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KINEMATIC ANALYSIS AND OPERATION FEASIBILITY OF A 3-DOF ASYMMETRIC PARALLEL MECHANISM

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Abstract. *Parallel mechanisms correspond to kinematic structures that become attractive due to some potential advantages over their traditional serial (single open-loop chain) counterparts. Among them, one can mention: high rigidity, lightness, fast dynamic response, precision and high load capacity. This work introduces a three-degree-of-freedom parallel mechanism, that has been purposely conceived as a robotic manipulator for pick-and-place operations. Initially, the mechanism topology is described not only by enumerating the employed joints in the active limbs, but also by defining the input and output motions. The following sections deal with important issues such as the position and velocity kinematics, singularities, and the workspace evaluation. Finally, a built prototype is presented to demonstrate the operation feasibility of the proposed parallel mechanism.*

Keywords: parallel robots; asymmetric mechanisms; kinematics

1 Introduction

Most of commercially available industrial robots are based on serial kinematic structures, i.e., their actuators and moving links are assembled serially, one after the other, resulting only one open-loop kinematic chain to position and orient a gripper or tool end-effector. However, both academic and industrial communities have demonstrated a growing interest on using another kind of kinematic structure, known as parallel manipulators, which is characterized by the presence of many independent limbs (kinematic chains), actuating in-parallel or simultaneously on the end-effector. This non-conventional architecture becomes attractive due to some potential advantages over its traditional serial counterpart. Among them, one can mention: high rigidity, lightness, fast dynamic response, precision and high load capacity [1, 20].

Different types of parallel architectures have been proposed to operate as robotic manipulators. The Neos Tricept [2] represents a tetrapod structure that contains one central passive limb to constrain the spatial end-effector's motion. Later, this architecture was applied as a milling machine-tool. Clavel [3] conceived the Delta robot, a 3-dof parallel mechanism that is based on spatial pantograph linkages. The H4 [4] and Adept Quattro [15] have a similar architecture of Delta but they use an intermediate module to provide three translations and one rotation. Tsai [5] modified the Delta robot by replacing the spherical joints by using universal joints with special relative orientation of their rotating axis, in order to constrain the end-effector's motion to only three translations. Ceccarelli [21] proposed an orientation parallel robot, named Capaman, whose three active limbs contain a sequence of parallelogram, prismatic and spherical joints. More recently, novel 3-dof architectures such as the Universal Cartesian Robot [6], Tripteron [7] and *3 PCC* [8] present some convenient features: their set of nonlinear position equations become linear and fully decoupled which is not only valid for the inverse but also for the direct kinematics. However, due to the fact that these robots are overconstrained mechanisms, they need to satisfy very special assembly conditions.

Most of the proposed parallel structures present only topologically symmetric active limbs, while their actuators are installed at or near the base. Despite the fact that there are very few works on asymmetric parallel mechanisms [9,10], they still represent an unexplored potential architecture in the robotics field [11].

In a design process, after selecting the type of parallel mechanism, the next step corresponds to the dimensional synthesis. This design phase is conducted by performing the kinematic, static and dynamic analysis in order to determine expected behavior with respect to the structural parameters. In general, the adopted criteria include performance indices such as the size and shape of the workspace , presence of singularities, and conditioning number of jacobian and stiffness matrices [16-19].

This work introduces a three-degree-of-freedom parallel mechanism, that has been purposely conceived as a robotic manipulator for pick-and-place operations. Initially, the mechanism topology is described not only by enumerating the employed joints in the active limbs, but also by defining the input and output motions. The following sections deal with important issues such as the position and velocity kinematics, singularities, and the workspace evaluation and optimization. Finally, a built prototype is presented to demonstrate the operation feasibility of the proposed parallel mechanism.

2 System description and position kinematics

2.1 System description. As shown in Fig.1, the kinematic structure of the proposed parallel mechanism is composed by three active limbs, connecting the moving platform with the fixed base. Two among them are *RSS* type, while the other is *PPaP*, that is located in the central region of the mechanism. The letters *R, P,* and *S* stand for revolute, prismatic, and spherical joints, respectively. Here, *Pa* denotes a parallelogram subchain. The underlined letter means an active joint. The proposed kinematic structure *2 RSS + PPaP* was conceived by applying an alternative type synthesis procedure outlined in [11, 14]. The central limb *PPaP* was chosen in such a way that the moving platform only performs three translations. In addition, the in-parallel actuation can be implemented by using one linear and two rotary motors. Constructively, the linear motor can be replaced by another rotary motor coupled to a pair of pulleys and a synchronous belt. An additional remark must be done regarding the peripheral limb *RSS*. Theoretically, that would be sufficient, in terms of mobility, to employ a sequence of revolute, universal (cardan) and spherical joints. Our preference for two spherical joints is based on practical reasons: easiness of assembly and motion.

2.2 Position Kinematics. In this analysis, the moving platform coordinates that are variables x_P, y_P, and z_P of point *P*, are assumed to be known, while the actuator coordinates as the angles q_j ($j=1,2,3$) are unknown. In the central limb, due to the geometry of the robot, the variable y_P coincides with q_3. In accordance with the notation in Fig.2, the coordinates of points C_1, C_2, B_1 and B_2 are the following

$$C_1 = [\,x_P + d,\ y_P,\ z_P\,]^T \qquad C_2 = [\,x_P - d,\ y_P,\ z_P\,]^T$$

$$B_1 = [\ell \cos q_1 + D,\ 0,\ \ell \sin q_1]^T \qquad B_2 = [\ell \cos q_2 - D,\ 0,\ \ell \sin q_2]^T$$

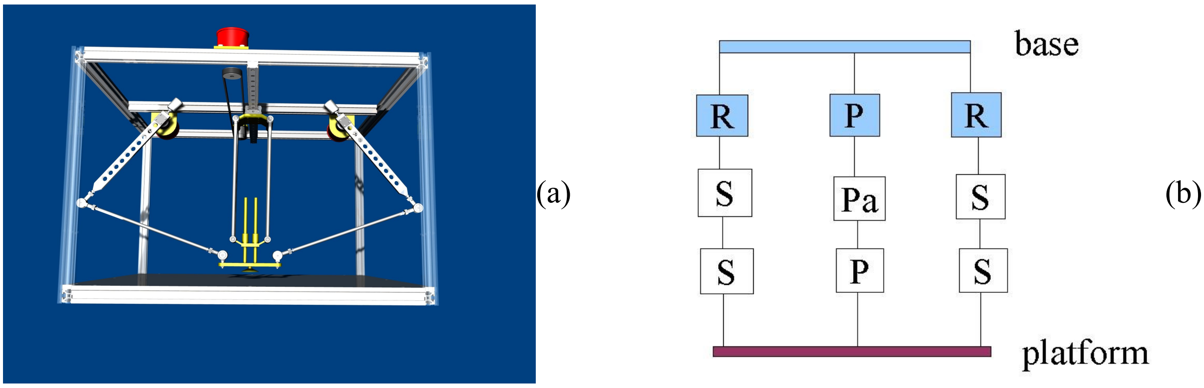


Fig.1 – Parallel mechanism: (a) CAD model; (b) graph representation.

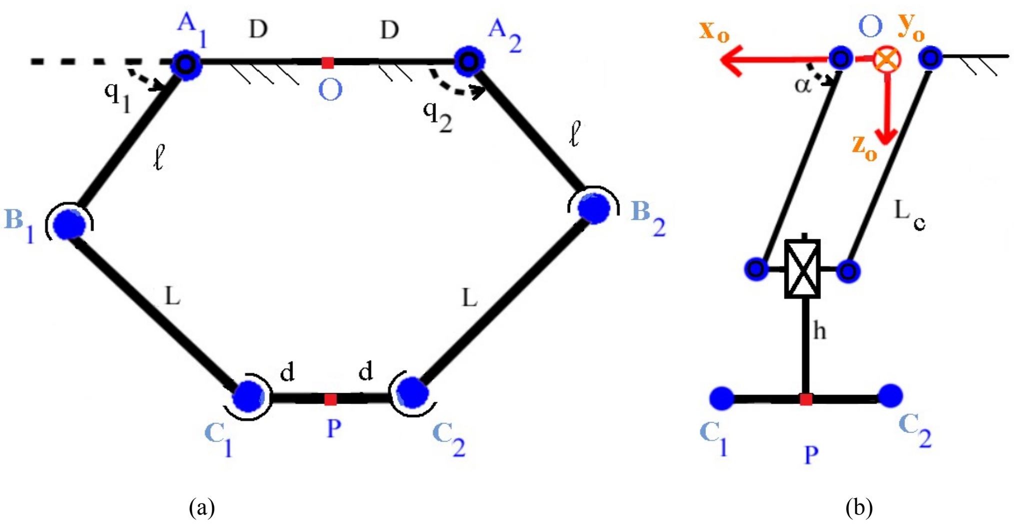


Fig.2– Variables and parameters in the kinematic diagrams of the proposed parallel mechanism: (a) peripheral limbs, (b) central limb.

For the *RSS* limb of the left side of the kinematic structure, once the lengths of the lower links in the limbs *RSS* are constant, then

$$(C_1 - B_1)^T (C_1 - B_1) = L^2 \tag{1}$$

By substituting the coordinate expressions of B_1 and C_1 in eq.(1), we obtain

$$\begin{aligned} & e_1 \cos q_1 + f_1 \sin q_1 + g_1 = 0 \\ & e_1 = -2.(x_P + d - D).\ell \\ & f_1 = -2.z_P.\ell \\ & g_1 = x_P^2 + y_P^2 + z_P^2 + \ell^2 - L^2 + (d - D)^2 + 2(d - D)x_P \end{aligned} \tag{2}$$

Eq. (2) can be modified into a 2th-order polynomial equation, where $u_1 = \tan(q_1/2)$.

$$(g_1 - e_1).u_1^2 + (2f_1).u_1 + e_1 + g_1 = 0 \tag{3}$$

Similarly, for the limb RSS on the right side, we can obtain

$$(g_2 - e_2).u_2^2 + (2f_2).u_2 + e_2 + g_2 = 0 \tag{4}$$

where

$$\begin{aligned} & e_2 = -2.(x_P - d + D).\ell \\ & f_2 = -2.z_P.\ell \\ & g_2 = x_P^2 + y_P^2 + z_P^2 + \ell^2 - L^2 + (-d + D)^2 + 2(-d + D)x_P \\ & u_2 = \tan(q_2/2) \end{aligned}$$

As one can notice, both eq. (3) and (4) may have up to two different solutions, and as a consequence, the mechanism itself may have up to four assembly modes. These assembly modes are only theoretically possible and, consequently, due to constructive reasons, one among the others is preferable. The chosen assembly mode represented in Fig.2 is adequate because it avoids interference between each peripheral active limb with the central one.

3 Velocity kinematics and singular configurations

By deriving the position equations with respect to time, one can obtain the mathematical relations between the end-effector velocity vector $\boldsymbol{V_P}$ and the actuators angular velocities $\dot{\boldsymbol{q}}$. Eq. (5) becomes important for two reasons: the motion planning of the end-effector path and the prediction of singular configurations.

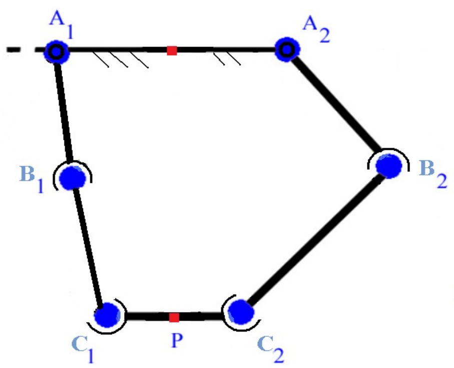
$$J_P \ \boldsymbol{V_P} = J_q \ \dot{\boldsymbol{q}} \tag{5}$$

where $\boldsymbol{V_P} = \begin{bmatrix} \dot{x}_P & \dot{y}_P & \dot{z}_P \end{bmatrix}^T$ $\quad \dot{\boldsymbol{q}} = \begin{bmatrix} \dot{q}_1 & \dot{q}_2 & \dot{q}_3 \end{bmatrix}^T$

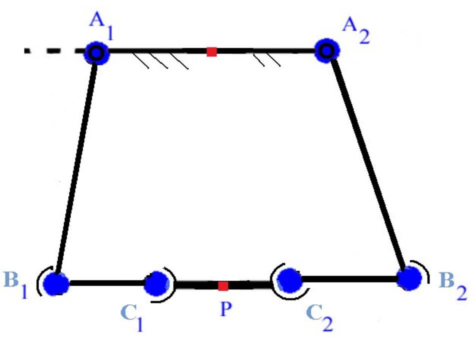
$$J_P = \begin{bmatrix} J_{P(1,1)} & J_{P(1,2)} & J_{P(1,3)} \\ J_{P(2,1)} & J_{P(2,2)} & J_{P(2,3)} \\ 0 & 1 & 0 \end{bmatrix} \qquad J_q = \begin{bmatrix} e_1 \sin q_1 - f_1.\cos q_1 & 0 & 0 \\ 0 & e_2 \sin q_2 - f_2 \cos q_2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$J_{P(1,1)} = -2\ell\cos q_1 + 2(d - D) + 2x_P$$
$$J_{P(1,2)} = 2y_P$$
$$J_{P(1,3)} = -2\ell\sin q_1 + 2z_P$$
$$J_{P(2,1)} = -2\ell\cos q_2 + 2(-d + D) + 2x_P$$
$$J_{P(2,2)} = 2y_P$$
$$J_{P(2,3)} = -2\ell\sin q_2 + 2z_P$$

In singular configurations, a parallel mechanism either reaches a locking position or even becomes uncontrollable. The conditions for occurrence of singularities can be investigated by the inspection of the determinants of Jacobian matrices, J_q and J_P [5]. Figure 3 shows two examples of singular configurations. When $\det(J_q)$ is null, the mechanism reaches the boundary of its workspace (Fig.3a). On the other hand, when $\det(J_P)$ equals zero, the mechanism might become uncontrollable. In Fig.3b one can notice such configuration: the actuators at A_1 and A_2 cannot withstand vertical forces acting upon the moving platform C_1PC_2 . Fortunately, this condition will occur only if the parameter ℓ is larger than L.



(a)



(b)

Fig.3 – Examples of singular configurations: (a) when det (J_q) equals zero; (b) when det (J_p) equals zero.

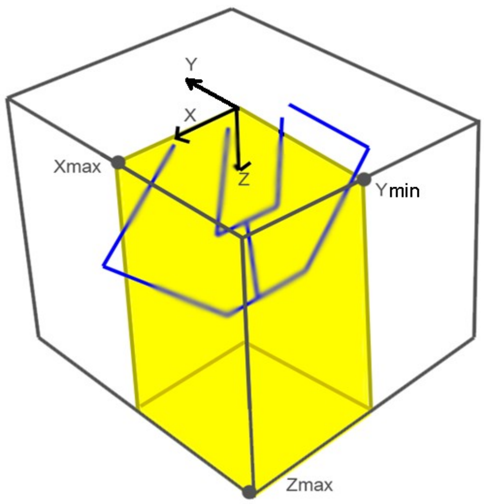


Fig.4 – The selected solid is one forth of the feasible parallelepiped

4 Workspace evaluation and optimization

The available workspace of the *3 RSS+PPaP* represents a 3D-region where end-effector point P, that belongs to the moving platform, can move. To determine this workspace, the discretization method [12, 13] is employed. This method considers that the workspace is determined from a solid, assumed larger than the feasible workspace, discretized by a regular mesh. Due to the symmetry of the manipulator, the selected solid is one forth of the feasible parallelepiped

(Fig.4). Then, a procedure checks whether or not each mesh node violates the physical and kinematic constraints. Consequently, workspace boundaries are composed by a set of nodes that has at least one neighbor node that does not belong to the workspace.

The physical constraints are represented by both the lengths of moving links and the passive joints limits. In addition, another important factor refers to the kinematic constraints. Our analysis calculates the determinants of Jacobian matrices J_q and J_P and verifies if their values are null, which correspond to singular configurations.

From Fig.2, one can observe that there are five dimensional parameters – ℓ, L, D, d, L_c - for the analyzed parallel mechanism. In addition, other two parameters Δh and Δy correspond to the strokes of prismatic joints near the end-effector and close to the base, respectively. Regarding the workspace optimization, we perform a reduction of the complexity of the problem: instead of employing seven design variables, we conduct the optimization by using only two among these variables. This will be explained in the following paragraphs.

The first assumption is that the strokes Δh , Δy and the length L_c remain constant even after the optimization. Two reasons justify that: firstly, to reduce the size of the problem and, secondly, the prismatic joints were already available for the built prototype (section 5). We also consider that the chosen design variables K_L and K_D correspond to the ratio between the lengths of moving links and the quotient of the lengths of the base and the moving platform, respectively.

$$K_L = \ell / L \tag{6}$$

$$K_D = D / d \tag{7}$$

The other assumption is that the sums of ℓ + L and D + d remain constant. The objective function *f*, only calculated for one forth of the workspace volume $\mathcal{V}$, due to geometric symmetry of the mechanism, is determined by the sum of small volumes Δv_i associated to each feasible node, as indicated in eq.(8). Once there are only two design variables, the numerical procedure comprises an exhaustive search [22] for the maximum value of the objective function in its feasible domain. Table 1 presents the initial and the optimum parameters for the parallel mechanism. Fig.5 shows the distribution of the objective function in the feasible domain of the design variables. The maximum value of the objective function is 3.5 dm^3, while K_L equals 0.5 and K_D is 1.0. The volume of the available workspace is approximately 14 dm^3 and Fig.6 presents the manipulator and the shape of the obtained workspace.

$$f(K_L, K_D) = \mathcal{V} = \sum_{i=1}^{n} \Delta v_i \tag{8}$$

Table 1 – Parameters of parallel mechanism

	ℓ [mm]	*L [mm]*	*D [mm]*	*d [mm]*	*Lc [mm]*	*Δ h [mm]*	*Δ y [mm]*
Initial values	350	450	280	80	364	100	700
Optimum values	267	533	180	180	364	100	700

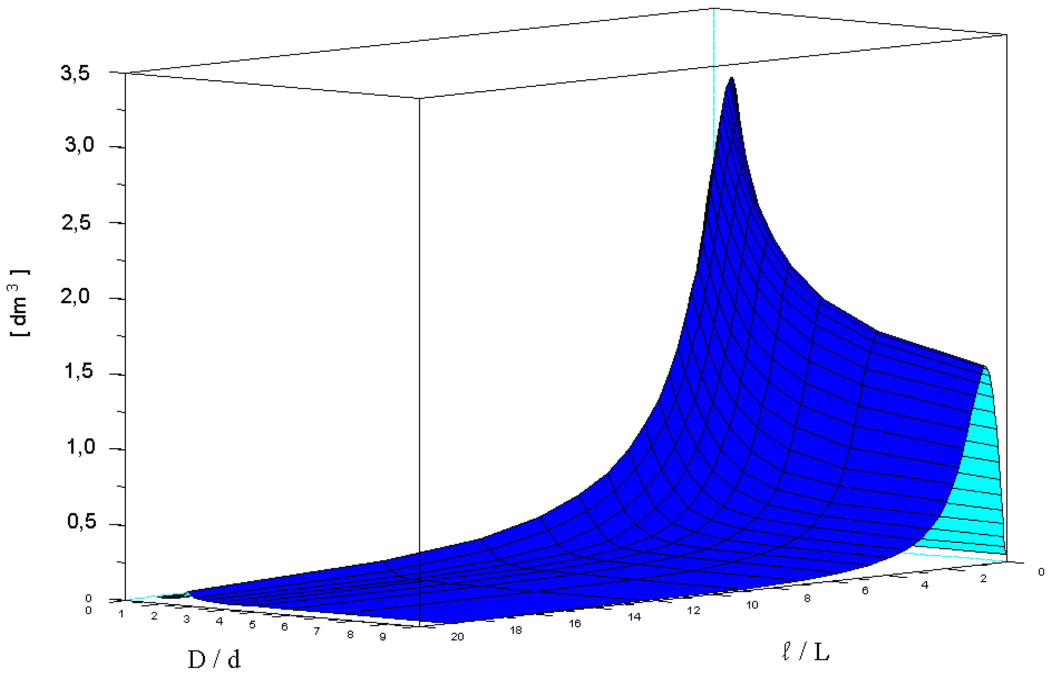


Fig.5 – The distribution of the objective function in the feasible domain of the design variables

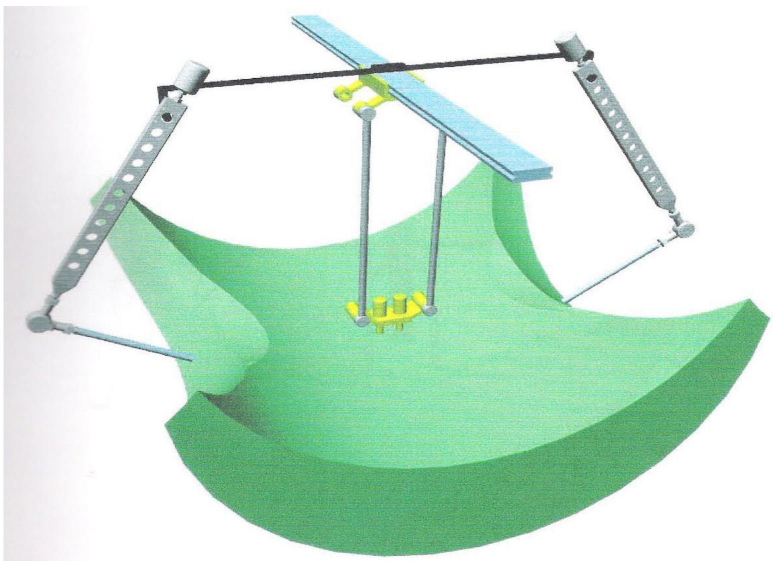
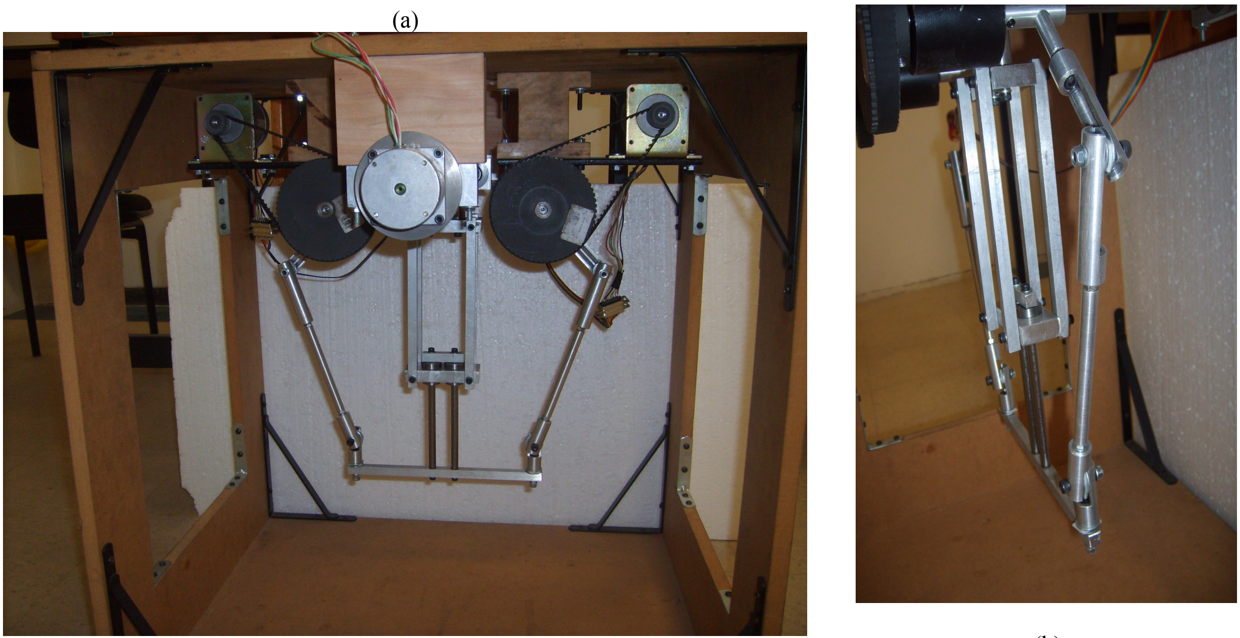


Fig.6 – The manipulator and the shape of the whole workspace

(a)



(b)

Fig. 7 - The built prototype: (a) general view; (b) detail of the spherical joints

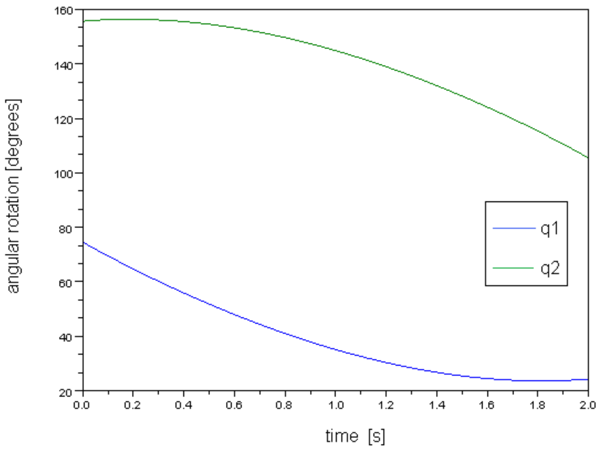


Fig. 8 – Simulation results: angular rotation q_1 and q_2

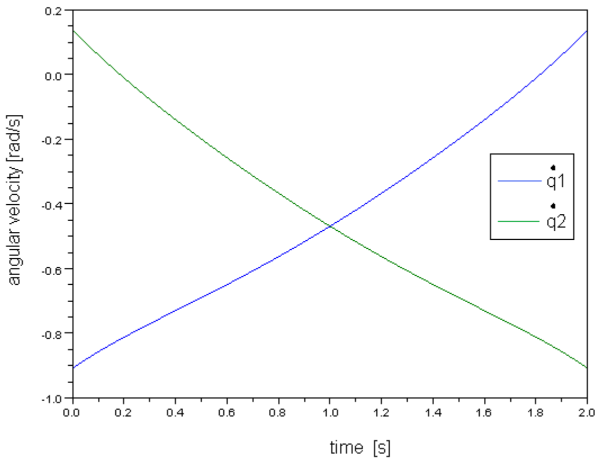


Fig. 9 – Simulation results: angular velocity $\dot{q}_1$ and $\dot{q}_2$

5 Experimental model: the built prototype

The built prototype, shown in Fig.7, is composed of the following subsystems: mechanical structure, actuator and control. The mechanical subsystem is composed by the parallel mechanism described in previous sections, built in ½ scale with respect to the optimum design parameters values. The actuator subsystem contains three stepping motors, one drive for each motor, and a power supply. The control subsystem includes a PC computer running EMC2 application under RT-Linux, communication cables connecting the drives with the computer through the parallel port, and a computational inverse kinematics model, written in Scilab environment that calculates the angular displacements of actuators from a specified sequence of the end-effector´s motion.

The selected tests for the built prototype consisted of pure translations of the moving platform along the axis x_o, y_o, and z_o by the action of the actuators of the three limbs. The simulation results for one of these tests are shown in Figs. 8-10. They correspond to the displacement of point P on the moving platform, along x-direction, from -0.113m to 0.113m, while the y and z coordinates remain unchanged at 0 and 0.24 m, respectively. The maximum speed, in these preliminary tests, was set in 0.113m/s. The actuator torques were obtained by performing a static analysis assuming a carrying load weight of approximately 4 N. After loading the computer program for a specific task in EMC2, the prototype started moving and, at this stage of the research, the authors observed qualitatively that the moving platform followed the predicted path for the point P.

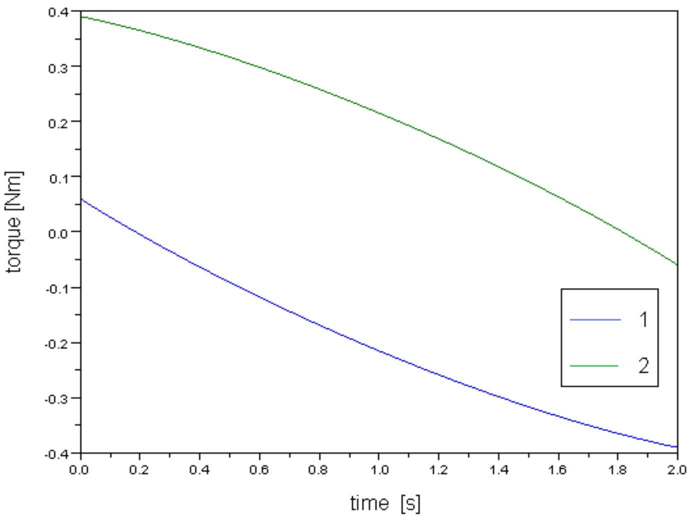


Fig.10. Simulation results: driving torques for the first and second actuators.

6 Conclusion

The kinematic behavior of a asymmetric parallel mechanism, the *2 RSS+PPaP* has been investigated by developing theoretical and experimental models. The main contributions of this paper include the design of a novel 3-dof parallel mechanism, together with the modeling and computation of its position and velocity kinematics, singularity analysis and the optimization of its available workspace.

Regarding the optimization procedure, the chosen design variables are related to the size of the base with respect to the size of the moving platform, and the ratio between the lengths of the connecting links. The objective function evaluates only one forth of the workspace volume, due to geometric symetry, by applying the discretization method, subject to physical and kinematic constraints.

Finally, the built prototype demonstrated the operation feasibility of the proposed mechanism as a translational parallel robot. The upcoming works will deal with the improvement of the presented prototype´s performance in order to become a fast pick-and-place robot, suitable to execute the required tasks in pharmaceutical, electronics and food industries.

References

[1] Merlet, J.P. "Still a long way to go on the road for parallel mechanisms", *ASME DETC Confer.*, Montreal, Canada, 2002
[2] Neumann, K.E. "Robot", US Patent No. 4,732,525, 1988
[3] Clavel, R. "Device for the movement and positioning of an element in space", US Patent no. 4,976,582, 1990
[4] Pierrot, F. and Company, O. "H4: a new family of 4-dof parallel robots", LIRMM – UMR 5506 CNRS / U.M. 2 Montpellier, France, 1998
[5] Tsai, L.-W. "Robot analysis: the mechanics of serial and parallel manipulators", John Wiley & Sons, New York, 1999
[6] Kim, H.S. and Tsai, L.-W. "Design Optimization of a Cartesian Parallel Manipulator", *ASME DETC Confer.*, Montreal, Canada, 2002
[7] Gosselin, C.M.; Kong, X.; Foucault, S. and Bonev, I.A. "A fully-decoupled 3-dof translational parallel mechanism", *In Proc. 4th Chemnitz Parallel Kinematics Seminar, PKS2004*, Verlag Wissenschaftliche Scripten, Reports from IWU, 24 : 595-610, 2004
[8] Di Gregorio and Parenti-Castelli, V. "Design of 3-dof parallel manipulators based on dynamic performances", *In Proc. of the 4th Chemnitz Parallel Kinematics Seminar, PKS2004*, Verlag Wissenschaftliche Scripten, Reports from IWU, 24: 385-397, 2004
[9] Li Q. and Huang Z. "Type Synthesis of 5-DOF Parallel Manipulators" *In Proc. of the 2003 IEEE Int. Conf. on Robotics and Automation* Taipei, Taiwan, September 14-19: 1203–1208, 2003
[10] Huang T., Li M., Zhao X.M., Mei J.P., Chetwynd D.G. and Hu S.J. "Conceptual Design and Dimensional Synthesis for a 3-DOF Module of the TriVariant – A Novel 5-DOF Reconfigurable Hybrid Robot" *IEEE Transactions on Robotics* 21(3): 449–456, 2005
[11] Hess-Coelho, T.A. "An alternative procedure for type synthesis of parallel mechanisms", *In Proc. 12th IFToMM World Congress*, Besançon, 2007
[12] Hess-Coelho, T.A. and Raszl, G. "Characterization of a prototype pf a robotic parallel structure considering its potential application as a machine-tool", *In Proc. 4th Chemnitz Parallel Kinematics Seminar, PKS2004*, Verlag Wissenschaftliche Scripten, Reports from IWU, 24: 421-

436, 2004
[13] Hess-Coelho, T.A. and Malvezzi, F., "Workspace optimization of 3 RSS+CP parallel mechanisms", *In Proc. 12th IFToMM World Congress*, Besançon, 2007
[14] Kumazawa, V.D. "Development of a parallel robot", Final Report, Escola Politecnica, Universidade de Sao Paulo, 2008 (available in portuguese)
[15] Adept tech Inc. "Adept Quattro s650H specifications", 2009
[16] Ottaviano E., Ceccarelli M. "Optimum Design of Parallel Manipulators for Workspace and Singularity Performances", *In Proc. Workshop on Fundamental Issues and Future Research Directions for Parallel Mechanisms and Manipulators*, Quebec City : 98-105, 2002
[17] Carbone G., Ottaviano E., Ceccarelli M., "Optimality Criteria for the Design of Manipulators", *IEEE Int. Confer. on Cybernetics & Intelligent Systems (CIS) and Robotics, Automation & Mechatronics (RAM) (CIS-RAM 2008)*, Chengdu, 2008
[18] Carbone G., Ottaviano E., Ceccarelli M., "An optimum Design Procedure for both Serial and Parallel Manipulators", *In Proc. IMechE Part C: Journal of Mechanical Engineering Science,* 221 (7): 829-843, 2007
[19] Ceccarelli M., Carbone G, "Numerical and experimental analysis of the stiffness performance of parallel manipulators", *In Proc. 2nd Int. Colloquium Collaborative Research Centre 562*, Braunschweig: 21-35, 2005.
[20] Ceccarelli, M., "Introduction to robotic manipulation", Kluwer, 2004
[21] Ceccarelli, M., 1997, "A new 3-dof spatial parallel mechanism", Mechanism and Machine Theory, 32(8): 895-902, 1997
[22] Nievergelt, J., "Exhaustive search, Combinatorial Optimization and Enumeration: Exploring the Potential of Raw Computing Power", SOFSEM 2000, LNCS 1963, Springer-Verlag Berlin Heidelberg : 18–35, 2000