Experimental and Numerical Analysis of a Sandwich Beam with Viscoelastic Layer

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Abstract. *In recent years, unconstrained and constrained viscoelastic damping materials have been largely used for improving the damping and, consequently, reducing the levels of vibration of structures. Extensive researches are being conducted at the Laboratory of Structures (COPPE/UFRJ) in this subject. In the present work, experimental and numerical analysis of the dynamics and damping properties of a sandwich beam with a viscoelastic layer is considered. The material functions of the viscoelastic core are first determined in order to obtain the required data to characterize the material. Then, a parameter estimation method built on the Levenberg-Marquardt and Genetic Algorithms is considered for determining the appropriated parameters of the Wiechert model for linear viscoelasticity. The numerical analysis of the simple and sandwich beams were carried out on the commercial finite element program ANSYS®. The results show the significant improvement on the damping characteristics of the structure, due to the viscoelastic damping layer. Besides, a satisfactory correlation between the numerical and experimental analysis in the frequency domain was achieved.*
Keywords: *Viscoelastic damping, Sandwich beam, Finite element model, Wiechert model.*

1. Introduction

The marine risers play an important role in the offshore industry, since they are used to bring oil and gas from the well on the sea floor to the platform on the water surface. Due to distributed loads from the ocean currents, the risers may experience undesirable vortex-induced vibrations (VIV) that may yield fatigue damage, shortening their operational life (Pantazopoulos, 1994; Le Cunff et al., 2002). Therefore, the prediction of the dynamic behavior and the suppression of the risers vibrations are important issues for improving their life span. In recent years, extensive research is been conducted on these subjects (How et al., 2008; Do and Pan, 2008).

Aiming at improving the damping mechanisms of the structures and, consequently, reducing the levels of their vibrations, unconstrained and constrained viscoelastic damping materials have been largely used (Trindade and Benjeddou, 2002). As the elastic and dissipative properties of viscoelastic materials depend, generally, on the frequency, operating temperature and amplitude and type of excitation, the analysis and design of high performance structures with viscoelastic layers require an accurate model to describe it dynamics. Extensive researches are being conducted at the Laboratory of Structures (COPPE/UFRJ) in obtaining experimental methods and computational tools for characterization and modeling of viscoelastic materials (Faísca, 1998; Silva, 2007). These researches have the final objective of improving the modeling and damping mechanisms of marine risers with constrained viscoelastic layers.

One of the earlier methods concerned to the analysis of constrained layer damping is the RKU method (Kervin et al., 1959). However, this model, based on the sandwich beam theory, may be not suitable when the viscoelastic core has a considerably lower modulus compared to the base and constraining layer moduli. The loss factor predicted by the RKU method, in such cases, may be much lower than that predicted by more accurate finite element models (Imaino and Harrison, 1991). However, there are some difficulties in deriving an appropriate finite element model capable of accurately describing the dynamic behavior of viscoelastic materials. Some of them are: To incorporate the experimentally derived material functions into the model, to model the material behavior dependency on the frequency, temperature and excitation, the relatively large model obtained due to the additional degrees of freedom incorporated into it.

In the present work, experimental and numerical analysis of the dynamics and damping properties of a sandwich beam with a viscoelastic layer is considered. The Wiechert model is used for modeling the viscoelastic material. Firstly, the complex relaxation modulus and creep compliance function is experimentally determined in order to appropriately characterize the viscoelastic material. Further, a parameter estimation method built on the Levenberg-Marquardt and Genetic Algorithms is considered for determining the appropriated parameters that fits the predicted material functions to those experimentally obtained. The modal properties, natural frequency and mode shape, of the sandwich beam were determined considering the Short Time Fourier Transform STFT algorithm (Bucher, 2001) and its displacement due to an impulsive excitation. The modal properties of the corresponding simple beam (i.e., the base beam without the viscoelastic core and constraining layer) were also determined for comparison purposes. All the numerical analysis was carried out on the commercial finite element program ANSYS®. The presented results show the significant improvement on the damping characteristics of the structure, due to the viscoelastic damping layer. Besides, a satisfactory correlation between the numerical and experimental analysis, performed in the time and frequency domains, was achieved

2. Mathematical Modeling of Linear Viscoelasticity

The uniaxial, non-aging and isothermal stress-strain equation for a linear viscoelastic material can be represented by a Boltzmann superposition integral, viz.

$$\begin{aligned} \sigma(t) &= \int_0^t G(t-\tau)\frac{d\epsilon(\tau)}{d\tau}\,d\tau \\ \epsilon(t) &= \int_0^t J(t-\tau)\frac{d\sigma(\tau)}{d\tau}\,d\tau \end{aligned} \tag{1}$$

where $\sigma(t)$ and $\epsilon(t)$ stands for stress and strain, respectively, $G(t)$ is the shear relaxation modulus and $J(t)$ is the creep compliance of the viscoelastic material. Different mechanical models, composed of springs and dampers, are provided in the literature in order to modeling the relaxation modulus and the creep compliance of viscoelastic materials.

The generalized Maxwell model or *Wiechert model*, which consists of a spring and n Maxwell elements connected in parallel, results in the following *Prony series* for the relaxation modulus

$$G(t) = G_\infty + \sum_{i=1}^{n} G_i e^{-t/\tau_i}\ , \tag{2}$$

where G_∞ is the equilibrium modulus, G_i and τ_i are the i-th relaxation strength and relaxation time, respectively.

The creep compliance, on the other hand, can be characterized more easily using the generalized Voigt model or *Kelvin model*, which consists of a spring and a dashpot and n Voigt elements connected in series. This model yields the creep compliance given by

$$J(t) = J_g + \sum_{i=1}^{n} J_i\left(1+e^{-t/\varrho_i}\right), \tag{3}$$

where J_g is the glassy compliance, J_i and ϱ are the i-th retardation strength and retardation time, respectively.

Considering Eq. (2), the corresponding relaxation function in the frequency domain may be obtained [**?**]

$$G(\omega) = G'(\omega) + jG''(\omega), \tag{4}$$

where $j=\sqrt{-1}$, $G'(\omega)$ is the storage function and $G''(\omega)$ is the loss function, given by

$$\begin{aligned} G'(\omega) &= G_\infty + \sum_{i=1}^{n}\frac{\omega^2\tau_i^2 G_i}{\omega^2\tau_i^2+1}, \\ G''(\omega) &= \sum_{i=1}^{n}\frac{w\tau_i G_i}{\omega^2\tau_i^2+1}. \end{aligned} \tag{5}$$

Therefore, for a given viscoelastic material, an inverse problem of parameter identification may be defined in order to fit the material functions in Eqs. (2-4) to a set of experimental data. A method of interconversion between the linear viscoelastic material functions is given in [**?**], such that only one of these functions need to be determined.

3. Characterization of the viscoelastic material

In order to characterize the viscoelastic material, one should determine the parameters of complex relaxation modulus $G(\omega)$ and that of the creep compliance $J(t)$, defined in Eqs. (4) and (3), respectively. As one may use an interconversion method to obtain the parameters of the creep compliance function from those of the complex relaxation, the following vector of unknown parameters may be defined

$$\mathbf{p} = [G_\infty \quad G_1 \quad G_2 \quad \ldots \quad G_n] \tag{6}$$

The present inverse problem of parameter estimation can then be stated as the minimization problem

$$\min_{\mathbf{p}}(E_\omega^2 + E_t^2), \tag{7}$$

where the frequency- and time-domain error functions are given, respectively, by

$$\begin{aligned} E_\omega^2 &= \sum_{j=1}^{N_t}\left[P_j^1\left(\bar{G}'_j - G'_j\right) - P_j^2\left(\bar{G}''_j - G''_j\right)\right], \\ E_t^2 &= \sum_{j=1}^{N_t}\left[P_j^3\left(\bar{J}_j - J_j\right)\right]. \end{aligned} \tag{8}$$

where N_t is the total number of samples considered in the estimation process, $\bar{G}'_j$, $\bar{G}''_j$ and $\bar{J}_i$ are, respectively, the experimental values of the storage, loss and creep compliance functions at the i-th sample; and G'_j, G''_j and J_i are the analytical counterparts, given by Eqs. (3) and (5).

In the present work, the Levenberg-Marquardt method and the Genetic Algorithms were used to solve the inverse problem defined in Eq. (7).

4. Experimental Setups

This section describes the experimental setups considered in the present work. The first one, depicted in Fig. 1, is used for determining the properties of the viscoelastic material VHB 4955, manufactured by 3M, that is used as the damping material in the sandwich beam. In this apparatus, an aluminum box is connected by eight viscoelastic patches to a support fixed to the shaking table. This assembly permits adding extra masses through the addition of metallic plates into the aluminum box. A long stroke shaker imposes displacements to the system and a load cell and four contactless sensors measure, respectively, the load and displacement suffered by the viscoelastic patches.

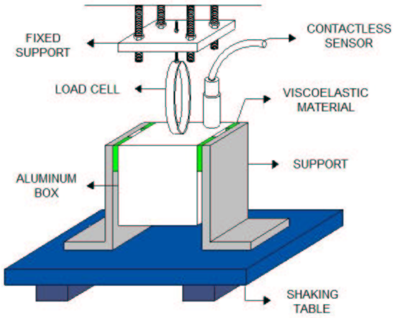

Figure 1. Experimental setup for characterizing the viscoelastic material.

The other experimental setup, depicted in Fig. 2, is concerned with the determination of the dynamics and the modal parameters of the simple and sandwich beam. The simple beam is the one without the viscoelastic and constraining layers. This apparatus is composed of a clamping mechanism and a contactless sensor used for measuring the displacements at a point in the beam due to initial conditions or impact excitations.

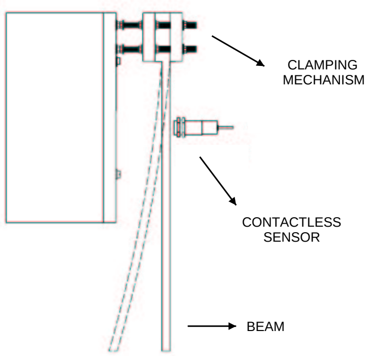

Figure 2. Experimental setup for determining the modal properties of the simple and sandwich beams.

5. Results

In the present work, the commercial finite element package ANSYS® (Release 11) was used to develop a 3-D model of the clamped-free sandwich beam with viscoelastic layer. The finite element model considers the host beam and the constraining layer as isotropic and purely elastic materials. The core is assumed a linear viscoelastic material with a frequency dependent shear modulus $G(\omega)$, as given by Eq. (4). Further, the layers are assumed to be perfectly bonded to each other.

The geometric properties of the sandwich beam are: length $L = 900$ mm, width $w = 50$ mm, thickness of the host beam and constraining layer $h_a = 5$ mm and thickness of the core $h_v = 2$ mm. In the structure, the host beam and constraining layer are made up of aluminum with the following material properties: Elastic modulus $E_a = 72.6$ Gpa and mass density $\rho_a = 2710$ kg/m^3. The elastic modulus of the aluminum was properly adjusted such that the first analytical natural frequency of the simple beam (i.e., without the core and constraining layers) fitted the corresponding experimental one. The dynamic modal properties, damping ratios and natural frequencies, of the simple beam are presented in Table 1. These parameters were determined using the experimental setup depicted in Fig. 2 and considering the displacement response of the beam due to an impulsive excitation. The decrement logarithmic and the Short Time Fourier Transform STFT algorithm were used for determining the modal parameters for the first and second modes, respectively.

Table 1. Experimental modal properties of the simple beam.

Mode Number	Frequency (Hz)	Damping Ratio (%)
1	5.16	0.23
2	31.54	0.05

The first two natural frequencies of the simple beam, experimental, analytical and predicted by a finite element model FEM, are shown in Tab. 2. The attributes of the FEM of the simple beam are the same as those of the sandwich beam and are stated in the sequel. The FEM predicted modal parameters were directly obtained using the modal analysis of ANSYS®.

Table 2. Natural frequencies of the simple beam.

Mode Number	Experimental (Hz)	Analitycal (Hz)	FEM Predicted (Hz)
1	5.16	5.16	5.16
2	31.54	32.34	32.34

In order to determine the parameters of the Wiechert model for the viscoelastic core, the complex relaxation modulus and creep compliance of the material was determined using the experimental setup depicted in Fig. 1. In that system, the geometric properties of the aluminum box are: 100 mm in length, 100 mm in width and 75 mm in height; yielding a mass of 73.4 g. The corresponding dimensions of each viscoelastic patch are: 20 mm in width, 19 mm in height and 2 mm in thickness.

In the determination of the complex relaxation function, a narrow-band noise excitation, with frequency band between 0 and 12.5 Hz, was applied by the shaker. Three different levels of force were considered: 1.4, 1.8 and 2.7 N with the aim at verifying the dependency of the relaxation modulus on the force intensity. The imposed force and the resulting displacement were measured, respectively, by the load cell and constactless sensors. The relaxation modulus is then derived from the frequency response function between the measured displacement and force (Faísca, 1998).

Using the same experimental setup, the creep compliance function, on the other hand, was determined measuring the displacement resulting from gradually adding up four weights of 5 N into the aluminum box. In this experiment, only the displacements related with the addition of weight was measured. Therefore, contactless sensors were used to measure the relative displacement between the aluminum box and shaking table and a load cell was used to measure the associated force. These experimental signals were then used for deriving the creep compliance function (Faísca, 1998).

Figure 3 depicts the experimentally determined storage modulus and loss function, along with the analytical ones predicted by the Wiechert model with eight elements, Eq. (5). The corresponding parameters are presented in Table 3. Besides, the elastic modulus and mass density of the viscoelastic core are $E_v = 6.88$ MPa and $\rho_v = 795$ kg/m^3, respectively.

The structural solid element SOLID186 was selected to mesh the entire sandwich beam. This higher order element exhibits quadratic displacement behavior and it is defined by 20 nodes, having three degrees of freedom per node: translations in the nodal x, y, and z directions. Besides, the selected element supports viscoelasticity. A relatively fine mesh was adopted, in order to accurately predict the dynamic behavior of the system. The total number of elements and nodes used to discretize the model is 4500 and 24267, respectively. Figure 4 shows the finite element mesh at the free end of the sandwich beam, where the magenta layer represents the viscoelastic core.

Aiming at obtaining the modal properties of the sandwich beam, an impulsive force, with constant magnitude $f = 10$ N and duration of 0.025 s, was applied at its free end. The displacement at the free end of the sandwich beam due to this impulsive excitation is depicted in Fig. 5, along with the corresponding one for the simple beam. From Fig. 5, one may clearly observe the high efficiency of the viscoelastic constrained layer in damping out the vibrations of the structure.

In order to obtain the displacement response for the simple beam, depicted in Fig. 5, a proportional damping was considered. The damping coefficients $\alpha = 0.15$ and $\beta = 2.74 \cdot 10^{-7}$, of the mass and stiffness matrices, respectively,

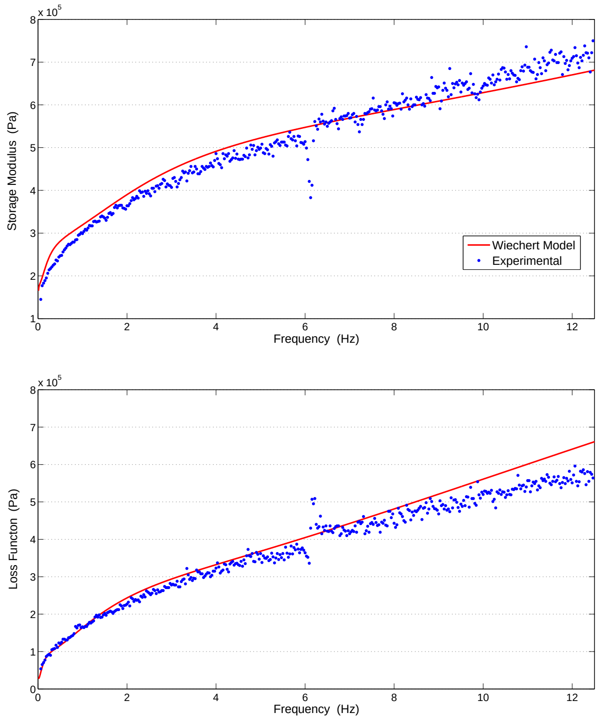

Figure 3. Storage modulus and loss function: Experimental and predicted by the Wiechert model.

Table 3. Parameters of the Wiechert model for the viscoelastic material.

i	G_∞	G_i	τ_i
–	0.05		
1		2.88	0.0028
2		0.27	0.06
3		0.11	0.71
4		0.007	6.33
5		0.03	29.2
6		0.02	48.9
7		0.05	277.8
8		0.023	3233.8

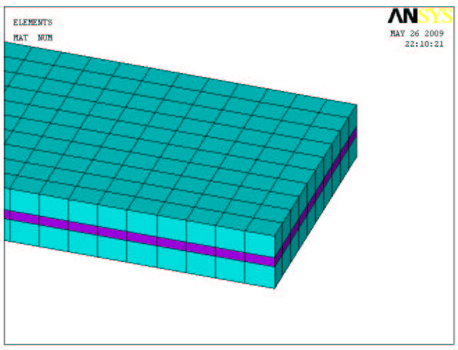

Figure 4. Finite element mesh at the free end of the sandwich beam.

were determined from the modal properties in Tab. 1 and, then, included in the finite element model. Both displacement responses were determined through the transient analysis in ANSYS®.

The modal properties of the sandwich beam, for the first mode, are shown Table 4. These properties where computed

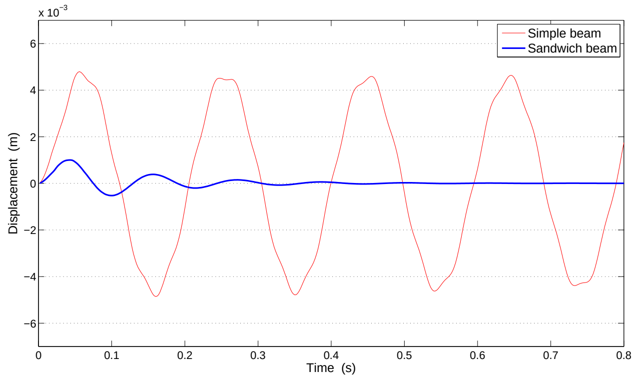

Figure 5. Displacement at the free end of the simple and sandwich beams due to an impulsive excitation.

considering the displacement response of the structure, depicted in Fig. 5, and the STFT algorithm. The displacement responses in Fig. 5 and the data presented in Tab. 4 corroborate with the fact that the constrained layer damping highly improved the damping mechanism of the system. Besides, the predicted modal parameter of the sandwich beam presented satisfactory agreement with the experimental ones.

Table 4. Modal parameters of the sandwich beam.

	First Mode	
Modal Parameter	Experimental	FEM Predicted
f (Hz)	8.25	8.37
ζ	14.22%	15.98%

6. Conclusions

The present work was concerned with the experimental and numerical analysis of a sandwich beam with a viscoelastic layer. The complex relaxation modulus and creep compliance of the viscoelastic material was experimentally determined and, then, used for estimating the appropriate parameters of the Wiechert model for linear viscoelasticity. The Levenberg-Marquadt and Genetic Algorithms methods were used to solve the corresponding inverse problem of parameter estimation such that the predicted material functions fitted the corresponding ones experimentally obtained. The finite element model of the sandwich beam was derived using the commercial finite element program ANSYS® and the element SOLID186 was used to mesh the entire system. The initial results, presented in this work, showed that viscoelastic constrained layers may be used as an efficient solution to reduce the vibration levels of flexible risers through the significant increase on the damping ratio. Besides, the modal parameters of the sandwich beam, predicted by the finite element model, showed satisfactory correlation with the experimental ones. Nevertheless, for future applications in centenary risers, the finite element model built on solid elements would present a huge number of degrees of freedom. Therefore, in order to solve this problem, simpler finite elements, such as 3-D frames, capable of modeling the sandwich beam are under development.

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