaternions. Considering the Euler angles, the attitude is represented by a matrix in terms of three independent parameters (Kaplan, 1976). The matrix is obtained through three basic consecutive rotations. The angles ϕ, θ and ψ that correspond to the rotation, are called Euler angles (see Fig. 2). The adopted sequence of rotations is: (i) $R_{z,\phi}$: rotation of ϕ around the axis Z of the known reference; (ii) $R_{x,\theta}$: rotation of θ around the axis X' (resulting of the first rotation); (iii) $R_{z,\psi}$: rotation of ψ around the axis Z'' (resulting of the second rotation).

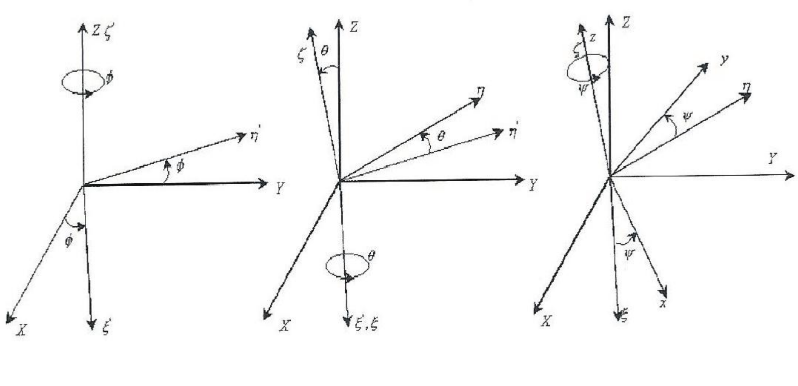

Figure 2. An example of sequence of rotations based on Euler angles.

The resulting rotation matrix A is given by:

$$
\begin{aligned}
A &= R_{z,\psi} \;\; R_{x,\theta} \;\; R_{z,\phi} \\
&= \begin{bmatrix} c\psi c\phi - s\psi c\theta s\phi & c\psi s\phi + s\psi c\theta c\phi & s\psi s\theta \\ -c\psi c\phi - c\psi c\theta s_{\phi} & -s\psi s\phi + c\psi c\theta c\phi & c\psi s\theta \\ s\theta s\phi & -s\theta c\phi & c\theta \end{bmatrix}
\end{aligned} \tag{1}
$$

where c represents the cosine function and s represents the sine function. The advantage of Euler angles for the attitude representation in comparison with the quaternions is the number of parameters, only three. The quaternions are 4-dimensional and the unitary norm condition. Nevertheless, Euler angles representation has problems with singularities in the motion modeling for some angles values (Suster, 1993). Quaternions can be seen as a generalization of the complex numbers concept, considering the 3-dimensional space (Kuipers, 2002; Bar-Itzhack and Oshman, 1985). As a complex number, the quaternion has an imaginary and a 3-dimensional real part, as given below:

$$\bar{q} = [q \quad q_1\hat{i} \quad q_2\hat{j} \quad q_3\hat{k}]^T \quad \text{or} \quad \bar{q} = [\cos\theta \;\; Q]^T \tag{2}$$

where $Q = \sin[n_1 \;\; n_2 \;\; n_3]^T$. A rotation of θ around a unitary vector n can be easily represented by $\bar{q}$.

The unitary module condition, which must be respected by the quaternion is:

$$\bar{q}^T\bar{q} = |Q|^2 + q^2 = 1 \tag{3}$$

Using the four parameters/components of the quaternion, the attitude representation becomes the shortest way to represent the attitude, without introducing problems of singularities in the attitude computation.

4. THE ATTITUDE DETERMINATION ALGORITHMS

The objective of the attitude determination procedure is to estimate the space orientation of the satellite, in a given instant, in terms of a known reference system, using measurements data obtained by sensors installed in the satellite body, and following a mathematical method to obtain the attitude matrix or attitude parameters (Shuster and Oh, 1981). One of the most important steps of the system development is the choice of determination algorithm to be adopted considering the microprocessor device, its limits in terms of memory, data processing speed, and computation precision. The three well-known algorithms of three axis attitude determination are: TRIAD, proposed by Harold D. Black, Q-Method, proposed by Paul Davenport, and QUEST (Quaternion Estimator) proposed by Malcolm D. Shuster (Shuster, 1978; Shuster and Oh, 1981; Shuster, 2007). The first one, TRIAD, constitutes a deterministic algorithm, and it has been used since the first satellites. The two others apply optimization methods to estimate the attitude. For all of them, two or more observation vectors are necessary as information about the space orientation that feeds the estimation procedure.

4.1 TRIAD algorithm

The TRIAD algorithm was implemented in systems of a number of satellites, e.g. Small Astronomy Satellite (SAS), Seasat, Atmosferic Explorer Missions (AEM) and Magsat (Shuster and Oh, 1981). The computation using TRIAD is the simplest and more direct way to obtain the attitude matrix, because it consists of the construction of two orthonormal vectors triads using two observation vectors and the two respective reference vectors.

In spite of the advantageous simplicity, the algorithm is strongly dependent of the observation noise level. One of the two pair observation-reference is taken as more confident, or considered less noise affected. It can be seen as limitation once it is not possible to assert about the best sensor information a priori. Another important limitation: even if more independent observation vectors are available, the algorithm can take only two vectors in each procedure computation

Two non parallel reference vectors, v_1 end v_2, and the respective observations w_1 and w_2, it is possible to determine the orthogonal matrix that fulfills the equations:

$$
\begin{aligned}
Av_1 &= w_1 \\
Av_2 &= w_2
\end{aligned} \tag{4}
$$

Therefore, two vectors triads are built as follow:

$$
\begin{aligned}
r_1 &= v_1 \\
r_2 &= (v_1 \times v_2)/(|v_1 \times v_2|) \\
r_3 &= (v_1 \times (v_1 \times v_2))/(|v_1 \times v_2|)
\end{aligned} \tag{5}
$$

$$
\begin{aligned}
s_1 &= w_1 \\
s_2 &= (w_1 \times w_2)/(|w_1 \times w_2|) \\
s_3 &= (w_1 \times (w_1 \times w_2))/(|w_1 \times w_2|)
\end{aligned} \tag{6}
$$

There is an orthogonal matrix A that satisfies:

$$Ar_i = s_i \quad i = 1, 2, 3 \tag{7}$$

given by:

$$A = \sum_{i=1}^{3} s_i r_i^T = [s_1 \; s_2 \; s_3][r_1 \; r_2 \; r_3]^T \tag{8}$$

The necessary and sufficient condition to make the matrix A the problem solution is:

$$v_1.v_2 = w_1.w_2 \tag{9}$$

The solution of TRIAD is not symmetric in terms of indexes 1 and 2. Part of information of the second vector is lost, and the precision becomes dependent of (w_1, v_1). In this case, the pair is taken as the most precise observation-reference vectors.

4.2 Q-Method algorithm

The Q-Method algorithm uses two observation vectors, and it can be described as follows (Shuster and Oh, 1981). Considering n unitary vectors w, $i = 1, \ldots, n$, where n corresponds to the number of independent sensors installed in the satellite body and wi the observation vectors. For each observed vector, a reference vector v_i is necessary to be used in the attitude computation. Therefore, the attitude matrix A associates the observed and reference one:

$$w_i = Av_i \tag{10}$$

A solution to this problem can be found using optimization methods. The matrix A is obtained when minimizing the error of Eq. (10). The Q-Method searches a solution that maximize the gain function $g(A)$ given by:

$$g(A) = 1 - L(A) = \sum_{i=1}^{n} a_i (w_i^T A v_i) \tag{11}$$

where a_i é the weight associated to each vector i, and $\sum_{i=1}^{n} a_i = 1$.

In order to make clear the procedure, it is convenient to Express A in terms of quaternions, as follows:

$$A(\bar{q}) = (q^2 - Q.Q)I - 2QQ^T + 2q\tilde{Q} \tag{12}$$

where I is the identity matrix and $\tilde{Q}$ é the anti symmetric matrix of Q, which is defined by:

$$\tilde{Q} = \begin{bmatrix} 0 & Q_3 & -Q_2 \\ -Q_3 & 0 & Q_1 \\ Q_2 & -Q_1 & 0 \end{bmatrix} \tag{13}$$

The cost function, in terms of quaternions, becomes:

$$\begin{array}{rcl} g(A) & = & (q^2 - Q.Q)trB^T + 2\,tr[Q.Q^T B^T] + 2q\,tr[\tilde{Q}B^T] \\ \text{or } \; g(\bar{q}) & = & \bar{q}^T K \bar{q} \end{array} \tag{14}$$

where K is a 4×4 matrix given by

$$K = \begin{bmatrix} S - \sigma I & Z \\ Z^T & \sigma \end{bmatrix} \tag{15}$$

with σ, S and Z defined as follows:

$$\begin{array}{rcl} \sigma & = & trB = \sum_{i=1}^{n} a_i w_i.v_i \\ S & = & B + B^T = \sum_{i=1}^{n} a_i (w_i v_i^T + v_i w_i^T) \\ Z & = & \sum_{i=1}^{n} a_i (w_i \times v_i) \end{array} \tag{16}$$

In this description, the problem of determining the attitude is reduced to finding the quaternion that maximizes the Eq. (14). It can be expressed by:

$$g'(\bar{q}) = \bar{q}^T K \bar{q} - \lambda\, \bar{q}^T \bar{q} \tag{17}$$

where λ is chosen in order to satisfy the constraint. And the expression becomes:

$$K\bar{q}_{opt} = \lambda\, \bar{q}_{opt} \tag{18}$$

that means the λ can be calculated as an eigenvalue of K and $\bar{q}_{opt}$ as an eigenvector of K. The attitude corresponds to the maximum eigenvalue λ_{max} and its eigenvector $\bar{q}_{opt}$, i.e. the optimal quaternion that provides the estimated attitude. These computations require intensive numerical processing. An alternative to the problem of determining the optimal eigenvalue and eigenvector is the method known as QUEST (Quaternion Estimator).

4.3 QUEST algorithm

The QUEST algorithm (Quaternion Estimator) estimates the optimal eigenvalue and eigenvector for the problem formulated in the Q-Method, without the necessity of an intensive processing. It was proposed by Shuster and Oh (1981), and it can be seen as an improvement of the Q-Method, searching the minimization of a cost function:

$$L(A) = \frac{1}{2}\sum_{i=1}^{n} a_i(\hat{W}_i - A\hat{V}_i)^2 \tag{19}$$

maximize the gain function

$$g(\bar{q}) = \sum_{i=1}^{n} a_i \hat{W}_i^T A \hat{V}_i = \lambda_{opt} \tag{20}$$

The solution for the optimal value of λ is:

$$\lambda_{opt} \approx \sum_{i=1}^{n} a_i - L(A) \tag{21}$$

An approximate solution, considering that the values of $L(A)$ is quite small, is given by:

$$\lambda_{opt} \approx \sum_{i=1}^{n} a_i \tag{22}$$

This approximation provides a result sufficiently precise for many applications. Taken the estimated eigenvalue, we compute the respective eigenvector, i.e. the quaternion that represents the estimated attitude. A simplified solution for the eigenvector problem can be given by:

$$p = \frac{\bar{q}}{q_4} \tag{23}$$

A alternative to the matrix inversion is a approximation using Gauss method or other linear equations systems to solve the following problem:

$$p = [(\lambda_{opt})I - S]^{-1} Z \tag{24}$$

The quaternion is finally obtained using the relations:

$$\bar{q} = \frac{1}{\sqrt{1 + p^T p}} \begin{bmatrix} p \\ 1 \end{bmatrix} \tag{25}$$

However, this method presents a problem when the rotation angles is π rad, i.e. $p^T p = 1$. For this case, it is not possible to obtain the optimal quaternion (Shuster and Oh, 1981). Nevertheless, for implementation purposes, a condition test $p^T p = 1$ must be included, and if this condition becomes true, the quaternion must be asserted as $[0\ \ 0\ \ 0\ \ 1]^T$.

5. RESULTS

The three algorithms have been implemented in ANSI-C, and the attitude determination has been performed for comparative results analysis. The provided inputs are the reference vectors (Sun direction and magnetic field direction) and the measurements vectors, taken from a fixed position/orientation in satellite body. The computed attitude results are presented in two distinct representations: attitude matrix and quaternions.

A routine of conversion between the different attitude representations (Euler angles, quaternions, matrix) was also developed, in order to evaluate the angular precision of the obtained results.

5.1 Evaluation tests and results analysis

Different tests were accomplished to evaluate the algorithms performance. The first one is described below:

Test 0 : The objective of this test is to verify if the three algorithms are obtaining the attitude, and compare the performance in terms of computing time and numerical errors of each algorithm. The attitude matrix was generated randomly, and using the reference vectors the measurements were obtained.

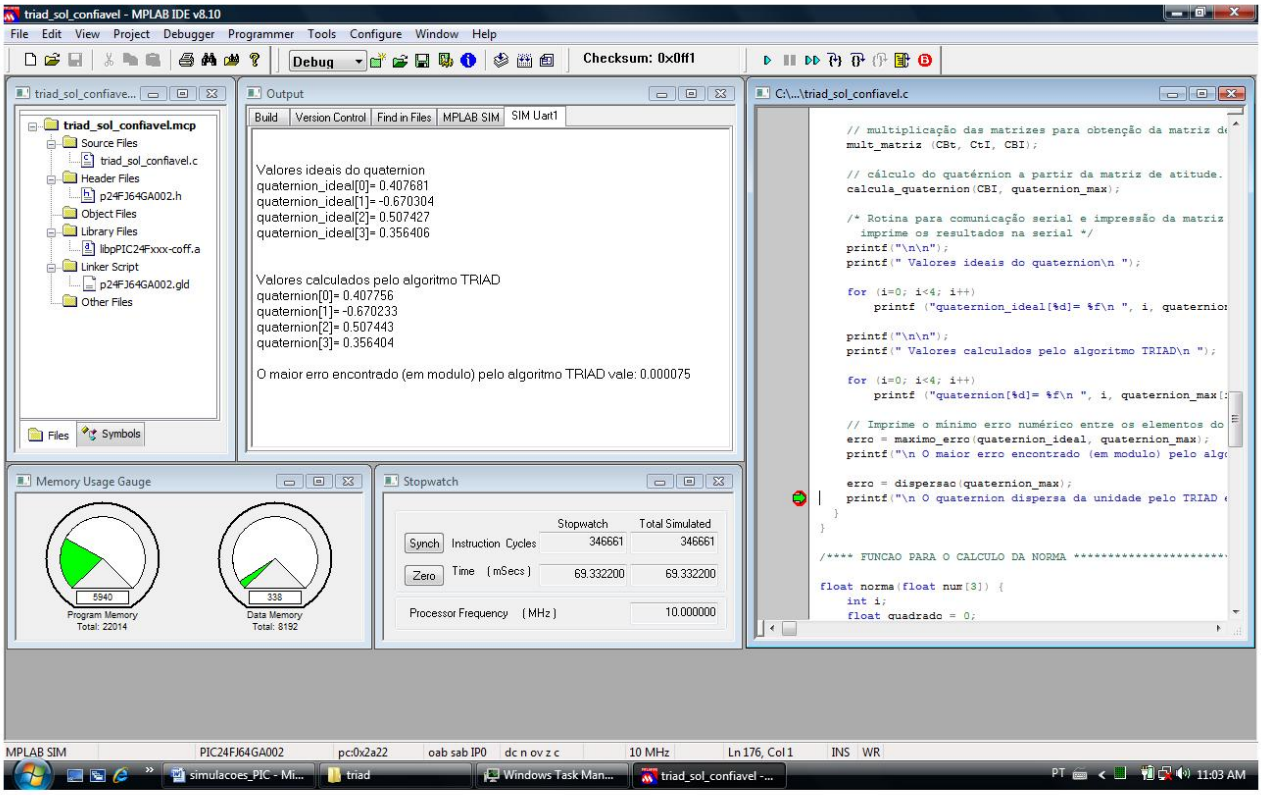

Figure 3. An example of screen of MPLAB turning TRIAD algorithm.

Table 1. Comparison of performance of algorithms implemented in a PIC24FJ64GA002, with $10MHz$ clock, in terms of quaternions and Euler angles (test 0).

Algorithm	Computing time	Clock cycles	Program memory	Data Data	max. error (abs.) quaternion	max. error (abs.) Euler angles
Q-method	$113.6ms$	$567,809$	$7109Bytes$	$338Bytes$	7.1×10^{-5}	3.3×10^{-3}
QUEST	$70.8ms$	$353,959$	$6792Bytes$	$350Bytes$	7.2×10^{-5}	3.3×10^{-3}
TRIAD (magn.)	$69.4ms$	$347,024$	$5940Bytes$	$338Bytes$	6.8×10^{-5}	3.2×10^{-3}
TRIAD (sun)	$69.3ms$	$346,661$	$5940Bytes$	$338Bytes$	7.5×10^{-5}	4.5×10^{-3}

The algorithms were implemented taking into account the adaptation to the microcontroller Microchip PIC24FJ64-GA002. The development environment used to emulate the microcontroller was the MPLAB IDE v8.10 with the Microchip C30 compiler in a non optimized mode. This environment provides data like profiling, processing time, data and program memory used and the maximum error for each implementation. The comparative results of Test 0 for the three algorithms are shown in Tab. 1.

It is possible to observe that TRIAD algorithm is the fastest one. TRIAD is also the algorithm that requires less program memory, and its requirement of data memory is one of the lowest. Nevertheless, as a deterministic method, TRIAD does not provide the less numerical error result, believably the essencial feature for the studied application. Q-Method and QUEST obtain much better errors results. TRIAD presents another drawback: its precision is dependent of the choice of one of the sensors measurements, taken as the most precise. We conclude that these facts eliminate TRIAD for the embedded attitude determination system, since during the flight, occasional errors can appear due to space environment producing a new uncertainty degree to the problem of finding the most reliable measured vector. Shuster (2007) presents a optimized version of TRIAD, where the algorithm obtain better results. The proposed solution is based on linear combination of estimated attitudes obtained using TRIAD, and taking different choices of measurements vectors. Better precision can be obtained, against costs of higher computing time, and it confirms the elimination of TRIAD candidate.

The other two algorithms, QUEST and Q-Method, both obtain close results in terms of computing precision. QUEST requires less memory for the program itself, but it demands 4% more data memory. Considering the computing time, the advantage of QUEST is approximately 60%, as already previewed by Shuster when he proposed the simplification on obtaining the eigenvalue and the eigenvector.

The logical conclusion based in this test results, is that the best candidate to the final implementation in the attitude determination embedded system is the QUEST algorithm. The main reason that conduct us towards this conclusion are the computing time and the required program memory, which are decisive for the performance of the embedded system.

Following this basic test, others tests were proposed in order to verify the reliability of the implemented algorithms. The 3 additional tests are defined below:

Test 1 : Verify the functioning and the precision of the algorithms. For $\phi = 10°$, $\theta = -10°$, $\psi = 90°$, the corresponding attitude matrix was computed, as well the quaternion. The matrix was used to generate the perfect measurements vectors (without any noise), by $w_i = Av_i$. The respective quaternion was also obtained to verify the error in terms of quaternions. The inputs for the three algorithms were the magnetic field and sun directions vectors simulated as the sensors observations. The results for the three algorithms were exactly the same, as expected because any noise was included in the measurements. In terms of quaternions, the absolute error was 0.000001.

Test 2 : Verify the effect on the maximum error of each quaternion component, produced by each algorithm due to changes in the attitude angles. For angles: $\phi = 10.5$, $\theta = -10.5$ and $\psi = 89.5$ it was calculated the quaternion, that hereafter we will refer as reference quaternion. It was used as input to the three algorithms in order to calculate the attitude and the error between the calculated and reference quaternion. The entries for the three algorithms of vectors sun and measured mag, obtained for $\phi = 10°$, $\theta = -10°$, $\psi = 90°$ were maintained. The run test result of the of the three implemented algorithms was the same. As expected, the biggest error among the components of quaternion calculated and the reference quaternion was equal to 0.003840. This value was obtained by the three implementations. This occurred because we used the same matrix to generate the sun measured vector and the magnetic field measured vector from the respective vectors of reference. [1]

Test 3 : Verify the angle deviation due to changes in the measurements of the magnetometer, adding $1mG$ in each component of the magnetic field (in terms of voltage, it is equivalent to $1mV$ in HMC2003 measurements), maintaining the reference quaternion of test 1. For the QUEST and the Q-Method algorithms, the biggest absolute error among the four components of the quaternion was 0.000499. For the TRIAD, taking the magnetometer as the most reliable sensor, the biggest error was 0.000675 (in terms of quaternion components) as expected. For the TRIAD, taking the sun sensor as the most confident sensor, the biggest error was 0.000322. In terms of Euler angles, the estimated attitude was $\phi = 10.468155$, $\theta = -10.004837$, $\psi = 89.569298$ for QUEST algorithm and $\phi = 10.467773$, $\theta = -10.00756$, $\psi = 89.569756$ for Q-Method.

Thus we could observe how a deviation of 1 mG, which is considered as a big noise deviation for crude measures of terrestrial magnetic field in a satellite in low earth orbit, applied to each component of the vector magnetic field obtained by the magnetometer, results of angular deviation in obtaining attitude. The largest deviation for the QUEST algorithm was $0.4682°$ and the highest deviation in the Q-algorithm method was $0.4678°$. As expected the precision of the Q-method is better than the QUEST algorithm, because the latter considers approximations to estimate the optimal quaternion. However, the processing time required for the attitude computation in QUEST is much smaller than the Q-method, not justifying the choice of Q-method and confirming that the QUEST selection as the best candidate for implementation in an embedded system to attitude determination of low orbit satellites.

6. CONCLUSION

This work has the purpose of implementing a reliable algorithm to estimate the satellite attitude, in the context of a project concerning the development of a low cost device based on microcontroller processing, and magnetometer and sun sensor measurements. The paper presented the details of the three selected algorithms, and discussed the implementation and testing of them in C programming language, using a development environment emulating the PIC24FJ64GA002 processor. The conclusive analysis points to the selection of the QUEST algorithm, that presents the smallest computing time, and less sensibility to the noise affecting negatively the sensors measurement data.

The next steps of the project include: (i) the study of the sensors bias, and the calibration procedures to compensate this bias; (ii) the study of sensors signal filtering, in order to reduce the effect of measurement noises; (iii) integration of the system, in terms of hardware, i.e. the processor, the magnetometers, the sun sensor, and so on; (iv) implementation of the fault tolerance techniques studied in another initiative of the project team.

7. ACKNOWLEDGEMENTS

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