A SWARM OPTIMIZATION ALGORITHM FOR OPTIMUM VEHICLE SUSPENSION DESIGN

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Abstract. *In this paper, it is investigated the design of a passive vehicle suspension system using a Swarm Optimization Algorithm in order to reduce vibrations. A simple quarter-car suspension model is used, where suspension spring, sprung mass, unsprung mass, tire spring and damping are modeled as a two degree of freedom system. The excitation of the road is assumed as a zero mean-random field and modeled as a single-sided Power Spectral Density in the form of a split power law. Variances of pavement load, the suspension deflection and dynamic loads are evaluated by means of the corresponding Power Spectral Densities in the frequency domain and used as objective function to be minimized. Some experiments are accomplished to evaluate the performance of the proposed algorithm. As result, a simple design example is compared with the literature regarding the optimum suspension parameters. The results show that the algorithm effectively leads to reliable results with low computational cost.*

Keywords*: swarm optimization algorithm, vehicle suspension design, optimization.*

1. INTRODUCTION

Generally speaking, vehicle's vibration depends mainly on the roughness of road surface and on the vehicle suspension system. Regarding the vehicle's vibration, this can lead to whole body uncomfortable sensations (low frequency vibrations), cause pain or injuries on lumbar area, hands and feet of the passenger/driver (low and high frequencies). In the vehicle durability's point of view, mechanical fatigue or excessive wear can occur as well. The design of vehicle's suspension systems should take into account these parameters. So, the design can be stated as: find bounded suspension parameters such as spring constant and viscous damping, in such a way that they will minimize vehicle's vibrations and the dynamic load received/applied by the vehicle to the road. This design is performed for a certain vehicle speed and road roughness profile. Such problem can be classified as a non-linear optimization problem with constraints. In the design process, the time domain approach is not suitable to solve such problem since very large time histories should be simulated in order to evaluate systems statistics like variance of the deflection vibration or dynamic loads. In this case, a frequency domain approach is worthwhile since statistical parameters can be easily evaluated in the frequency domain; besides road surface roughness, which is by itself a random process, can be better characterized in terms of Power Spectral Densities and wave numbers. The optimization process can be accomplished using several methods available in the literature since they account for the non-linear nature of the optimization problem. In this case, the proposed approach by a swarm optimization algorithm is suitable since this algorithm accounts for non-linearity and local optima in the optimization process by only using functions evaluations instead of function gradients.

2. BIBLIOGRAPHICAL REVIEW

The optimization of suspension parameters has been investigated by several authors in the last decades. The main idea behind such research is the minimization of the vehicle vibrations or the load applied into the road in order to avoid stress fatigue, human vibration discomfort or even extend the road life time. There are many ways to model the dynamic behavior of vehicles. The simplest one is to model as a quarter car suspension that represents all the vehicle mass, suspensions system and wheels. This approach, although simple, can model and predict some global dynamic behavior and serves as a first approach to study the phenomenon. More sophisticated models that use a mode elaborated system of connected spring, masses and dampers can be used in order to predict more sophisticated behavior like dynamic modes of vibration, dynamic interaction between parts of the vehicle. But this approach can lead to a hard problem to be solved mainly if it is modeled in the time domain. Sun (2002) presented the concept of the design of "road-friendly" vehicles stating that the load applied to the pavement as the main variable to be minimized. This research used a walking-beam suspension model which allows a series of dynamic behavior for the vehicle. It was used a one dimensional road profile in the frequency domain. The paper revises three commonly used objective functions to be optimized: ride quality, suspension stroke and road adhesion, but the probability of peak value of tire load exceeding a given value is taken as objective function of suspension design. It is found a well-known behavior for vehicles: for high air pressure and suspension systems with small damping this will lead to large dynamic tire loads. All the analyses were performed in the frequency domain. Spetzas and Kanarachos (2002) developed a methodology to design an active/hybrid car suspension system taking as control parameter the passenger comfort (minimization of passenger accelerations). It was used a heuristic algorithm (Neural Network) to learn the unknown control function. The heuristic algorithm was trained using a semi-stochastic optimization method and latter validated with two cases of minimization of passenger accelerations. The numerical comparisons were accomplished in the time domain and the vehicle was modeled as a 2 DOF model. Yldirim and Uzmay (2004) proposed a neural network scheme (Model Reference Adaptive Control) for controlling a bus suspension system. It was used traditional controllers (PI, PD and PID) to compare the results with the neural controller and the simulations showed a superior performance of the later controller against the traditional ones. It was used the time domain to perform the control of the suspension and the relative displacements as the objective function to be minimized. The vehicle was modeled as a 1 DOF system. The highlighted advantage in the use of such controller was the unnecessary knowledge of the system parameters in a precise way which leads to a more robust control. Sun *et al.* (2006) proposed a methodology to account for the contact area between tire and road surface. It is stated that the tire-pavement should be modeled as a contact area instead of a point contact. It was found that the distributed contact acts as a low pass filter governed by the weight function and contact interface. So, statistical parameters like spectral densities could be evaluated based on the counterparts of the original distributions. It was also found that compared with the one point contact, the distributed contact model smoothes the high frequency components of the road random field. It is said that the methodology could be easily coupled with optimum designs of vehicle suspensions system.

Regarding the algorithms used in the optimization tasks, Sun *et al.* (2007) used a genetic based algorithm to design and optimize a vehicle suspensions system. It was used the dynamic load applied to the road as the parameter to be minimized. The suspension model was a simple quarter car and the optimization was accomplished in the frequency domain. The road surface was modeled as a one variable stochastic process with a power spectral density function indicated by ISO 8606 Standard (1995). In the paper it is concluded that the numerical method of the genetic algorithms can be used as an optimization tool for such hard non-linear optimization problem.

Regarding the road surface roughness simulation, Andrén (2006) described a literature survey of power spectral densities approximations that can be used to represent road profiles. They were proposed by several authors and some are adopted by Standards. It was found ten approximations for the PSD representation. Based on the least square residual, the fit of approximations to measured data increases as the complexity adopted for the PSD. The author concludes that a two or three parameter PSD is probably enough for road rough identification.

3. VEHICLE SUSPENSION MODEL

In this paper a simple quarter-car suspension model is used to model the dynamic vehicle behavior. It is composed of an equivalent sprung mass m_1 that models vehicle's chassis/ body, passengers, engine, etc., an equivalent unsprung mass m_2 that represents wheels and axles, the suspension stiffness k_1, the tire stiffness k_2, suspension damping c_1 and tire damping c_2. The roughness along the road (x) related to the mean surface road level is indicated by $\xi(x)$. The vertical absolute sprung mass displacements are measured by variable $y_1(t)$ and the absolute vertical unsprung mass displacements are measured by variable $y_2(t)$. In Fig. 1 it is sketched this quarter car suspension model with the definition of the used variables.

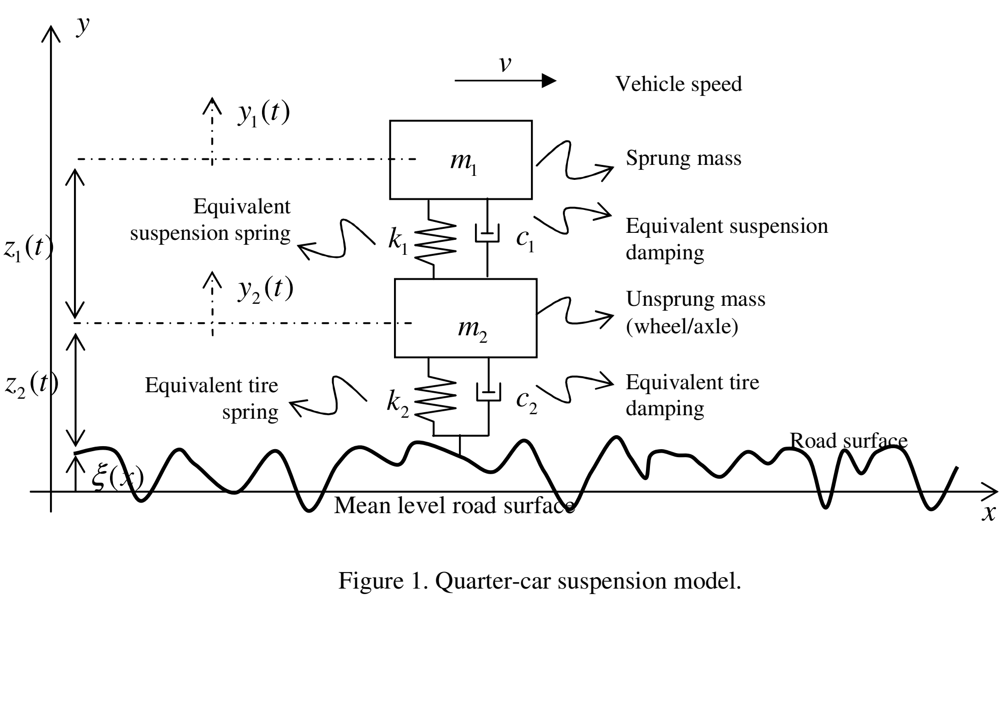

Figure 1. Quarter-car suspension model.

The related equations regarding to force equilibrium in Fig. 1 in the vertical direction are:

$$m_2\ddot{y}_1 + c_1(\dot{y}_2 - \dot{y}_1) + c_2(\dot{y}_2 - \dot{\xi}) + k_1(y_2 - y_1) + k_2(y_2 - \xi) = 0$$
$$m_1\ddot{y}_2 - c_1(\dot{y}_2 - \dot{y}_1) - k_1(y_2 - y_1) = 0 \tag{1}$$

If it is named $z_1(t) = y_2(t) - y_1(t)$ and $z_2(t) = \xi - y_2(t)$, respectively the relative displacements for unsprung and sprung masses and if this new variables are substituted in the previous equation, this yields:

$$-\ddot{z}_2 - \frac{c_2}{m_2}\dot{z}_2 - \frac{k_2}{m_2}z_2 - \ddot{z}_1 + \frac{c_1}{m_2}\dot{z}_1 + \frac{k_1}{m_2}z_1 + \ddot{\xi} = 0$$
$$-\ddot{z}_2 \qquad - \frac{c_1}{m_1}\dot{z}_1 - \frac{k_1}{m_1}z_1 + \ddot{\xi} = 0 \tag{2}$$

If it is applied the Fourier Transform to the previous equations, assuming a excitation in the sinusoidal form of $\xi = e^{i\omega t}$, the following equation in the frequency domain is obtained:

$$\begin{bmatrix} \omega^2 - i\omega c_2/m_2 - k_2/m_2 & i\omega c_1/m_2 + k_1/m_2 \\ \omega^2 & \omega^2 + i\omega c_1/m_1 + k_1/m_1 \end{bmatrix} \begin{bmatrix} H_2(\omega) \\ H_1(\omega) \end{bmatrix} = \begin{bmatrix} \omega^2 \\ \omega^2 \end{bmatrix} \tag{3}$$

where $H_1(\omega)$ and $H_2(\omega)$ are the Frequency Response Functions (FRF) for sprung and unsprung masses, respectively. These functions are indicated below:

$$H_1(\omega) = \frac{\omega^3 i\alpha_2 + \omega^2\beta_2}{-\omega^4 + \omega^3(-i\alpha_1 + i\alpha_2 + ih\alpha_1) + \omega^2(\beta_2 - \beta_1 + h\beta_1 - \alpha_1\alpha_2) + \omega(i\alpha_1\beta_2 + i\alpha_2\beta_1) + \beta_1\beta_2} \tag{4}$$

$$H_2(\omega) = \frac{\omega^4 + \omega^3(i\alpha_1 - ih\alpha_1) + \omega^2(\beta_1 - h\beta_1)}{-\omega^4 + \omega^3(-i\alpha_1 + i\alpha_2 + ih\alpha_1) + \omega^2(\beta_2 - \beta_1 + h\beta_1 - \alpha_1\alpha_2) + \omega(i\alpha_1\beta_2 + i\alpha_2\beta_1) + \beta_1\beta_2} \tag{5}$$

where, for the sake of simplicity, it was defined $\alpha_1 = c_1/m_1$, $\alpha_2 = c_2/m_2$, $\beta_1 = k_1/m_1$, $\beta_2 = k_2/m_2$, $h = m_1/m_2$.

4. ROAD ROUGHNESS MODELLING

As indicated by Andrén (2006), the PSD of the road profile can be used both to assess the road roughness and as input to a vehicle model. ISO 8608 (1995) specifies the way to report the measured data for road roughness as the correct notation. A series of models can be found in the literature that approximates the road roughness profiles by simple formulae. These formulations use a series of parameters that are road dependent. The ISO model, based on Dodds and Robson (1973) works, is taken as:

$$G_\xi(\Omega) = C(\Omega)^{-w} \tag{6}$$

where G_ξ is the single sided power spectral density for road roughness (m^2 / cycle / m), Ω means is the wave number (cycle/m), C is the general road roughness coefficient (m^3/ cycle), which is related to the road surface condition, w is the wavelength distribution. A better formulation is the BSI (1972) recommendation that states for the road profile roughness the following formula:

$$G_\xi(\Omega) = \begin{cases} C(\frac{\Omega}{\Omega_0})^{-w_1} & for\ \Omega \le \Omega_0 \\ C(\frac{\Omega}{\Omega_0})^{-w_2} & for\ \Omega \ge \Omega_0 \end{cases} \tag{7}$$

where the single sided PSD was split into two straight lines at the discontinuity frequency Ω_0 (cycle/m). The discontinuity frequency is usually set as $\Omega_0 = 1/2\pi \cong 0{,}16$ cycle/m, which corresponds to a wavelength of about 6.3 m. w_1 and w_2 are wavelength parameters distributions. Other sophisticated models are available but they use more parameters to describe the road profile roughness. Instead of following this way, in this paper the two split model with $\Omega_0 = 0{,}1$ cycle/m, distributions parameters $w_1 = w_2 = 2.0$ and the general road roughness coefficient as $C = 0.01$ m^3/cycle will be used.

5. SYSTEM RESPONSE IN THE FREQUENCY DOMAIN

In order to evaluate the system response in the frequency domain due to inputs, according to the random vibration theory, the Power Spectral Densities of the displacements responses of the unsprung $S_{z_2}(\omega)$ and sprung masses $S_{z_1}(\omega)$ are respectively written as:

$$
\begin{aligned}
S_{z_2}(\omega) &= \left|H_2(\omega)\right|^2 S_\xi(\omega) \\
S_{z_1}(\omega) &= \left|H_1(\omega)\right|^2 S_\xi(\omega)
\end{aligned}
\tag{8}
$$

where $S_\xi(\omega)$ is the Power Spectral density of the elevation of the road surface profile (roughness). The Power Spectral Densities are defined for all frequencies. In order to evaluate $S_\xi(\omega)$ it is necessary to transform the given single sided Power Spectral Density (defined for positive frequencies) of the road roughness $G(\Omega)$, in the space domain, into this double sided Power Spectral Density. Remembering that a constant speed v the wave number Ω and the angular frequency ω are related through $\omega = 2\pi v\Omega$, the following relationship yields for the transformation:

$$
S_\xi(\omega) = \frac{1}{4\pi v} G_\xi(\omega) \tag{9}
$$

The power spectral density of the dynamic load as function of the power spectral density of the road profile is given, as indicated by Sun and Deng (1998) as:

$$
S_P(\omega) = \left|(k_2 + ic_2\omega)H_2(\omega)\right|^2 S_\xi(\omega) \tag{10}
$$

In order to evaluate parameters that represent such stochastic processes, the variance of the corresponding values will be used. In this case, the corresponding values of the variance of the vehicle deflections and dynamic loads are evaluated, respectively as:

$$
\sigma_{Z_1}^2 = \int_{-\infty}^{\infty} S_{Z_1}(\omega)d\omega = \int_0^{\infty} G_{Z_1}(\omega)d\omega = \int_0^{\infty} 2S_{Z_1}(\omega)d\omega = \int_0^{\infty} 2\left|H_1(\omega)\right|^2 S_\xi(\omega)d\omega = \int_0^{\infty} 2\left|H_1(\omega)\right|^2 \frac{1}{4\pi v} G_\xi(\omega)d\omega \tag{11}
$$

and

$$
\sigma_P^2 = \int_0^{\infty} G_P(\omega)d\omega = \int_0^{\infty} 2S_{P_1}(\omega)d\omega = \int_0^{\infty} 2\left|(k_2 + ic_2\omega)H_2(\omega)\right|^2 S_\xi(\omega)d\omega = \int_0^{\infty} 2\left|(k_2 + ic_2\omega)H_2(\omega)\right|^2 \frac{1}{4\pi v} G_\xi(\omega)d\omega \tag{12}
$$

Such integrals are evaluated over the frequency range of 0 to 500($2\pi v\Omega_0$). The algorithm used for these one-dimensional integrations was the Romberg Method. This method uses trapezoidal approximations over an even number of subintervals and then compares sequential estimates by summing the areas of the trapezoids. The method terminates when the four most recent estimates differ by less than the value of a tolerance (*10^{-6}*). This integration is suitable for this purpose since the functions are not periodic functions. Figure 2 shows a plot of the single sided power spectral density for vehicle deflections in the frequency domain and vehicle velocity of 20m/s.

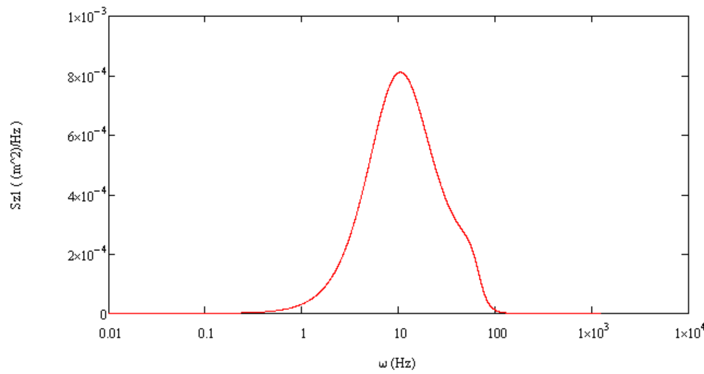

Figure 2. Single sided power spectral density for vehicle deflection.

More details about the theory of Power Spectral Densities generation and evaluation can be found in Bendat and Piersol (1986).

6. SWARM OPTIMIZATION ALGORITHM

The particle Swarm optimization (PSO) has been inspired by the social behavior of animal behavior such as fish schooling, insects swarming and birds flocking. This method is used to search for the global optimum of wide variety of arbitrary problems. It was first introduced by Kennedy and Everhart (1995). The initial intent of the particle swarm concept was to graphically simulate the graceful and unpredictable choreography of a bird flock, the aim of discovering patterns that govern the ability of such bird flock to fly synchronously and suddenly change direction with regrouping in an optimal formation. Rigorously speaking, it is a stochastic, population based evolutionary computer algorithm. The basis for the method relies on the social influence and social learning which enable persons to maintain cognitive consistency. So, the exchange of ideas and interactions between individuals may lead them to solve problems. The particle swarm simulates this social plot. As stated by Li *et al.*(2007), the method involves a number of particles, which have a defined position and velocity and they are initialized randomly in a multidimensional search space of an objective function. Each particle represents a potential solution of the problem and the measure of this potentiality is its objective function. The set of particles are generally referred as "swarm". These particles fly through the multidimensional space and have two essential reasoning capabilities: their memory of their own best position and knowledge of the global or their neighborhood's best. In a minimization optimization problem, "best" simply means the position of the particle ($\mathbf{x}_i$) with the smallest objective value, $\min f(\mathbf{x}_i)$. Members of a swarm communicate good positions to each other and adjust their own position and velocity based on this information of good positions. So, related to each particle there are a set of design variables ($\mathbf{x}_i$) and the respective velocities ($\mathbf{v}_i$) that represents the potential solution of the optimization problem.

At each iteration, the basic swarm parameters position and velocity are updated using the following equations:

$$\begin{aligned} v_{i,j}^{k+1} &= \chi[\varpi v_i^{k+1} + \lambda_1 r_1 (xlbest_{i,j}^k - x_{i,j}^k) + \lambda_2 r_2 (xgbest_j^k - x_{i,j}^k)] \\ x_{i,j}^{k+1} &= x_{i,j}^k + v_{i,j}^{k+1} \end{aligned} \tag{13}$$

where ϖ is the inertia weight for velocities (previously set between 0 and 1, in this paper ser as 0,7), $x_{i,j}^k$ is the current value (k) of design variable j of particle i, $v_{i,j}^{k+1}$ is the updated velocity of design variable j of particle i, $xlbest_{i,j}^k$ is the best design variable j ever found by particle i, $xgbest_j^k$ is the best design variable j ever found by the swarm, r_1 and r_2 are uniform random numbers in the [0,1] range, λ_1 means the cognitive component (self confidence of the particle) and λ_2 means the social component (swarm confidence) and they are constants that influence how each particle is directed towards good positions taking into account personal best and global best information , respectively. They usually are set as $\lambda_1 = \lambda_2 = 1.5$. The role of the inertia weight ϖ is crucial for the P.S.O. convergence. It is employed to control the impact of previous velocities on the current particle velocity. A general rule of thumb indicates to set a large value initially to make the algorithm explore the search space and than gradually reduce it in order to get refined solutions. In this paper it is initially set as $\varpi = 0.8$ and updated based on coefficient of variation ($cov = \sigma / \mu$) of the swarm objective function accordingly to $\varpi = 0.4[1 + min(\text{cov}, 0.6)]$. The χ parameter is used to avoid divergence behavior in the algorithm and it is given by the following expression, which was developed based on convergence assumptions for the algorithm, as indicated by Bergh and Engelbrecht (2006).

$$\chi = \frac{1.6}{\left|2-(\lambda_1+\lambda_2)-\sqrt{(\lambda_1+\lambda_2)^2-4(\lambda_1+\lambda_2)}\right|} \tag{14}$$

This coefficient is crucial to keep the algorithm stable and avoid divergence in the iteration process. There are variations in the algorithm that add a third term in the previous velocity update that accounts for neighborhood information. This requires the user to set an influence region to define the neighborhood. In this paper the just the simple algorithm was used in order to reduce the number of heuristic parameters. For the generation of initial particles of swarm it is common to set randomly distributed particles across the design space, so:

$$x_{i,j}^0 = x_{j\,min} + r(x_{j\,max} - x_{j\,min}) \tag{15}$$

$$v_{i,j}^0 = 0 \tag{16}$$

where $x_{i,j}^0$ means the initial position for design variable *j* of particle *i*, *r* means an uniformly random generated number in the [0,1] range, $x_{j\,min}$ and $x_{j\,max}$ means the lower and upper bounds for design variable *j.* It is implicit in this formulation that the iterations mean the time step of the process. A simple way to understand this updating procedure is depicted by Hassan *et al.*(2005) and indicated in the following Fig. 3.

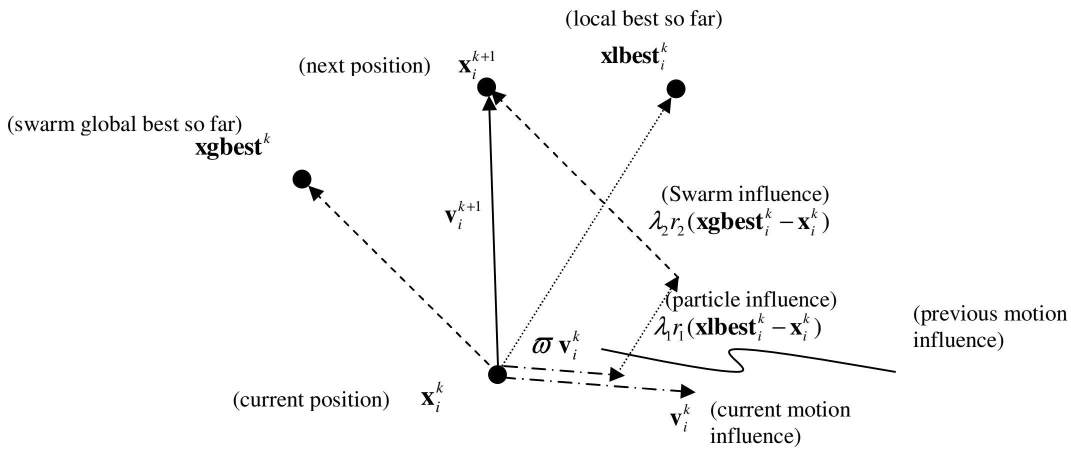

Figure 3. Vector representation of velocity and position updates in Particle Swarm Optimization Algorithm (Hassan *et al.,* 2005).

The easier way to set a convergence criterion for the algorithm is monitoring the differences in the global best design variables between iteration or even the global best objective function. However a more effectively one can be built based on the Coefficient of Variation of objective function in the swarm. In this paper a combination of the three criteria was simultaneously employed. In the following a pseudo-code for the implemented Swarm Optimization Algorithm is depicted in the Fig. 4.

7. EXAMPLES

7.1 Minimization of the applied load

In this example it is desired the dynamic vehicle loads to be minimized in order to decrease the damage on the pavement. It is intended to compare the solutions obtained with those obtained by Sun *at al.* (2007). So it is assumed the following vehicle constant parameters: $m_t = 550$ kg, $m_s = 4450$ kg, $c_t = 0\,Ns/m$, $v = 20\,m/s$ ($72\,km/h$) and for the pavement a roughness parameters in the split power law, it is assumed the following parameters $C_{sp} = 10^{-2}$, $\Omega_l = 0.1 cycle/m$ and a $w_1 = w_2 = 2.0$.

Set the algorithms parameters: number of particles *n*, number of design variables *m*, cognitive parameter c_1, social parameter c_2, velocity momentum ω, coefficient to avoid divergence χ, minimum coefficient of variation cov_{min}, upper and lower bound for design variables $\mathbf{x}_{min}$ and $\mathbf{x}_{max}$.
Create initial random Swarm and initialize the local best values
For each particle *i* in the swarm
  For each design variable *j*
    r=*uniform*[0,1]
    $x^0_{i,j} = x_{j\,min} + r(x_{j\,max} - x_{j\,min})$
    $v^0_{i,j} = 0$
    Set the local best design variable as the current one
    $xlbest_{i,j} = x_{i,j}$
  End
  Set the local best objective function as the current one
  $f_i(\mathbf{xlbest}_i) = f(\mathbf{x}_i)$
End
Iterates with the Swarm to find particle with design variables that lead to a minimum objective function
Loop until convergence criterion of Coefficient of Variation($cov < cov_{min}$), global best objective function ($f(\mathbf{xgbest}_{i+1})$ - $f(\mathbf{xgbest}_i)$ <tolerance) or global best design variable ($|\mathbf{xlbest}_{i+1} - \mathbf{xlbest}_i|$ <tolerance) of the Swarm is met
  Evaluate for each particle the objective function $f_i(\mathbf{x}_i)$
  Update the local best and their objective function
  For each particle *i*
    If $f(\mathbf{x}_i) <$ $f_i(\mathbf{xlbest}_i)$ then $f_i(\mathbf{xlbest}_i) = f(\mathbf{x}_i)$ and $\mathbf{xlbest}_i = \mathbf{x}_i$
  End
  Find the minimum particle objective function *min*($f(\mathbf{x}_i)$)
  If *min*($f(\mathbf{x}_i)$) < $f(\mathbf{xgbest}_i)$ then $f(\mathbf{xgbest}_i) =$ *min*($f(\mathbf{x}_i)$) and $\mathbf{xgbest}_i = \mathbf{x}_{index[\min(f_i(\mathbf{x}_i))]}$
  For each particle *i* in the swarm
    r_1= *uniform*[0,1]
    r_2= *uniform*[0,1]
    $\mathbf{v}_i^{k+1} = \omega \mathbf{v}_i^k + c_1 r_1 (\mathbf{xlbest}_i^k - \mathbf{x}_i^k) + c_2 r_2 (\mathbf{xgbest}_i^k - \mathbf{x}_i^k)$
    $\mathbf{x}_i^{k+1} = \mathbf{x}_i^k + \chi \mathbf{v}_i^{k+1}$
  End
End

Figure 4. Pseudo-code for the simple Particle Swarm Optimization.

Similarly to Sun *et al.* (2007) example, the following constraints to the design variables were set:

$$\begin{aligned} &Find \quad k_1, k_2, c_1 \\ &to\ Minimize \quad \sigma_P^2 = \int_{-\infty}^{\infty} S_P(\omega) d\omega \\ &s.t. \quad 1x10^5\ N/m \le k_1 \le 3x10^6\ N/m \\ &\qquad 1.5x10^6\ N/m \le k_2 \le 2x10^6\ N/m \\ &\qquad 0\,Ns/m \le c_1 \le 3x10^5\ N/m \\ &\qquad \sigma_{Z1} \le 0.2\ m \end{aligned} \tag{17}$$

The constraints in the optimization problem are treated as usual, using the penalty methodology with high penalization coefficient (in this work 1000). The used parameters for the swarm optimization algorithm are indicated by Tab. 1.

Table 1. Swarm optimization parameters.

Velocity momentum ω	Constant c_1	Constant c_2	Number of Particles m	Tolerance tol
0.5	2.0	2.0	30	$1x10^{-6}$

This optimization took 16 algorithm's iterations to find a minimum of $\sigma_P^2 = 1.88x10^{11}\ N^2$. The total number of functions evaluations was 320. No information regarding the number of function evaluations is reported by Sun *et al.*(2007). The final design variables were: $k_1 = 1x10^5\ N/m$, $k_2 = 1.5x10^6\ N/m$ and $c_1 = 2.355x10^4\ Ns/m$. The variance of the suspension deflection using the optimum variables is $\sigma_{Z_1}^2 = 0.165\,m^2$. IN this example, the two initial design variables, k_1, k_2 assumed lower bound values and the suspension damping c_1 assumed an intermediate value. Table 2 compares the obtained results with those by Sun *et al.* (2007).

Table 2. Comparison between Swarm results and Genetic Algorithm results (Sun *et al.*, 2007).

Velocity momentum ω	$k_1\,(N/m)$	$k_2\,(N/m)$	$c_1\,(Ns/m)$	$\sigma_P^2\,(N^2)$	$\sigma_{Z_1}^2\,(m^2)$
Sun *et al.*(2007)	$6.2218x10^5$	$1.705449x10^6$	$2.6582x10^4$	$2.9x10^{11}$ (*)	-(**)
Present work	$1x10^5$	$1.5x10^6$	$2.355x10^4$	$1.88x10^{11}$	0.165

(*) estimated from Figure.
(**) not specified

Figure 5 shows the objective function (dynamic vehicle load) decrease behavior during swarm optimization process.

Figure 5. Objective function decrease (dB) during swarm optimization iterations.

Figure 6 shows the design variables convergence during swarm optimization iterations.

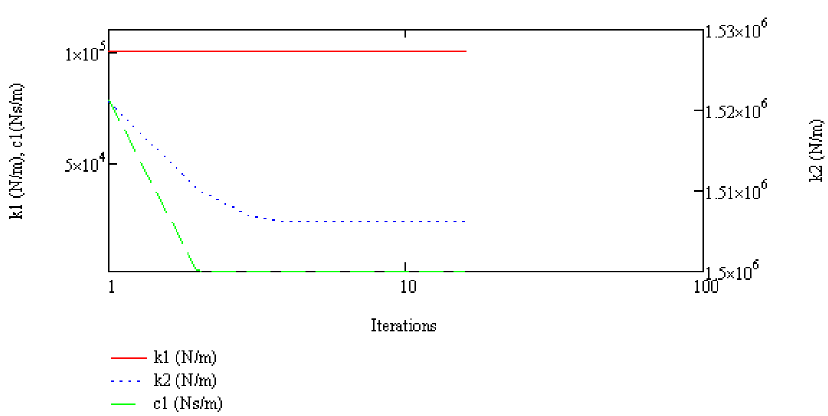

Figure 6. Design variables k_1, k_2 and c_1 during swarm optimization iterations.

Figure 7 shows the variance in the dynamic vehicle load during iterations of the swarm optimization.

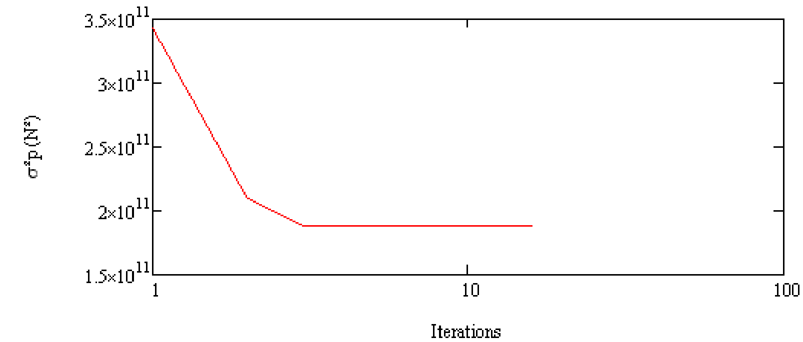

Figure 7. Variance in the dynamic vehicle load during swarm optimization process iterations.

7.1 Minimization of suspension deflections

In this example it is desired the vehicle's deflections (z_1) to be minimized in order to increase the passenger's comfort during the journey. In order to accomplish this intent the objective function is set as the variance of the vehicle suspension deflection ($\sigma^2_{z_1}$). So, it is assumed the same vehicle's parameters, and road roughness from the previous example. The same design variable constraints from the previous example were used. Again, the constraints in the optimization problem are treated as usual, using the penalty methodology with high penalization coefficient (in this work 1000). The swarm optimization parameters used were the same indicated for the previous example indicated in Tab. 1. This optimization took 17 algorithm's iterations to find a minimum of $\sigma^2_{Z_1} = 1.713x10^{-3}\ m^2$. The total number of function evaluations was 340. No information regarding the number of function evaluations is reported by Sun *et al.*(2007). The final design variables were: $k_1 = 3x10^6\ N/m$, $k_2 = 1.5x10^6\ N/m$ and $c_1 = 3.0x10^5\ Ns/m$. The variance of the suspension deflection using the optimum variables resulted in a dynamic load of $\sigma^2_P = 1.07x10^{11}\ N^2$. The two initial design variables k_1, k_2 converged to upper and lower bound values, respectively, and the suspension damping c_1 converged to the upper bound value. Figure 8 shows the variance in the vehicle's deflection during iterations of the swarm optimization.

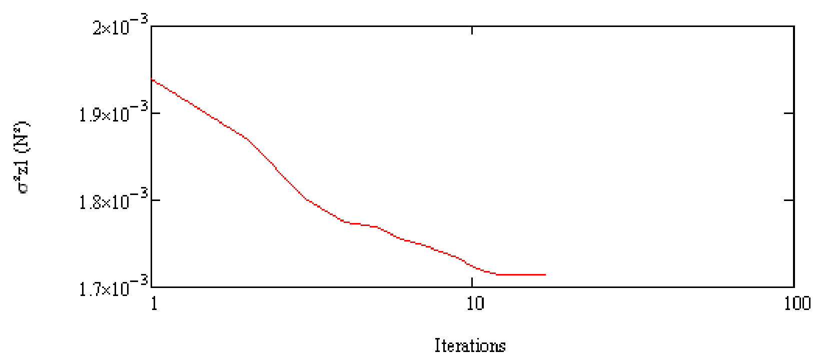

Figure 8. Variance in the vehicle's deflections during swarm optimization process iterations.

8. CONCLUSIONS

This paper presented a methodology to improve the design of suspension systems of vehicles. The design of the suspension system was treated in the frequency domain and the vehicle's suspension system modeled as quarter car 2 degree of freedom model. The road roughness was modeled as a random process with prescribed PSD following the ISO8606 Standard. The parameters to be optimized were the suspension and tire stiffness and suspension damping. A new algorithm was used to perform the optimization. Since the design problem is a hard non-linear problem, a Swarm Optimization algorithm was selected based on its capabilities dealing with this kind of problems and low number of heuristic parameters to be fitted. The main steps of the method were explained and the pseudo-code for the algorithm used in the evaluations was depicted. Two examples of design optimization of a suspension system were developed. On the first one, dynamic load applied of into the road by the vehicle suspension was used as objective function to be minimized. This example was compared with a literature and presented better results. In the second example, suspension deflections were used as objective function to be minimized. The two initial design variables converged to upper and lower bound values, respectively, and the suspension damping converged to the upper bound value. Both examples showed the robustness and the low number of function evaluations required by the algorithm in the optimization.

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