STUDY OF SUPERSONIC AIRCRAFT LONGITUDINAL MOTION REGARDING THE CENTER OF GRAVITY POSITION

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Abstract. *This work deals with the study of longitudinal motion and control for a supersonic aircraft regarding its center of gravity positioning. Supersonic aircrafts show dynamics variation depending on the Mach number, while the center of gravity position control is a usual method to compensate this effect. This work aims to estimate a conceptual aircraft stability derivatives, to verify the center of gravity position influence in these derivatives and to propose and simulate a center of gravity control system. The stability derivatives are estimated (as a function of center of gravity position and Mach number) using the USAF Stability and Control Digital Datcom software. The influence of the center of gravity on the stability derivatives and longitudinal motion equations is verified. The control system is implemented using a linear quadratic regulator (LQR). Simulations are performed using the estimated stability derivatives and the non-linear equations of motion. The results compare three control systems. The first with a fixed center of gravity, the second with a constant matrix gain and the third with gain schedule.*

Keywords: *Longitudinal Control, Center of Gravity, Stability Derivatives, Supersonic Aircrafts.*

1. INTRODUCTION

This paper summarizes the work carried out at the Instituto Tecnológico de Aeronáutica (ITA) in the Master in Engineering Postgraduate Program in the Aeronautical & Mechanical Engineering course in the area of Flight Mechanics and Control.

1.1 Objective

The purpose of the work is to study the longitudinal motion and control of a conceptual supersonic aircraft regarding its center of gravity (cg) positioning (besides, of course, control surfaces). To achieve this objective it is necessary to determine the equations of longitudinal motion considering this new variable and to estimate an aircraft model which represents the aircraft behavior. In other words, it is needed to estimate the aircraft stability derivatives identifying how they are affected by the cg position and Mach number. Once the equations are determined and the aircraft model is estimated, it becomes possible to simulate the aircraft acceleration phase response and a control system can be designed, simulated and its results can be analyzed.

1.2 Motivation

One important role of the scientific research is to indicate solutions that could bring benefits not only for the companies but also for the society. In spite of using a supersonic aircraft, the methodology herein described is also valid and can be applied for conventional aircrafts.

2. EQUATIONS

The longitudinal motion equations taking into account changes in the cg position are determined based on the equations extensively demonstrated on the literature (Etkin, 1982) and (Stevens and Lewis, 2003). Since, in this case, the moment of inertia I_y is not a constant but a function of the cg position, special attention is paid to the pitch moment equation, which describes that the angular momentum derivative $\frac{d(I_y q)}{dt}$ is equal to the sum of aerodynamic and thrust moments $M + M_F$ applied about its axis.

Applying the chain rule, the derivative $\frac{dI_y}{dt}$ is given by:

$$\frac{dI_y}{dt} = \frac{dI_y}{dX_{cg}} \frac{dX_{cg}}{dt} \tag{1}$$

As the moment of inertia I_y is a quadratic function of the cg position, that is $I_y = AX_{cg}^2 + BX_{cg} + C$, the longitudinal motion is represented by the set of equations denoted by Eq. (2) to Eq. (7), between them the cg position changes are reflected in the Eq. (5).

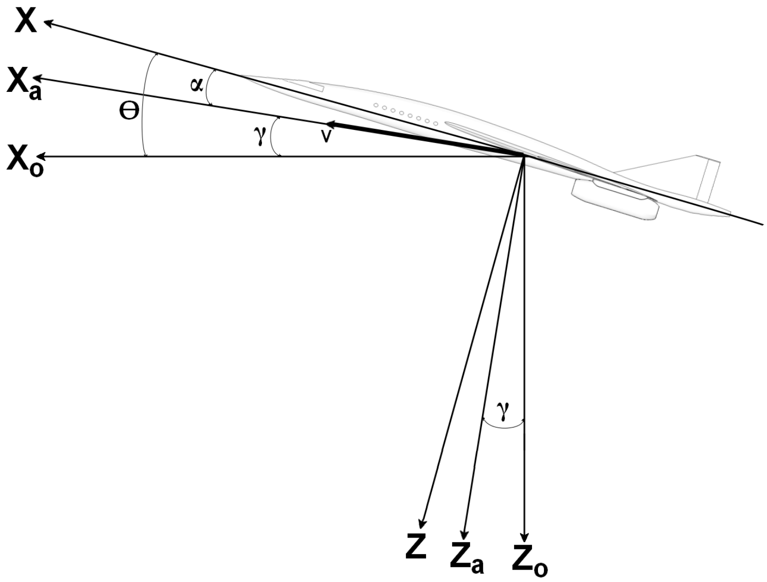

Figure 1. Definition of axes and aerodynamic angles

$$\dot{V} = -g\sin\gamma - \frac{D}{m} + \frac{F\cos(\alpha+\alpha_F)}{m} \tag{2}$$

$$\dot{\gamma} = \frac{-g}{V}\cos(\gamma) + \frac{L + F\sin(\alpha+\alpha_F)}{mV} \tag{3}$$

$$\dot{\alpha} = q - \dot{\gamma} \tag{4}$$

$$\dot{q} = \frac{M + M_F - q(2AX_{cg}+B)\dot{X_{cg}}}{I_y} \tag{5}$$

$$\dot{H} = V\sin\gamma \tag{6}$$

$$\dot{x} = V\cos\gamma \tag{7}$$

3. AIRCRAFT MODEL & STABILITY DERIVATIVES

The conceptual aircraft utilized in this study is a preliminary version of a work developed during the 8th Embraer Engineering Specialization Program. The aircraft called "Strider" carries six passengers plus a crew of three and cruises at Mach 1.6 above 50,000 ft. The figure 2 presents its three view, the main characteristics are the double-delta wing with a wing spam of 18 meters, underwing engines, absence of horizontal tail and a variable cross section along its length. The fuel system with its multiple tanks constitutes the means by which the aircraft cg is moved back or forward.

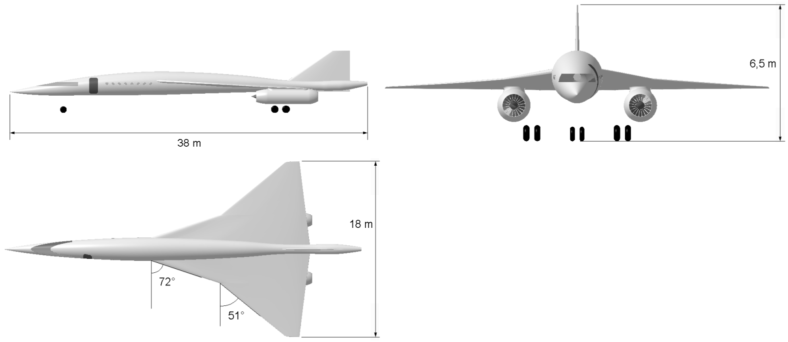

Figure 2. Three view

During the preliminary design phase it is common to make changes in the aircraft configuration, therefore, in some cases new stability derivatives estimates become necessary. Dealing with this concern and automating the process, a software environment is developed, which comprises of an Excel Sheet, a Macro and a Java stand-alone application. In the sheet are stored the aircraft parameters and stated the cg range and its discretization step. While the Macro generates the DATCOM input files, one for each cg position, and stores the DATCOM outputs. The stand-alone Java application sweeps through the DATCOM outputs looking for the longitudinal stability derivatives and creates the aircraft model in a Matlab file. The whole process is represented in Fig. 3.

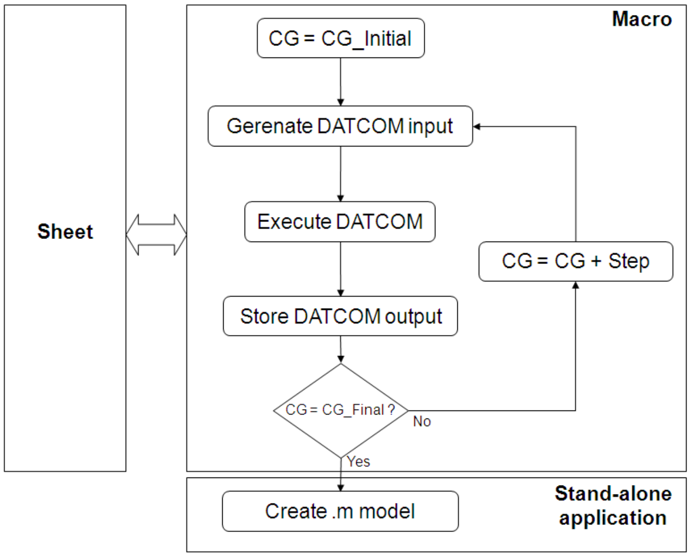

Figure 3. Software environment diagram

The derivatives are estimated for a range of Mach numbers, cg positions, angles of attack (stability derivatives only) and elevon deflection angles (control derivatives only). Table 1 presents the conditions in which the derivatives are estimated. The reference point for cg location is the aircraft nose. Since there are some restrictions on the DATCOM methods mainly due to wing double-delta geometry and control surfaces position regarding Mach waves, an equivalent delta-wing was used to estimate the $Cl_{\dot{\alpha}}$, Cl_q, $Cm_{\dot{\alpha}}$ and Cm_q stability derivatives. It is opportune to observe that this equivalent wing adoption may lead to values different from those that a double-delta wing would present and that the Cm_{δ_e} and Cl_{δ_e} derivatives were estimated by the conceptual aircraft aerodynamics team using the software Panair.

Table 1. Conditions

Altitude [ft]	40,000
Mach	0.35 0.6 0.9 1.1 1.25 1.4 1.6
Total weight [kg]	41,900
Angle of attack [°]	-5 -4 -3 -2 -1 0 1 2 3 4 5
Elevon deflection [°]	-24 -20 -16 -12 -10 -8 -4 -2 -1 0 1 2
CG position [m]	22.0 23.0 24.0 25.0 26.0

Considering that on the altitude of 40,000 ft the aircraft speed is on the range of 0.9 to 1.6 Mach, the range on Tab. 1 presents low values which, in spite of not being used for the simulation, are important to indicate how the stability derivatives are affected by Mach.

The Figure 4 surface graphs present the Cl_α, $Cl_{\dot{\alpha}}$, Cl_q, Cm_α, $Cm_{\dot{\alpha}}$ and Cm_q estimates showing how they vary with Mach and cg position. According to the utilized method and its results, the stability derivatives present a small variation with α (only in the subsonic regime), which is negligible when comparing to the other variables (Mach and cg position), therefore the surfaces were plotted for a constant $\alpha = 3°$ (which is a consistent value for equilibrium in subsonic flight).

The Cm_α is the most important derivative related to longitudinal stability, it can be observed that Cm_α reaches a minimum near sonic speed. Moreover, Cm_α assumes more negative values in the supersonic than in the subsonic regime, which means that in supersonic regime the aircraft becomes more stable (demanding greater control surface deflections to be controlled). As the cg position is moved backwards the Cm_α value becomes less negative. Interesting effects are observed at $Cm_{\dot{\alpha}}$ and $Cl_{\dot{\alpha}}$ derivatives, which present opposite behaviors between subsonic and supersonic regimes. As the cg position is moved backwards the $Cm_{\dot{\alpha}}$ derivative has its value increased in the subsonic regime, while the opposite happens in supersonic speeds. The $Cl_{\dot{\alpha}}$ is subjected to a change on its value from positive to negative during the aircraft acceleration. The results related to Cm_q and Cl_q derivatives indicate the presence of peak values near sonic speed.

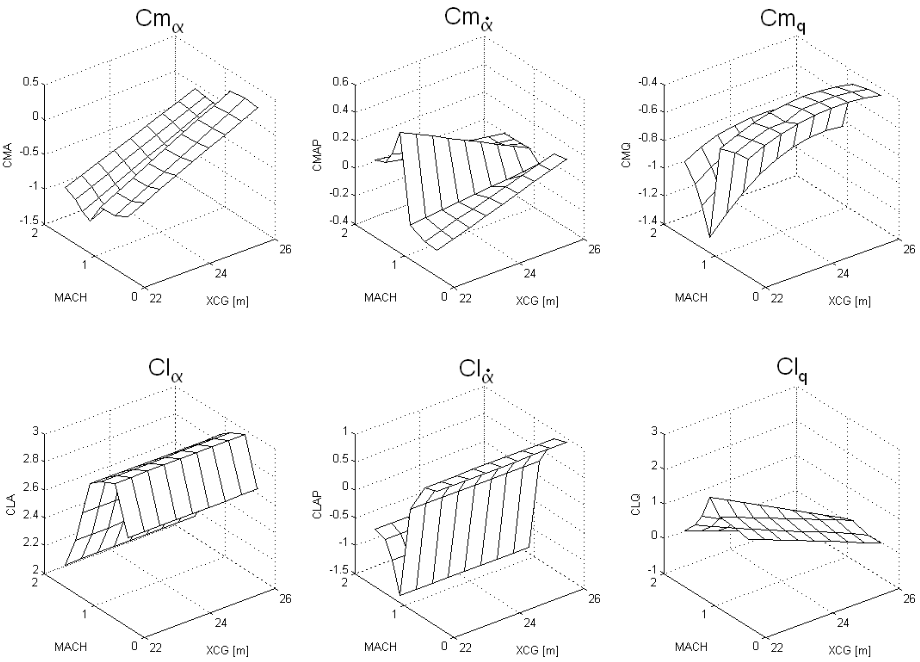

Figure 4. $Cl_\alpha, Cl_{\dot\alpha}, Cl_q, Cm_\alpha, Cm_{\dot\alpha}, Cm_q$ stability derivatives at $\alpha = 3°$, as a function of Mach and cg position

The elevon related derivatives are presented in Figure 5 as a function of Mach and cg position. Since they vary also with elevon deflection (δ_e) (the derivatives absolute values become smaller as the absolute value of the deflection is increased, in other words, they become less effective as the deflection is increased), a constant value of $\delta_e = 2°$ was used for plotting the surfaces (such value is consistent to the average value achieved for equilibrium establishment). The peak presence is also observed during transition from subsonic to supersonic regime.

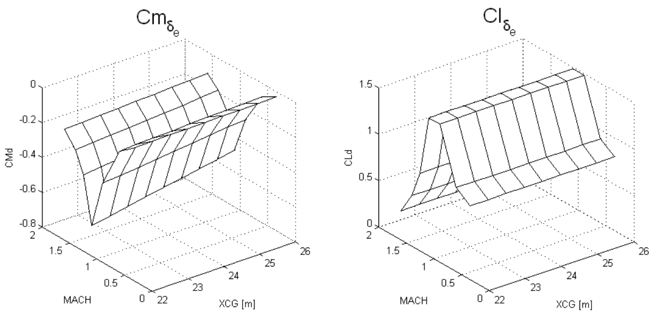

Figure 5. Cm_{δ_e} and Cl_{δ_e} derivatives as a function of Mach and cg position at $\delta_e = 2°$

The Figure 6 presents the aircraft thrust model, which is the maximum available thrust as a function of altitude and Mach number.

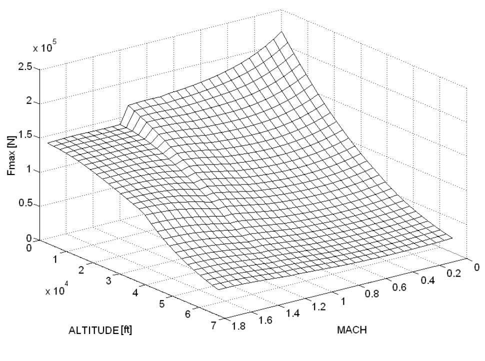

Figure 6. Thrust model

As presented in the equations section, the moment of inertia about Y axis is a necessary parameter to perform the longitudinal motion simulation and control. Basic physics concepts are applied to estimate this parameter. The aircraft is represented by discrete mass elements (since it is known all the aircraft components mass and location) and a summation of each individual moment of inertia is performed regarding the cg position, which is given by the fuel distribution. The summation is performed for five fuel distributions, this way the moment of inertia versus cg position is obtained. Figure 7 presents the result. The moment of inertia as a function of cg position is given by the Eq. 8.

$$I_y = 29312X_{cg}^2 - 1408598X_{cg} + 17992057 \tag{8}$$

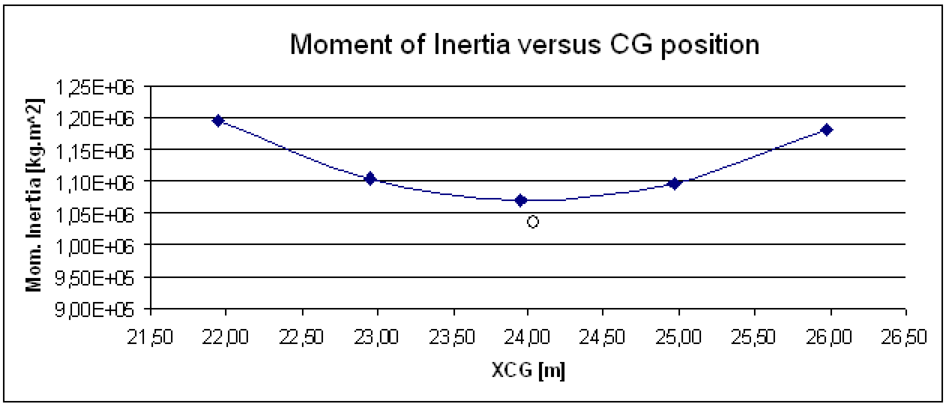

Figure 7. Moment of inertia as a function of cg position

4. CONTROL SYSTEM DESIGN

The point of interest for the control system design is the flight phase called acceleration, in which the aircraft departs from equilibrium at Mach 0.9 and, at constant altitude, accelerates to establish a new equilibrium at a supersonic speed. As verified through the stability derivatives, the aircraft is subjected to different behaviors depending on Mach. Between them is important to emphasize that the static margin is increased as Mach increases (due to a backward change in the aircraft aerodynamic center), that is, the aircraft becomes more stable (static stability) this way demanding greater elevon deflections to stablish the equilibrim. In other words, a nose-down movement is experienced in case the elevons are not deflected to compensate this change during the acceleration.

The control system is implemented using a Linear Quadratic Regulator (LQR), which comprises of state-feedback through a gain matrix computed in a way that minimizes a cost function in which weighting factors are defined by the designer. One difficulty in this method is to specify these weighting factors, therefore the work of Stevens and Lewis (2003) as well as the rule of thumb (known as Bryson's rule) are utilized as starting point. The Bryson's rule defines the weight factor as equal to the inverse of the square of maximum allowable deviation for each variable. To obtain the gain matrix, it is necessary to linearize the equations about an equilibrium point, however the simulations are performed using the non-linear equations this way verifying the controller robustness.

4.1 Actuator Dynamics

The aircraft is represented by the state space:

$$\dot{x} = Ax + Bu \tag{9}$$

where $x = [\ \triangle V \quad \gamma \quad \triangle\alpha \quad q \quad \triangle H]^T$. The elevon and cg position actuators dynamics are included in the system, this way the state space augmented by two states is given by:

$$\dot{x_a} = A_a x_a + B_a u_a \tag{10}$$

where:

$x_a = [\ \triangle V \quad \gamma \quad \triangle\alpha \quad q \quad \triangle H \quad \triangle\delta_e \quad \triangle X_{cg}\ ]^T$;

$$A_a = \begin{bmatrix} A & B \\ [0]_{1x5} & -20 \quad 0 \\ [0]_{1x5} & 0 \quad -1 \end{bmatrix}, B_a = \begin{bmatrix} [0]_{5x2} \\ 20 \quad 0 \\ 0 \quad 1 \end{bmatrix} \text{ and } u_a = \begin{bmatrix} u\delta_e \\ uX_{cg} \end{bmatrix}.$$

The elevon dynamics transfer function (Stevens and Lewis, 2003) is given by:

$$\frac{20}{s+20} \tag{11}$$

The cg position actuator (fuel pumps) dynamics transfer function is implemented based on engineering judment, concerned about avoiding the extremes of too slow and too fast responses, it is given by Eq. 12:

$$\frac{1}{s+1} \tag{12}$$

Figure 8 presents both actuators step response.

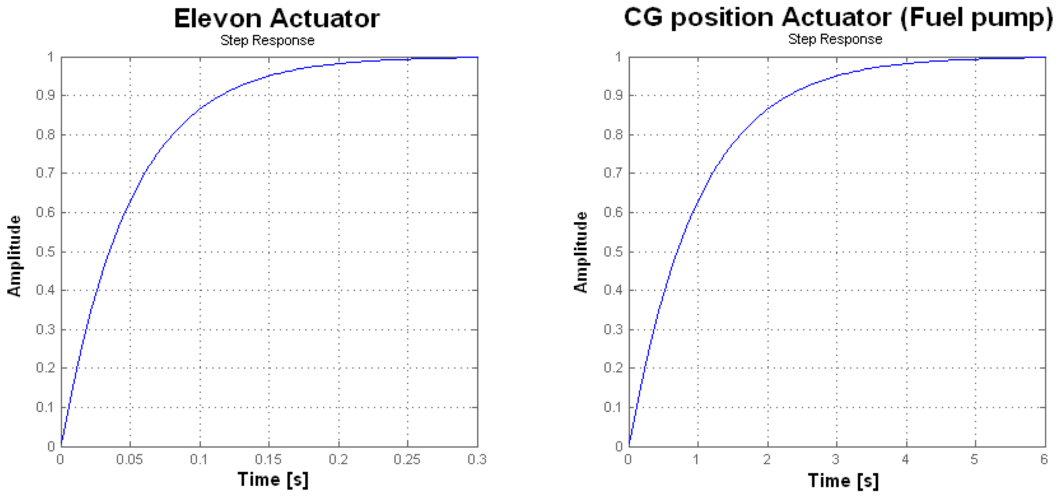

Figure 8. Elevon and cg position actuators step response

Position and velocity constraints are implemented to perform the simulations. They are presented in Tab. 2. The elevons actuator constraints are based on the work of Stevens and Lewis (2003) and cg position actuator on Morency and Stockin (2005). Figure 9 presents the system including the actuators dynamics, constraints and the aircraft dynamics.

Table 2. Actuators constraints

	Elevon	Center of gravity
Position constraints	± 25 [°]	$23,5$ a 26 $[m]$
Velocity constraints	± 60 $[°/s]$	$\pm 0,01$ $[m/s]$

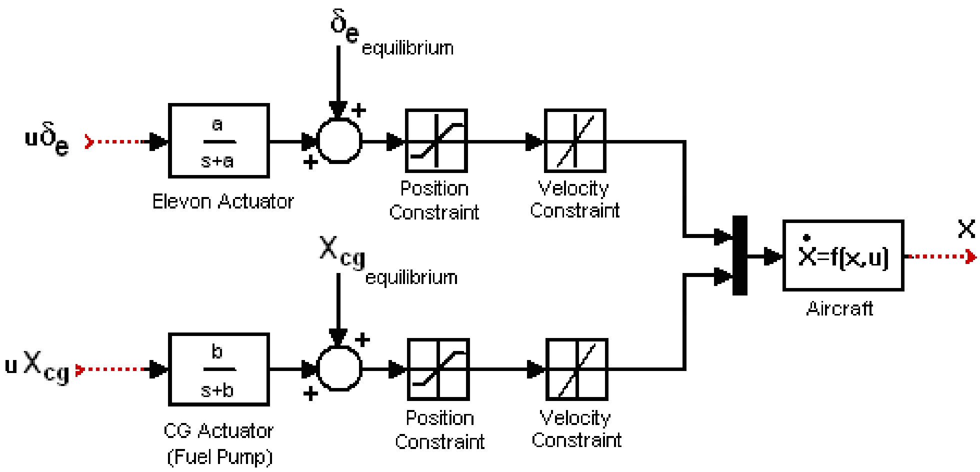

Figure 9. Plant to be controlled, with actuator dynamics and constraints

5. SIMULATION RESULTS

5.1 Equilibrium

Figure 10 presents the conditions for equilibrium stablishment at 40,000 ft, in which can be noticed that angle of attack and elevon deflection are strongly affected due to cg position, while thrust and thrust lever are not significantly affected.

As the cg position moves forwards, the angle of attack absolute value increases as well as the elevon deflection. In the supersonic regime the angle of attack becomes negative.

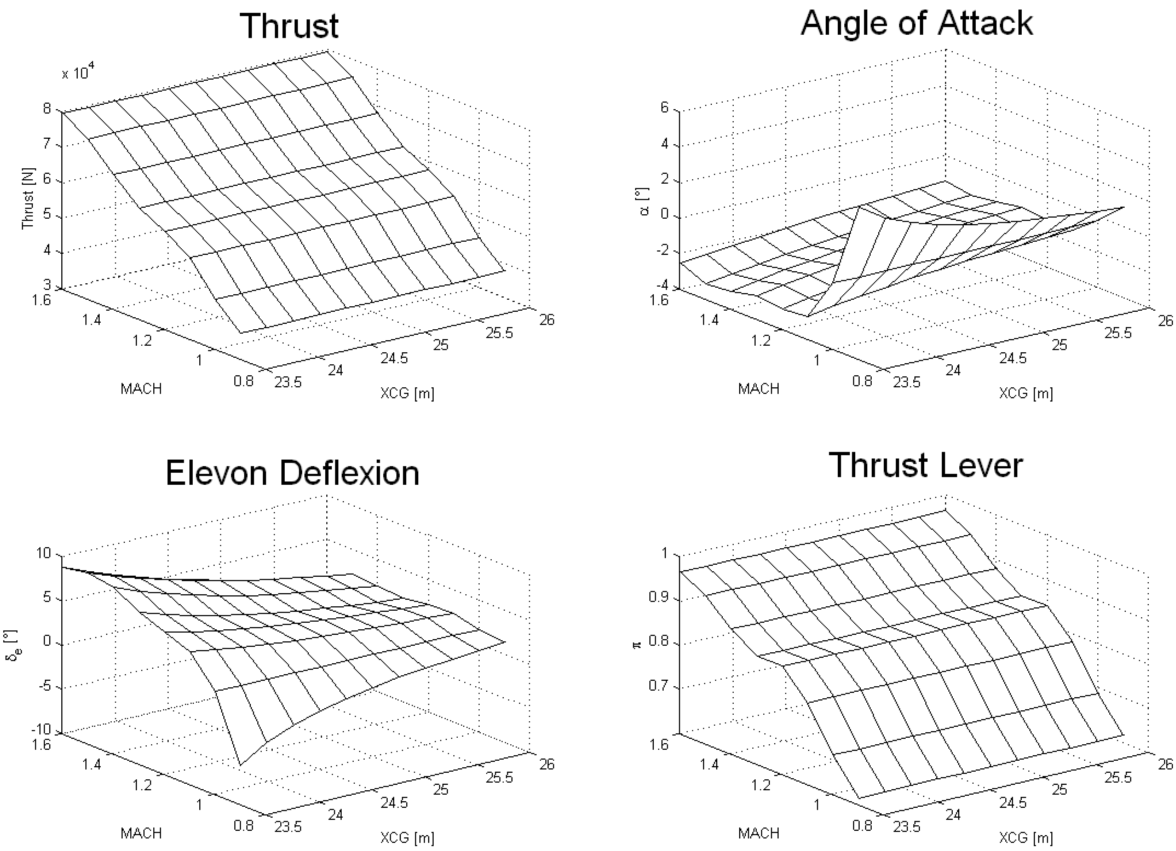

Figure 10. Equilibrium conditions as a function of Mach and cg position

5.2 Acceleration with fixed cg position

Figure 11 presents the simulation for acceleration with a fixed cg position. It is observed that the altitude leveled off about 17 meters below the initial one, while angle of attack stabilized about $-2.5°$ and elevon deflection about $7.3°$.

5.3 Acceleration controlled by constant feedback gain matrix

Figure 12 presents the simulation for acceleration controlled by a constant gain matrix. It is observed that the altitude leveled off about 45 meters below its initial value, while angle of attack stabilized about $-1.6°$ and elevon deflection about $2°$.

5.4 Acceleration controlled by gain-scheduled feedback matrix

Figure 13 presents the simulation for acceleration controlled by the gain-scheduled matrix. It is observed that the altitude leveled off about 15 meters below its initial value, while angle of attack stabilized about $-2,9°$ and elevon deflection about $8.5°$. By comparing this elevon deflection with the one obtained by the constant feedback gain matrix controller, it is clear that this is an elevated deflection. This is explained by the forward position that cg assumed during the acceleration. This cg forward movement occurred due to the chosen weighting factors, it is expected that increasing the weight related to this control (elevon), its deflection becomes smaller, at the cost of a greater variation on cg position.

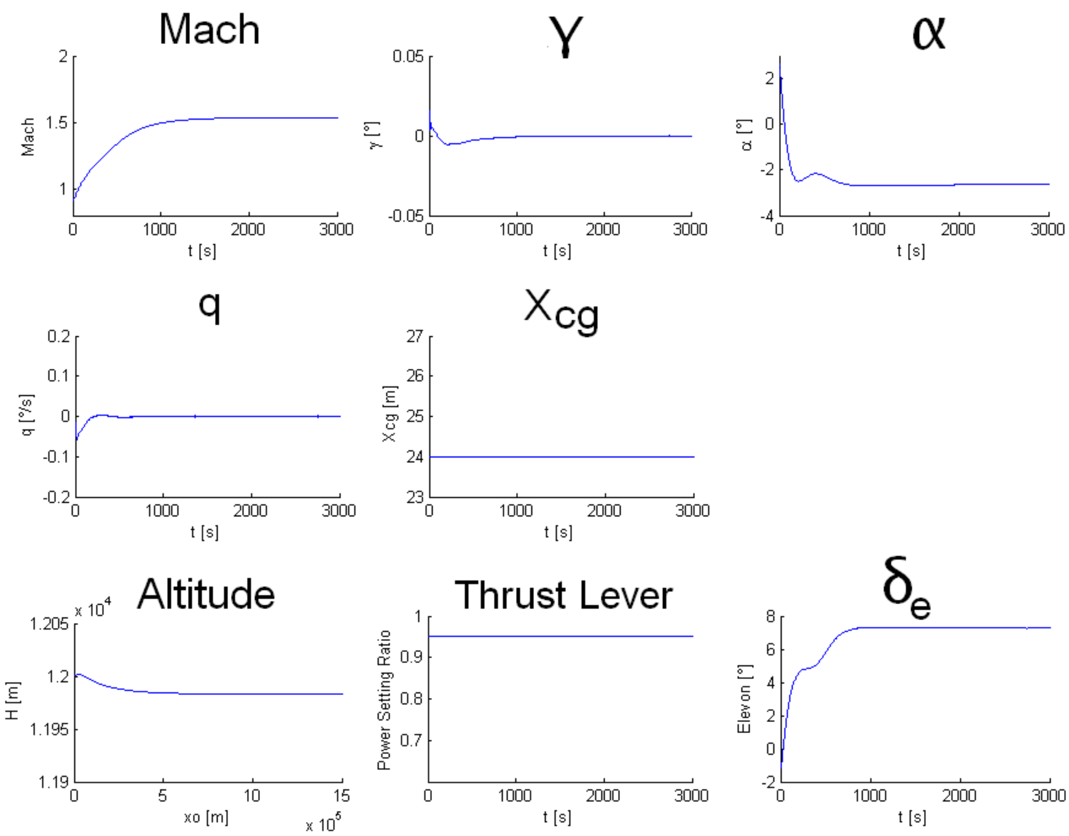

Figure 11. Acceleration with fixed cg position

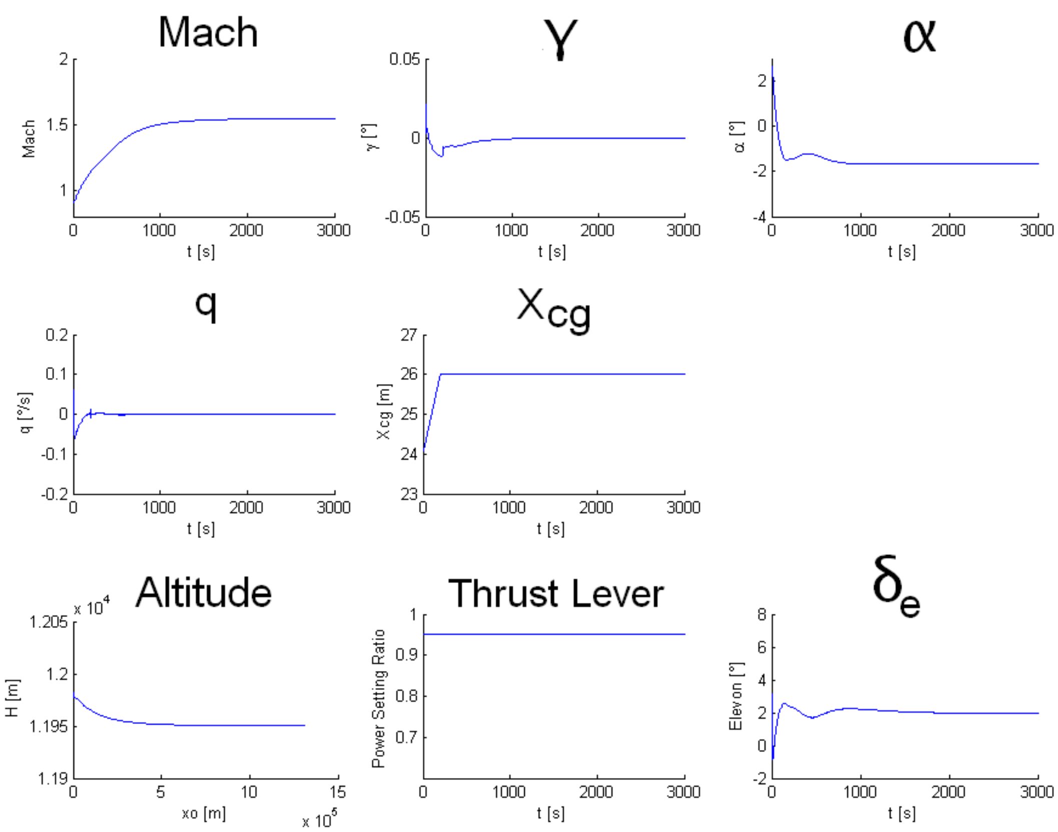

Figure 12. Acceleration controlled by constant feedback gain matrix

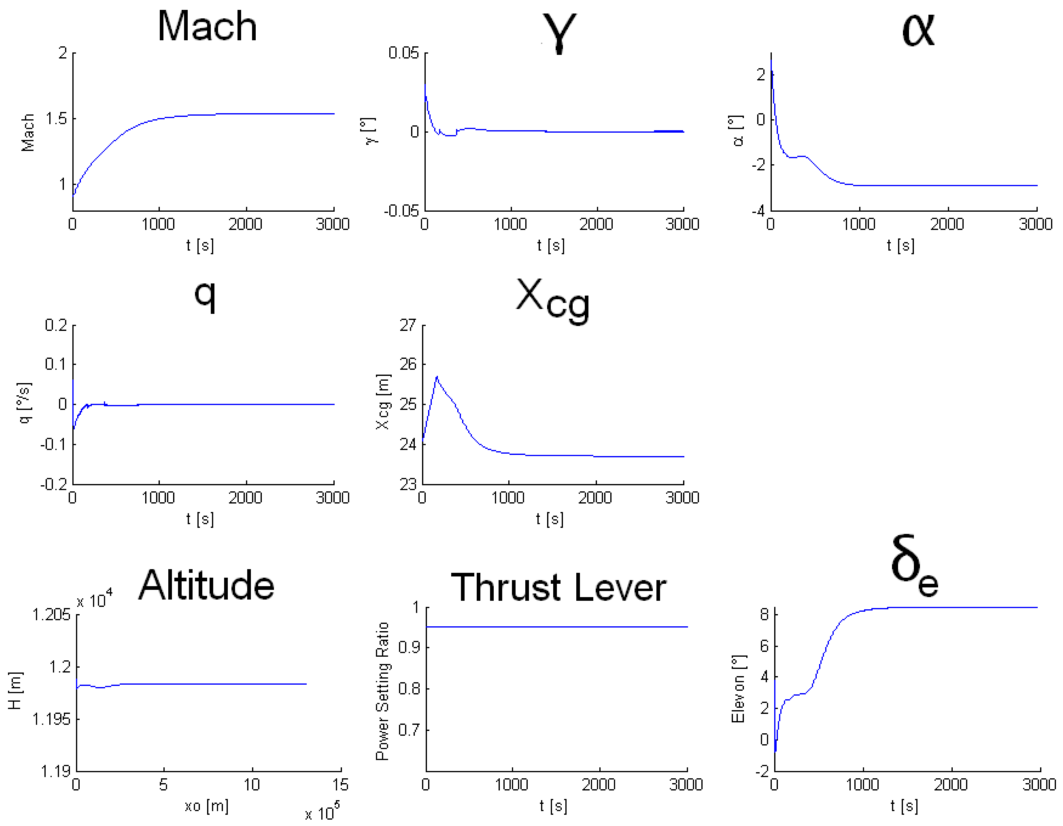

Figure 13. Acceleration controlled by gain-scheduled feedback matrix

6. FINAL REMARKS

The software environment tool revealed itself very useful creating automatically the Matlab model file. The stability derivatives were estimated as a function of cg position and Mach. Due to some DATCOM method restrictions, it was necessary to use an equivalent delta-wing for obtaining the $Cl_{\dot{\alpha}}$, Cl_q $Cm_{\dot{\alpha}}$ e Cm_q derivatives. It was verified how Mach and cg position affect the stability derivatives, in special the Cm_α, which is related to the static stability. During subsonic to supersonic transition was observed how the derivatives behave, some of them, such as $Cl_{\dot{\alpha}}$ e $Cm_{\dot{\alpha}}$ have their values changed from positive to negative (or vice-versa).

The well known equations of longitudinal motion were analyzed and cg-related terms were added. The moment of inertia about Y axis was determined discretizing the aircraft in mass elements and using basic physics concepts.

Regarding the control system design, dynamics and constraints were added to the actuators. The gain matrices were obtained with linearized models, but the simulations were performed using the non-linear equations.

The simulation results show that the objective was accomplished, since the aircraft accelerated from Mach 0.9 at constant altitude and established the equilibrium in the supersonic speed about Mach 1.6. The constant gain matrix controller presented a cg commanded backwards until reaching the position constraint. While the gain-scheduled matrix controller kept the cg inside its allowed range and presented the smaller altitude variation. It is opportune to observed that as the cg moves backwards the elevon deflection decreases.

Many others control systems techniques are possible and interesting to be applied. In this work only a stability augmentation system was implemented. Once the influence and possibilities of the cg are verified, future projects can focus on aircraft performance or emergency longitudinal control in case of primary controls are lost.

7. REFERENCES

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