ON THE INFLUENCE OF A COMPANION MODE ON THE NONLINEAR OSCILLATIONS OF FLUID-FILLED CYLINDRICAL SHELLS

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Abstract. *The aim of this work is to investigate the influence of the companion mode on the nonlinear vibrations of a simply-supported fluid-filled cylindrical shell under time dependent loads. Donnell nonlinear shallow shell theory in terms of in-plane and transversal displacements is used to study the nonlinear vibrations of the shell. The fluid is modeled as non-viscous and incompressible. The irrotational motion of the fluid is described by a velocity potential which satisfies the Laplace equation and the proper boundary conditions at the shell-fluid interface. To discretize the partial differential equations of motion first, using a perturbation procedure, a general expression for the transversal displacement field is obtained identifying the coupling between linear and companion modes through quadratic and cubic terms in the equations of motion. Then, a particular solution is selected which ensures the convergence of the response up to very large deflection. Finally, the in-plane displacements are obtained as a function of the transversal displacement by solving the in-plane equations analytically. So the proposed solution satisfies all the boundary, continuity and symmetry conditions. By substituting this solution into the equation of motion in the transversal direction and applying the Galerkin method, a discrete system in time domain is obtained and solved by numerical techniques. The influence played by the internal fluid on the natural frequencies, critical loads and bifurcations is examined. A detailed bifurcation analysis using continuation techniques shows the influence of the companion mode on the resonance curves and stability of the steady-state response in the main resonance region.*

Keywords*: cylindrical shell, low-dimensional models, companion mode, nonlinear analysis, time dependent loads***.**

1. INTRODUCTION

Thin-walled cylindrical shells are one of the most common structural elements with applications in nearly all engineering fields. The study of the nonlinear vibrations of cylindrical shells began in the middle of the last century with the works by Chu (1961), Nowinski (1963), Evensen (1963, 1967), Olson (1965), Dowell and Ventres (1968), Atluri (1972) and Chen and Babcock (1975), among others. In these works either the Ritz or the Galerkin method are used to discretize the shell. In all cases, either a simply-supported shell or an infinitely long shell with a periodic displacement field in the axial direction is considered. For this a modal expansion for the displacement field is necessary. The development of consistent modal solutions capable of describing the main modal interactions observed in cylindrical shells has received much attention in literature. A detailed review of this subject, including more than 350 papers, was published in 2003 by Amabili and Païdoussis (2003).

Chu (1961) and Nowinski (1963) observed that the natural frequency increases with the vibration amplitude, a behavior observed previously for thin plates and beams. This hardening behavior was questioned by Evensen (1963), who showed that the analyses of Nowinski (1963) and Chu (1961) were not accurate, due to the fact that the modal solution did not satisfy the shell boundary conditions. The work of Evensen (1963) was confirmed by Olson (1965) who, contrary to the known literature results, observed experimentally a frequency-amplitude relation different from that observed by Chu (1961) and Nowinski (1963). Olson (1965) showed that cylindrical shells exhibit a softening behavior, that is, the nonlinear frequency decreases with increasing vibration amplitude. Evensen (1967) observed, through an experimental analysis, the same behavior as that observed by Olson (1965), and obtained a good correlation between theoretical and experimental results. This was possible due to the correct consideration of the nonlinear modal coupling between the basic linear vibration mode and a circumferentially axi-symmetric mode. The axi-symmetric mode in the modal expansion was chosen in such a way that the transverse displacement becomes null at the boundaries. Knowing the importance of axi-symmetric modes, Dowell and Ventres (1968) and Atluri (1972), both using different modal expansions, but containing axi-symmetric modes, obtained a frequency-amplitude relation with hardening behavior. The error was due to the incorrect representation of axi-symmetric modes on the modal expansion, as showed by Varadan et al. (1989).

The consideration of axi-symmetric modes is necessary to describe the inherent in-out asymmetry observed in the nonlinear vibration of the shell at large amplitudes. The symmetry-breaking effect of the axi-symmetric component was also observed in the nonlinear post-buckling analysis of cylindrical shells under lateral pressure and axial compression (Hunt et al., 1986). This leads to strong quadratic nonlinearities in the equations of motion, showing the importance of this modal coupling on the shell nonlinear behavior.

In some forced vibration experiment, the presence of traveling waves in the circumferential direction was observed. This phenomenon was modeled by considering together with the vibration mode directly excited by the external harmonic forcing, named driven mode in the technical literature, another mode with the same wave numbers but with a different phase (sine and cosine functions around the shell circumference), referred to as companion mode. Chen and Babcock (1975), using a perturbation techniques, studied the participation of both driven and companion modes in the dynamic response of the cylindrical shell, corroborating the experimental results. This is one of the main contributions to the study of the companion mode on the modal solution for transversal displacements of cylindrical shells. So it was concluded that both driven and companion modes plus axi-symmetric modes should be included in the forced vibration analysis of cylindrical shell. However, the modal expansion is usually obtained in an empirical way, without a mathematical basis.

Gonçalves and Batista (1988), using a perturbation technique derived a mathematically consistent displacement field for an infinitely long shell. Through this methodology all possible modes that couple with the driven mode through the quadratic and cubic nonlinearities are identified. Using a finite number of modes and imposing the boundary conditions, the authors deduced a consistent reduced-order model and used it to study the nonlinear free vibrations of empty and fluid-filled simply supported cylindrical shell. They showed that in both cases a softening behavior is observed and that the fluid increases the softening characteristic of the problem. They did not consider the participation of the companion mode.

Amabili et. al. (1998), considering the participation of the companion mode, presented a detailed analysis of the nonlinear vibrations of fluid-filled cylindrical shells. The modal solution was, based on these previous works, satisfied both the boundary conditions for the transversal displacements and the continuity of the displacements in the circumferential direction. The boundary conditions for the axial displacements were satisfied on the average. It was observed that, for the laterally excited shell, the participation of the companion mode is important in the main resonance region. They showed that, due to the interaction between driven and companion modes, there are certain frequencies ranges where no stable forced solution occurs. Amabili and co-workers in the following years developed more refined modal expansions and studied both numerically and experimentally a large number of shell problems. Their contributions can be found in the recent book by Amabili (2008).

Recently, the authors of this paper, based on the perturbation technique proposed by Gonçalves and Batista (1988), proposed a new solution method for cylindrical shells well adapted to the derivation of efficient reduced order models (Gonçalves et. al. 2008). In the present work this methodology is extended to include the influence of the companion mode on the modal expansion. Based on the obtained modal expansion, a reduced order model is obtained to study the nonlinear vibrations of a simply supported fluid-filled cylindrical shell subjected to lateral harmonic pressure. The consistent modal solution for the displacements field for u, v and w satisfies all boundary conditions and continuity for a simply supported cylindrical shell, including the nonlinear boundary conditions of the shell not satisfied in previous works (Gonçalves et. al. 2008). The obtained results pay attention to the influence of the companion mode on the resonance curves of the cylindrical shell.

2. PROBLEM FORMULATION

2.1 Shell equations

Consider a cylindrical of radius R, thickness h and length L, made of a linear elastic material with Young's modulus E, Poisson coefficient υ and mass density ρ. The three displacement components u, v and w are related to the cylindrical co-ordinate system x, θ and z, as shown in Fig. 1.

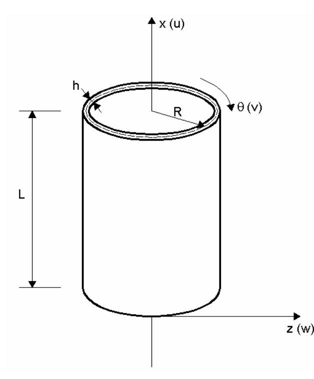


Figure 1. Shell geometry and coordinate system

For an isotropic shell the constitutive law is given by:

$$\begin{Bmatrix} \bar{\sigma}_x \\ \bar{\sigma}_\theta \\ \bar{\tau}_{x\theta} \end{Bmatrix} = \frac{E}{(1-\upsilon^2)} \begin{bmatrix} 1 & \upsilon & 0 \\ \upsilon & 1 & 0 \\ 0 & 0 & \frac{(1-\upsilon)}{2} \end{bmatrix} \begin{Bmatrix} \bar{\varepsilon}_x \\ \bar{\varepsilon}_\theta \\ \bar{\gamma}_{x\theta} \end{Bmatrix} \tag{1}$$

The shell deformations at an arbitrary point are given in terms of the middle surface strain and change of curvature components by:

$$\left[\bar{\varepsilon}_x, \bar{\varepsilon}_\theta, \bar{\gamma}_{x\theta}\right] = \left[\varepsilon_x + z\,\chi_x, \varepsilon_\theta + z\chi_\theta, \gamma_{x\theta} + 2\,z\chi_{x\theta}\right] \tag{2}$$

These middle surface quantities are given in terms of the displacement components, according to Donnell shallow shell theory, by:

$$\begin{gathered} \left[\varepsilon_x, \varepsilon_\theta, \gamma_{x\theta}\right] = \left[u_{,x} + \frac{1}{2} w_{,x}^2, \frac{1}{R}\left(v_{,\theta} + w\right) + \frac{1}{2R^2} w_{,\theta}^2, v_{,x} + \frac{1}{R}\left(u_{,\theta} + w_{,x}\, w_{,\theta}\right)\right] \\ \left[\chi_x, \chi_\theta, \chi_{x\theta}\right] = \left[-w_{,xx}, -\frac{1}{R^2} w_{,\theta\theta}, -\frac{1}{R} w_{,x\theta}\right] \end{gathered} \tag{3}$$

The shell is subjected to a harmonic lateral pressure of the form:

$$p = P\,h\cos\left(n\,\theta\right)\sin\left(\frac{m\pi\,x}{L}\right)\cos\left(\omega t\right) \tag{4}$$

where P is the pressure magnitude; n and m are, respectively, the number of waves in the circumferential direction and the number of half-waves in the axial direction, ω is the excitation frequency and t is time.

The nonlinear equations of motion, considering only the transversal inertia and damping forces, are given in terms of the force and moments resultants by:

$$N_{x,x} + \frac{N_{x\theta,\theta}}{R} = 0 \tag{5a}$$

$$N_{x\theta,x} + \frac{N_{\theta,\theta}}{R} = 0 \tag{5b}$$

$$\rho\,h\,\ddot{w} + 2\eta\,\rho\,\omega_0\,h\,\dot{w} - p - p_H - \frac{1}{R^2}\left[R\,M_{x,xx} + 2\,M_{x\theta,x\theta} + \frac{M_{\theta,\theta\theta}}{R} + R\,N_x\,w_{,xx} + N_\theta\left(\frac{w_{,\theta\theta}}{R} - 1\right) + 2\,N_{x\theta}\ \mathrm{w}_{,x\theta}\right] = 0 \tag{5c}$$

where η is the linear viscous damping, ω_0 is the lowest vibration frequency of the empty shell, p_H is the hydrodynamic forces of internal fluid and the force and moments resultants are obtained by the integration of the stress components along the shell thickness as follows:

$$\left[N_x, N_\theta, N_{x\theta}, M_x, M_\theta, M_{x\theta}\right] = \int_{-h/2}^{h/2} \left[\bar{\sigma}_x, \bar{\sigma}_\theta, \bar{\tau}_{x\theta}, z\,\bar{\sigma}_x, z\,\bar{\sigma}_\theta, z\,\bar{\tau}_{x\theta}\right] dz \tag{6}$$

For a simply-supported shell, the following boundary conditions must be satisfied:

$$v(0,\theta) = v(L,\theta) = 0 \tag{7a}$$

$$w(0,\theta) = w(L,\theta) = 0 \tag{7b}$$

$$M_x(0,\theta) = M_x(L,\theta) = 0 \tag{7c}$$

$$N_x(0,\theta) = N_x(L,\theta) = 0 \tag{7d}$$

The boundary condition, Eq. (7d), is a nonlinear boundary condition when written in terms of the displacements, that is:

$$N_x = \frac{E\,h}{1-\nu^2}\left[u_{,x} + \frac{1}{2}w_{,x}^2 + \frac{\upsilon}{R}\left(v_{,\theta} + w + \frac{1}{2R}w_{,\theta}^2\right)\right] \tag{8}$$

The displacement field, in this work, is also required to satisfy the following conditions:

$$u(L/2,\theta)=0 \qquad \text{and} \qquad v(x,0)=v(x,2\pi) \tag{9}$$

In the foregoing, the following non-dimensional parameters have been introduced:

$$W = \frac{w}{h};\; \varepsilon = \frac{x}{L};\; \tau = \omega_o t;\; \Omega = \frac{\omega}{\omega_o};\; \Gamma = \frac{P}{P_{cr}};\; P_{cr} = \frac{E\,h}{R}\left[\frac{\left[(\pi R/L)^2+n^2\right]^2}{n^2}\frac{(h/R)^2}{12\left(1-\upsilon^2\right)} + \frac{(\pi R/L)^4}{n^2\left[(\pi R/L)^2+n^2\right]^2}\right] \tag{10}$$

Here P_{cr} is the classical static critical lateral pressure of the shell.

2.2 General solution of the shell displacement field by a perturbation

The numerical model is developed by expanding the transversal displacement component *w* in series in the circumferential and axial variables. From previous investigations on modal solutions for the nonlinear analysis of cylindrical shells under axial loads (Hunt et al. 1986; Gonçalves and Batista, 1988; Gonçalves and Del Prado, 2002; Awrejcewicz and Krys'ko, 2003; Gonçalves et. al. 2008) it is observed that, in order to obtain a consistent modeling with a limited number of modes, the sum of shape functions for the displacements must express the nonlinear coupling between the modes and describe consistently the unstable post-buckling response of the shell as well as the correct frequency-amplitude relation.

Gonçalves and Del Prado (2005) and Gonçalves et. al. (2008) trough perturbation techniques and satisfying the boundary conditions, deduced a consistent modal solution for the transversal displacements of a simply supported cylindrical as defined in Eq. (7b,c). Using these works as basis, it was deduced a new modal solution including the companion mode. This is necessary because, in the linear analysis of a perfect shell, there are two different modes with the same natural frequency.

Table 1 shows the first four vibration modes for an empty cylindrical shell obtained with a finite element program ABAQUS. It is considered a shell with radius R = 0.2 m, length L = 0.4 m and thickness h = 0.002 m. The material is considered as homogeneous and isotropic with elasticity modulus E = 210 GPa, Poisson ratio ν = 0.3 and density ρ = 7850 kg/m³. It is observed that the natural frequencies occur in pairs with the same number of longitudinal half-waves (*m*) and circumferential waves (*n*). One can observe that each two modes with the same natural frequency have different angular orientations. If one is described by *cos(nθ)* the other is described by *sin(nθ)*. This phenomenon is related to the axial symmetry of the shell geometry. So, in this case one has 1:1 internal resonance. In the harmonically forced system, if one mode is directly driven, the other, the companion mode, can be excited through the nonlinear coupling terms. The simultaneous excitation of both modes gives rise to traveling waves in the circumferential direction.

So, in order to use perturbation techniques, analysis should be started by considering both the driven and its companion mode as the seed or basic initial solution, that is (Gonçalves and Del Prado, 2005):

$$W = \zeta_{11}(\tau)\cos(n\theta)\,\sin(m\pi\varepsilon) + \xi_{11}(\tau)\sin(n\theta)\,\sin(m\pi\varepsilon) \tag{11}$$

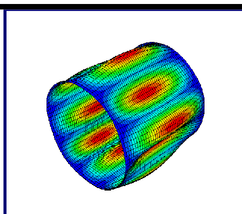
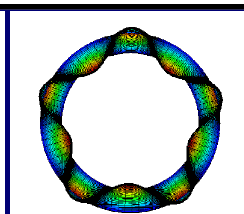
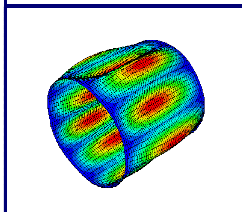
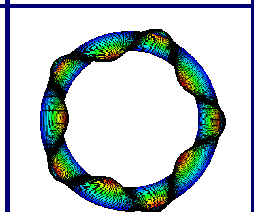
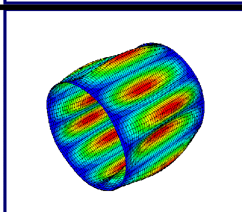
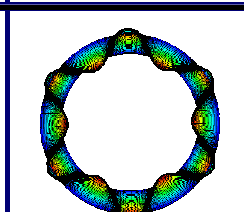
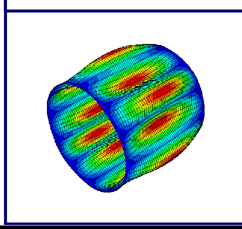
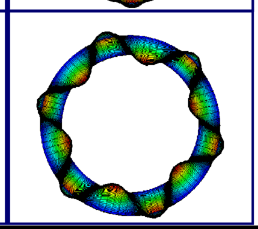
By applying the perturbation procedures described in Gonçalves et. al. (2008) and by considering the boundary conditions for a simply supported cylindrical shell, Eq. (7b,c), it is found a modal solution that accounts for the internal resonance. It is given by:

$$\begin{aligned} W = {} & \sum_{i=1,3,5}^{\infty}\sum_{j=1,3,5}^{\infty}\left[\zeta_{ij}(\tau)\cos(in\theta) + \xi_{ij}(\tau)\sin(in\theta)\right]\sin(jm\pi\varepsilon) \\ & + \sum_{\alpha=0,2,4}^{\infty}\sum_{\beta=0}^{\infty}\left[\zeta_{\alpha(2+6\beta)}(\tau)\cos(\alpha n\theta) + \xi_{\alpha(2+6\beta)}(\tau)\sin(\alpha n\theta)\right]\times \\ & \left\{-\frac{(3+6\beta)}{(4+12\beta)}cos(6\beta m\pi\varepsilon) + \cos((2+6\beta)m\pi\varepsilon) - \frac{(1+6\beta)}{(4+12\beta)}\cos((4+6\beta)m\pi\varepsilon)\right\} \end{aligned} \tag{12}$$

From studies of convergence related to the driven modes, $\zeta_{ij}(\tau)$ presented in Gonçalves et. al. (2008), the following 11 dof modal expansion, including the companion mode participation, $\xi_{ij}(\tau)\neq 0$, is proposed:

$$\begin{aligned} W = & \left[\zeta_{11}(\tau)\cos(n\theta)+\xi_{11}(\tau)\sin(n\theta)+\zeta_{31}(\tau)\cos(3n\theta)+\xi_{31}(\tau)\sin(3n\theta)\right]\sin(m\pi\varepsilon) \\ & +\left[\zeta_{13}(\tau)\cos(n\theta)+\xi_{13}(\tau)\sin(n\theta)+\zeta_{33}(\tau)\cos(3n\theta)+\xi_{33}(\tau)\sin(3n\theta)\right]\sin(3m\pi\varepsilon) \\ & +\left[\zeta_{02}(\tau)+\zeta_{22}(\tau)\cos(2n\theta)+\xi_{22}(\tau)\sin(2n\theta)\right]\left[-\frac{3}{4}+\cos(2m\pi\varepsilon)-\frac{1}{4}\cos(4m\pi\varepsilon)\right] \end{aligned} \tag{13}$$

Table 1. Natural frequencies and vibration modes for the empty cylindrical shell.

Vibration mode	Frequency (Hz)	Graphic representation	
1º	490,82		
2º	490,82		
3º	539,63		
4º	539,63		

The in-plane displacements u and v are obtained by substituting Eq. (13) into the in-plane equilibrium Eq. (5a) and Eq. (5.b) and solving the system of linear partial differential equations in u and v and imposing the relevant boundary, symmetry and continuity conditions. Based on this procedure one selects the necessary number of in-plane modes and write their modal amplitudes in terms of the modal amplitudes ζ_{ij} and ξ_{ij} in Eq. (13) (Gonçalves et al. 2008). It is important to notice that the harmonic terms in the modal expansion for u and v derived by this procedure are similar to those derived by the perturbation procedure. Finally, by substituting the adopted expansion for the transversal displacement w together with the obtained expressions for u and v into the equation of motion in the transversal direction, Eq. (5c), and by applying the standard Galerkin method, a consistent discretized system of ordinary differential equations of motion is derived.

2.3 Fluid equations

The fluid is assumed to be incompressible and non-viscous and the flow to be isentropic and irrotational. From the linearized Bernoulli equation, the hydrodynamic pressure is given by:

$$p_H = -\rho_F \left.\dot{\phi}\right|_{r=R} \tag{14}$$

where ρ_F is the density of the internal fluid.

The velocity potential of the fluid, ϕ, should satisfy the following Laplace equation:

$$\phi_{,rr} + \frac{1}{r}\phi_{,r} + \frac{1}{r^2}\phi_{,\theta\theta} + \phi_{,xx} = 0 \tag{15}$$

and the boundary conditions in the fluid and structure interface.

Considering null fluid potential at the both ends of the shell and that the transversal velocity of the fluid should be equal to the shell velocity at the shell-fluid interface, the fluid potential should satisfy the following boundary conditions:

$$\phi = 0\Big|_{x=0,L} \quad \text{and} \quad \left.\frac{\partial \phi}{\partial r} = \dot{w}\right|_{r=R} \tag{16}$$

The following potential velocity:

$$\phi = \sum_{i=0}^{3}\sum_{\overline{m}=1}^{\overline{M}} \left[A_{i\overline{m}}(\tau)\cos(in\theta) + B_{i\overline{m}}(\tau)\sin(in\theta)\right]\cos\left(\frac{\pi(2\overline{m}-1)(L+2x)}{2L}\right)\mathrm{I}_{i\times n}\left(\frac{(2\overline{m}-1)\pi r}{2L}\right) \tag{17}$$

where I_n is the modified Bessel function of first class and order n. It was first used by Kim et. al. (2004); it satisfies the boundary conditions at $x=0,L$, Eq. (16), and the Laplace equation, Eq. (12).

The amplitude of each mode $A_{i\overline{m}}(\tau)$, $B_{i\overline{m}}(\tau)$ are determined by applying the impenetrability condition, second condition in Eq. (16), and by using the Galerkin method and assuming as weight function the trigonometric term of Eq. (17). The modal amplitudes can be written as functions of the lateral displacements of the shell as:

$$A_{i\overline{m}}(\tau), B_{i\overline{m}}(\tau) = \left.\frac{\int_0^L \left[\left(\frac{\partial w}{\partial t}\right)\cos\left(\frac{\pi(2\overline{m}-1)(L+2x)}{2L}\right)\right]dx}{\int_0^L \left[\left(\frac{\partial \phi}{\partial r}\right)\cos\left(\frac{\pi(2\overline{m}-1)(L+2x)}{2L}\right)\right]dx}\right|_{r=R} \tag{18}$$

Finally, the hydrodynamic pressure is given by:

$$\begin{aligned} p_H = -\rho_F \sum_{i=0}^{3}\sum_{\overline{m}=1}^{\overline{M}} \Bigg\{ & \left(\frac{\partial A_{i\overline{m}}(\tau)}{\partial \tau} A_{i\overline{m}}(\tau)\cos(in\theta) + \frac{\partial B_{i\overline{m}}(\tau)}{\partial \tau} B_{i\overline{m}}(\tau)\sin(in\theta)\right) \\ & \times\cos\left(\frac{\pi(2\overline{m}-1)(L+2x)}{2L}\right)\mathrm{I}_{i\times n}\left(\frac{(2\overline{m}-1)\pi R}{2L}\right)\Bigg\} \end{aligned} \tag{19}$$

3. NUMERICAL RESULTS

A simply supported cylindrical shell with the same geometric and physical properties given previously in item 2.2 is considered. For this geometry, the combination m = 1 e n = 5 gives the smallest buckling and natural frequency (Gonçalves e Del Prado, 2005) as can be verified in the first two vibration modes of Table 1. It is considered an internal fluid with density ρ_F= 1000 kg/m³ and the viscous damping considered is η = 0.001.

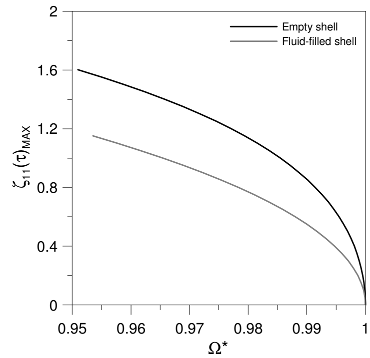
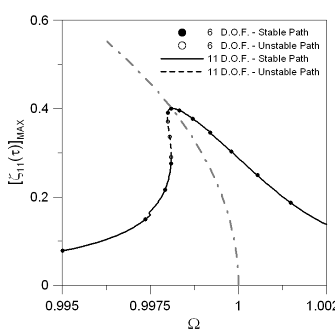


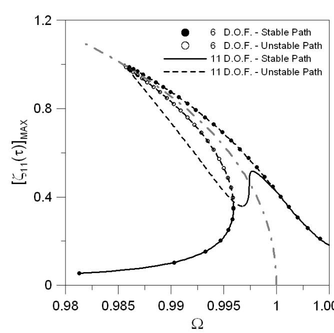
Figure 2. Frequency-amplitude relations for the fluid-filled and empty shell. ($\Gamma = 0$, $\eta = 0.0$).

In Fig. 2 the frequency-amplitude relations (backbone curves) for the empty and fluid-filled shell are compared. These curves are obtained by the shooting method (Li e Xu, 2005), with no necessity of temporal discretization of the modal amplitudes. Amplitude $\zeta_{11}(\tau)_{MAX}$ is the maximum amplitude. The non-dimensional frequency of the shell, Ω^*, is defined as the ratio between the linear and nonlinear natural frequency. As can be seen, the frequency decreases with increasing vibration amplitude. This softening behavior is typical of cylindrical shell and is intensified by the internal fluid.

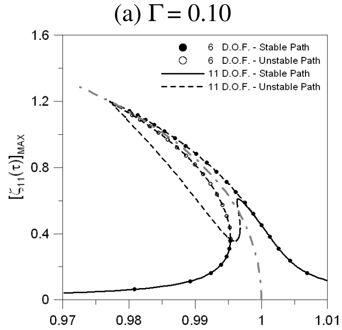
Figures 3 and 4 show the resonance curves for respectively empty and fluid-filled cylindrical shell, subjected to a harmonic lateral pressure. Two different modal expansions, one with 6 dof ($\xi_{ij}(\tau)=0$ in Eq. (13)) and one with 11 dof ($\xi_{ij}(\tau)\neq 0$ in Eq. (13)) are considered. Continuation techniques associated with the Newton-Raphson method are used to obtain these results. These figures show the variation of the coordinate $\zeta_{11}(\tau)_{MAX}$ with the normalized excitation frequency Ω. The excitation frequency parameter is normalized with respect to the natural frequency of the empty shell. The natural frequency of the empty shell is higher than that of the fluid-filled shell due to the added mass of the fluid. In these figures, continuous lines represent stable solutions and dotted lines represent unstable solutions. Symbols (●) and (○) represent stable and unstable solutions considering only the nonlinear driven mode in Eq. (13) as $\xi_{ij}(\tau) = 0$. As can be seen the shell shows a nonlinear softening behavior and, as expected, the ressonance curves follow the frequency-amplitude relation indicated in gray line.



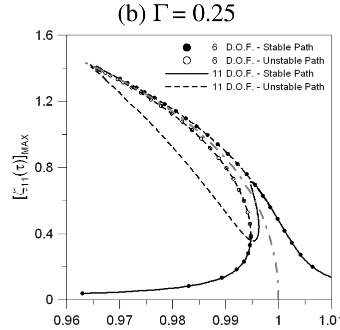
(a) $\Gamma = 0.10$



(b) $\Gamma = 0.25$

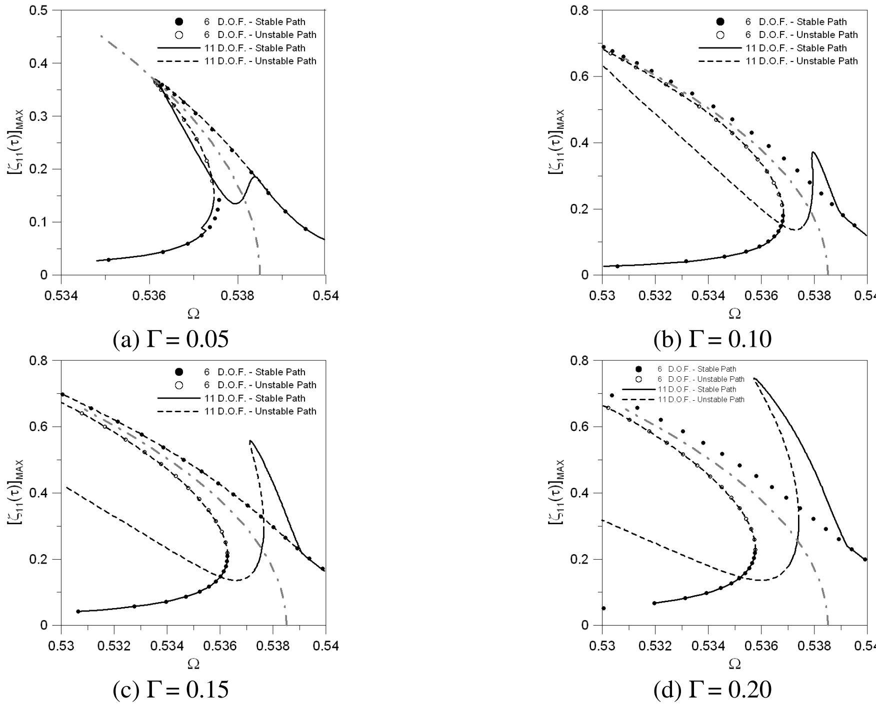


(c) $\Gamma = 0.30$



(d) $\Gamma = 0.35$

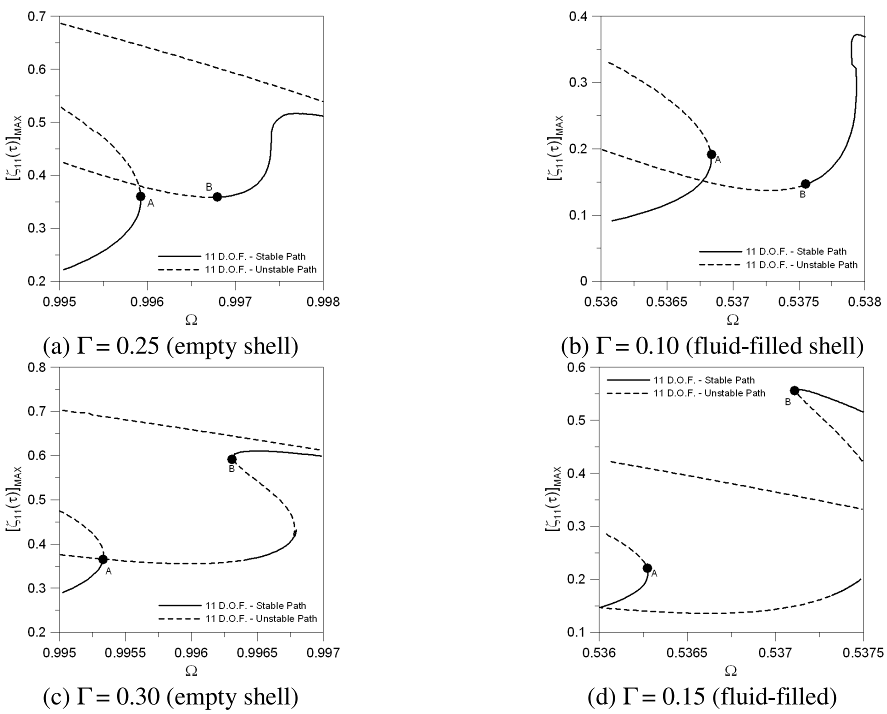
Figure 3. Resonance curves for empty shell. Influence of companion modes on the nonlinear response of the shell.



(a) $\Gamma = 0.05$ (b) $\Gamma = 0.10$

(c) $\Gamma = 0.15$ (d) $\Gamma = 0.20$

Figure 4. Resonance curves for the fluid-filled shell. Influence of companion modes on the nonlinear response of the shell



(a) $\Gamma = 0.25$ (empty shell) (b) $\Gamma = 0.10$ (fluid-filled shell)

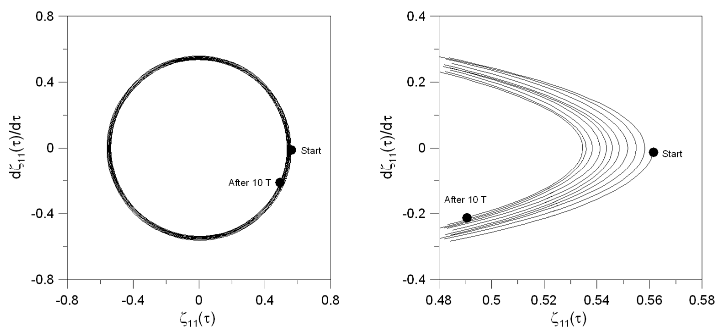
(c) $\Gamma = 0.30$ (empty shell) (d) $\Gamma = 0.15$ (fluid-filled)

Figure 5. Resonance curves for empty and fluid-filled shell showing regions with no stable solution.

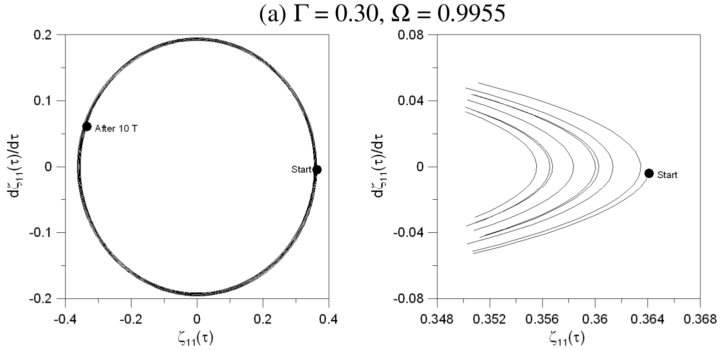
For small values of Γ, resonance curves obtained with either the 6 or the 11 dof models are the same. For the empty shell and for any value of the lateral pressure, the path located at right hand side of the backbone curve is always stable in the 6 dof model, while along the curve on the left hand side of the backbone curve there is a saddle-node bifurcation separating the stable and unstable branch. As the value of Γ increases, the inclusion of the companion mode modifies the stability of the solution and generates new equilibrium paths, modifying completely the behavior of the shell in the resonance region. The same is observed in Fig. 4 for the fluid-filled cylindrical shell.

The influence of the companion modes is not limited only to the main changes observed in the resonance curves showed previously. The companion modes generate, for certain combinations of Γ and Ω, regions with no stable solutions (Amabili et. al. 1998). Figure 5 shows some of the regions with only unstable solutions in the resonance curves for the cylindrical shell. As can be seen, for a certain value of the lateral pressure Γ, and for any value of the frequency of excitation Ω, between points A and B, there are not stable solutions (continuous lines).

Figure 6 shows the phase portraits for a certain combination of Γ and Ω between points A and B. To obtain these portraits, the time response of the shell is obtained with initial conditions given by an unstable point from Fig. 5c and Fig. 5d. Since the initial conditions have always a small error, the response does not return to the initial point after one excitation period but diverges slowly with time. Since no stable region is found in this range the response goes to infinity.



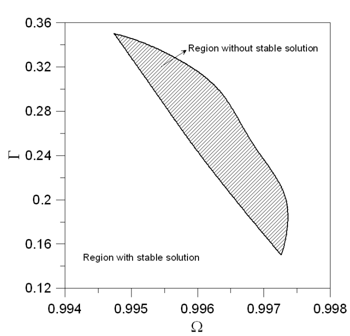
(a) $\Gamma = 0.30$, $\Omega = 0.9955$



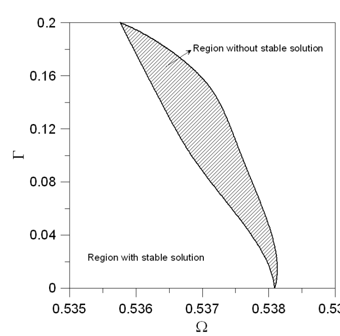
(b) $\Gamma = 0.15$, $\Omega = 0.5366$

Figure 6. Phase portraits for cylindrical shell (a) empty and (b) fluid-filled. Initial conditions on the unstable resonance branch.

Figure 7 shows, in the Γ x Ω space, the region where there are only unstable solutions for both empty and fluid-filled shell. This figure was obtained from the maximum values of the unstable solutions in the bifurcation diagrams as shown, for example, in points A and B of Fig. 5. As can be seen in Fig. 7, the fluid-filled shell shows unstable solutions for extremely low values of parameter Γ.



(a) empty shell



(b) fluid-filled shell

Figure 7. Region with unstable solution. Companion mode influence.

4. CONCLUSIONS

In the present paper, Donnell shallow shell theory has been applied to model the dynamics of a thin-walled circular cylindrical shell under lateral pressure. Based on a perturbation technique, a general solution for the displacement field with companion mode participation is obtained satisfying all boundary, symmetry and continuity conditions of a simply-supported shell. The results also corroborate the coupling between asymmetric and symmetric modes. The results show that this technique can be used to derive consistent low-dimensional models for the nonlinear dynamic analysis of cylindrical shells. Then low dimensional models considering or not the nonlinear companion mode are derived and the results are compared. Both empty and fluid-filled shells are analyzed. The results show that companion mode modifies substantially the resonance curves of the cylindrical shell and may even create completely unstable regions. So, for a perfect shell under external forcing, the inclusion of companion modes should always be considered in the shell modeling.

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