

## THREE-DIMENSIONAL SURFACE RECONSTRUCTION USING NURBS

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**Abstract.** *In this paper is presented a new methodology for 3D surface reconstruction from unorganized cloud of points. In the first step of the proposed methodology the number of acquired points is reduced by substituting a set of points closed in a defined region by one at its center of mass, considering a point as a unitary mass. From the new set of points the contour of the object cross-section is drawn using NURBS. The analysis of consecutive cross-sections enables to define a new number of knots and control points when the object cross-section is bigger or smaller than the previous ones, in order to simplify the data processing when the cross-section presents a reduction or an augmentation, as in a conical object. From all defined points the surface can be meshed and then, using commercial software the object rendering can be made for its visualization. From simulations results it can be shown that the proposed method is very effective for clustering unorganized point clouds for meshing surfaces. Examples are shown in order to explain the methodology.*

**Keywords:** *Surface reconstruction, Unorganized 3D points, 3D shape recovery, NURBS, 3D reconstruction*

### 1. INTRODUCTION

Due to the increasing interest in using the computer aided design and manufacturer's concept (CAD/CAM) in several areas like as medical science, biomedical engineering, geographical data processing, paleontology and reverse engineering, the digitalization and reconstruction techniques of objects having complex shape and/or free-form have been presented an important development.

The velocity and precision of the digitalization techniques are due to the improvement in areas of physics, electrical, engineering, the lasers' development, CCD cameras, and the high velocity acquisition cards. The graphical reconstruction of a 3D surface is done from a set of points representing the surface coordinates, using an acquisition data process which acquire thousand or millions of data. These acquired data consist in a cloud of points in general, not organized, with gaps and noise, requiring efficient and trustful algorithms capable to obtain computational models from those samples (Curless, 1997).

A surface reconstruction always consists of getting a computational model that resembles more faithfully to the real object and this must be done from a cloud of points obtained from systems based on objects measurements or whose systems that not use measurements. Systems based on measurements uses techniques based on images or on active 3D sensors. Systems not based on measurements are commercial computer animation software that allow the generation of 3D models starting from simple elements like polygonal boxes. In all cases the reconstruction method must identify the correct geometry, the topology and the object shape from a finite ensemble of points (Remondino, 2003).

Several works have proposed techniques to overcome the difficulty involved in the problem of surface reconstruction from unorganized points (Yu, 1999), (Mangiameli et al., 1996), (Fritzke, 1993a, 1993b, 1996), (Ivrissimtzis et al., 2003), (Kawahara and Saito, 1996), (Hoppe et al., 1992), (Amenta et al., 1998).

There is a vast literature on geometry extraction from point clouds. It includes local polynomial fits, global smooth approximations, Voronoi/Delaunay-based methods, and level set methods, like in (Alexa et al., 2003), (Amenta and Kill, 2004), (Edelsbrunner and Mücke, 1994), (Gopi et al., 2000), (Bertalmo et al., 2001), (Xie et al. 2004). Earlier heuristic approaches gave no guarantees, but new powerful methods to give guarantees of faithfulness to the underlying continuous shape in the limit of infinitely dense data, under somewhat restrictive assumptions on object smoothness and the evenness of the sampling (Dey and Sun, 2005a,b), (Kamberov and Kamberova, 2007).

A lot of methods have been developed in order to create a regular and continuous mesh representation from a cloud of points (Mencl, 2001). From the meshed surface others techniques can be used to the pos-processing (textured, shading and rendering) and visualization of the 3D model (Patias, 2001).

The digitalized points obtained through the acquisition system and image analysis techniques like computed tomography (CT), magnetic resonance imaging (MRI) and ultrasound imaging, have allowed to build computational models that reduce the costs and the time of product creation. Splines and NURBS (Non-Uniform Rational B-Splines) have been useful to model the surfaces, enabling a good description of the object surface (Remondino, 2003).

Although the various methods to reconstruct a surface from a cloud of points, in general they have been applied to a specific area or have data restrictions. Problems like noise, shadow (failure of reading due to hidden details), sinuous edges and contours restrict each method have been made difficult the development of a generic and efficient method.

In this paper is presented a new methodology for 3D surface reconstruction from unorganized cloud of points. In the first step of the methodology the number of acquired points is reduced by substituting a set of points closed in a defined region by one at its center of mass, considering a point as a unitary mass. The process is made considering slices of the object. Then, the new set of points of a slice should be organized and using NURBS the object cross-section can be defined. The analysis of consecutive cross-sections enables to define a new number of knots and control points when the object cross-section is bigger or smaller than the previous ones, in order to simplify the data processing when the cross-section presents a reduction or an augmentation, as in a conical object. From all defined cross-section, knots and control points, the surface can be meshed and then, using commercial software the object rendering can be made for its visualization. Examples are presented in order to explain the methodology.

## 2. METHODOLOGY

In a 3D reconstruction process, the results can be an approximated surface or an interpolated surface depending of the used method and the desired objective. The methods are chosen by considering the data processing time and the quality of the final graphics reconstruction which, in general, use polygon (triangulation, for example) or functions like NURBS (Curless, 1997).

The reconstruction using slices (cross section of the object), associated to NURBS method, is a good method because reproduce with accuracy the object surface, and the application of the contour parameters are more simple (Mencl, 2001).

### 2.1. NURBS: Non-Uniform Rational Basis Splines.

NURBS is a standard industrial tool for the representation and design of simple and complex geometry and can interpolate the control points reproducing the object shape with accuracy and allows to change its shape as well (Xie et al., 2004).

The interest to use NURBS is due to the following characteristics: the common mathematical model used to represent standard analytical forms and free-form; they offer good flexibility for the complex shape design; the data processing time is small, and they are generalization of the Bezier and B-splines curves and surfaces.

The NURBS surfaces are a curve set of the same class and is the generalization of a tensor-product B-spline surface. It is defined over the parametric variables  $u$  and  $v$  as:

$$S(u,v) = \frac{\sum_{i=0}^m \sum_{j=0}^n P_{i,j} \cdot w_{i,j} \cdot B_{i,k}(u) \cdot B_{j,l}(v)}{\sum_{i=0}^m \sum_{j=0}^n w_{i,j} \cdot B_{i,k}(u) \cdot B_{j,l}(v)} \quad (1)$$

where  $B_{i,k}$  and  $B_{j,l}$  are B-splines basis functions of degree  $k-1$  and  $l-1$ , respectively. The NURBS surface has  $m \times n$  control points  $P_{i,j}$  and weights  $w_{i,j}$  that define the influence of the control points on the curve. The number of knots is  $(m+k)+(n+l)$ . The number of the control points  $m$  and  $n$  are defined by the surface order on each parametric direction  $u$  and  $v$ , respectively. Each control point has its own weight that is positive, in general.

The B-splines basis functions are knot's functions  $t_i$  in an increasing sequence. Then, the B-splines basis functions can be defined as:

$$B_{i,1}(u) = \begin{cases} 1 & \text{if } t_i \leq u \leq t_{i+1} \\ 0 & \text{otherwise} \end{cases} \quad (2)$$

With

$$B_{i,k}(u) = \frac{u-t_i}{t_{i+k-1}-t_i} \cdot B_{i,k-1}(u) + \frac{t_{i+k}-u}{t_{i+k}-t_{i+1}} \cdot B_{i+1,k-1}(u) \quad (3)$$

The knots are represented by a number list that is commonly called as knot vector that must be an equal or increasing number sequence and, the repetition of the each knot cannot be higher than the surface degree on the respective direction. More details about NURBS curves can be found in Piegl and Tiller (1996).

## 2.2. The reduction method

In order to simplify the reconstruction method and to reduce the data processing time, it had been proposed a method to reduce the number of the acquired points. This method substitutes a set of points closed in a defined region by one at its center of mass (CM), considering a point having a unitary mass.

From simulations the better geometry of the region is a parallelepiped which is defined by dimensions “ $dx$ ”, “ $dy$ ” and “ $dz$ ”. As the geometry is simple, it is easy to identify its inner points and elaborate the algorithm leading to a small data processing time. Furthermore, it is easy change de values of the dimensions in order to obtain the better set of points that gives a good surface reproduction. Nevertheless, an indiscriminate changing of the region dimensions can give unprofitable results as shown in section 3. In this way the dimensions of a region is called as “resolution”. From the point clouds one can obtain the number of points,  $n$ , at each region and the  $x_i$ ,  $y_i$  and  $z_i$  coordinates of each  $i$ -th point.

Once both the region dimensions and the number of points in the region are known the center of mass of each region,  $x_{CM}$ ,  $y_{CM}$  and  $z_{CM}$ , can be obtained as

$$x_{CM} = \frac{\sum_{i=1}^n x_i}{n} ; \quad y_{CM} = \frac{\sum_{i=1}^n y_i}{n} ; \quad z_{CM} = \frac{\sum_{i=1}^n z_i}{n} \quad (4)$$

In Figure 1a is represented a set of a planar acquired points in which regions are defined by  $dx$  and  $dy$ , and in Fig. 1b its correspondent center of mass (CM).

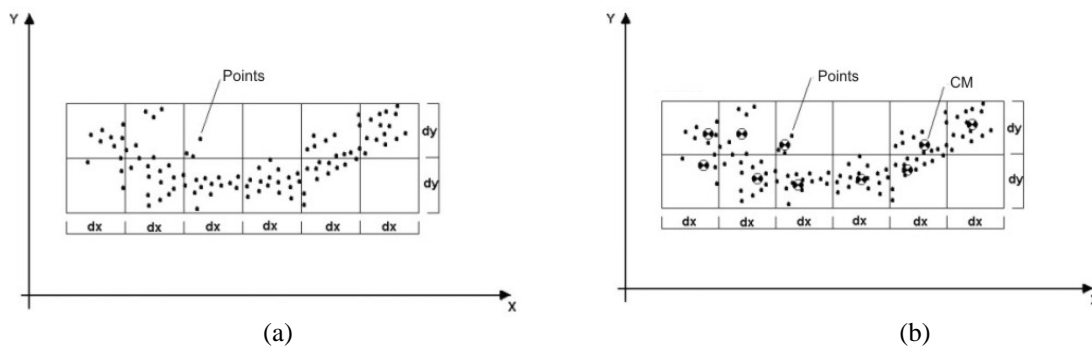


Figure 1. Method for reducing the acquired points. a) The acquired points and the regions; b) Regions and its center of mass (CM).

The final surface quality is function of the chosen resolution (the dimensions of the region), principally for conical surfaces. If the resolution is big, details of the surface will be not represented and the vertices could be truncated. If the resolution is very small, the contour of the cross-section will be irregular and waved.

Numerical simulations have been shown that the resolution can be based on the width of point clouds and the local radii of the section. As result, for radii bigger than ten times the width of cloud points a resolution as two times the width of cloud points can be used. For smaller radii we have been used a resolution equal of the width of cloud points.

In order to show the behavior of the curve when the resolution of the proposed method is not adequate, an example is presented in Fig. 2. The represented section has a medium diameter of 100 mm and the width of the acquired cloud of points is 2 mm. In Figure 2a is represented the curve where the used resolution is 0.8 mm, i.e., less then the recommended. One can see that the curve is not uniform and presents an irregular contour. In Figure 2b an adequate resolution is used, i.e., 4 mm, where one can note the smooth behavior of the curve.

As the methodology uses the cross-section of the object to mesh the surface, one coordinate (for example the  $z$  coordinate) can be obtained from a common coordinate using the least square method, for example, for each slice.

It is important to note that the width of cloud of points is function of the reading equipment and the used resolution defines the number of CM for defining the contour of the object.

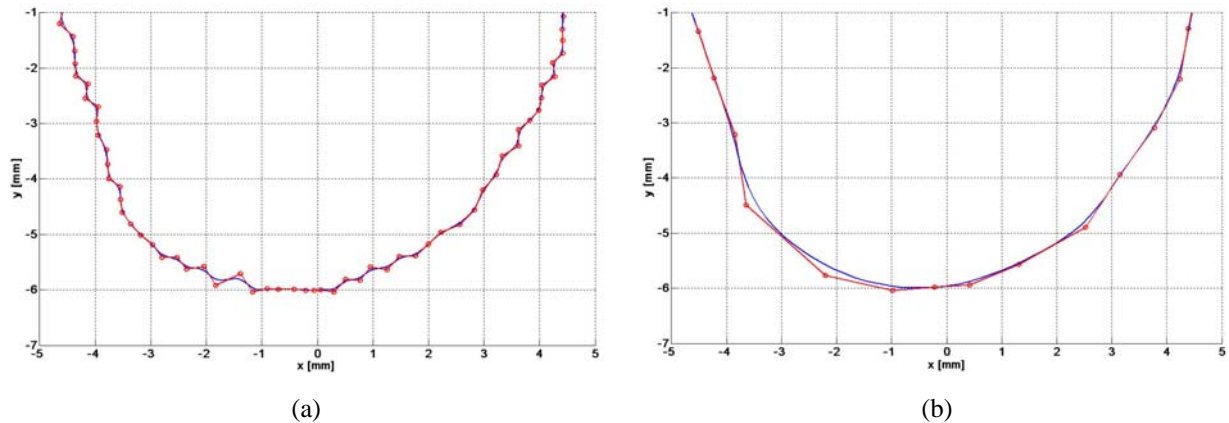


Figure 2. Methodology for reduction the acquired points related to the resolution process.  
 a) Curve using an inadequate resolution; b) Curve using an adequate resolution.

### 2.3. Approach to select the CM to define the contour of the object

To apply NURBS a sequential set of points must be defined. Then, in the proposed methodology it must be defined the sequential CM that will define the cross-section of the object. The selected CM to form the contour of the cross-section of the object must represent the major probability of that CM being a point of the object surface.

From numerical simulations two parameters have been chosen: the number of acquired points in the region (more the number of points in a region bigger is the probability of its CM be a point of the object surface), and the distance between CM in a local macro-region as described in following.

The first step to define the sequential CM consists in choosing a CM to be the initial reference CM. Then a local macro-region should be defined in which is composed by all neighbor regions from the reference CM, as shown in Fig. 3. In the figure, the region that was chosen the CM as reference is noted as 1.

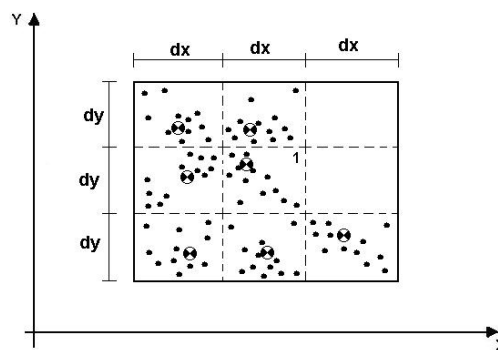


Figure 3. A local macro-region.

Once the local macro-region had been defined, the two parameters, the number of acquired points of each region and the distance between the reference CM and the others one, can be computed. Then, the bigger value of the product of these two parameters will define the next CM belonging to the contour, and that will be the next reference CM. The others regions of the analyzed local macro-region are eliminated for the next step. Thus, the process repeats till close the contour. At the end of the process the CM that will define the contour of the object cross-section in a connectivity order is defined yet.

### 2.4. Algorithm to mesh the surface

After the two previous approaches, the CM which defines the cross-section of the object are known. Thus it is possible to draw a curve for each slice by computing the control points and knots necessary to represent the object surface by using NURBS. In Figure 4 is shown a set of acquired points, the CM used as control points and segments connecting them, and the obtained NURBS curve. In the example the regions are defined for resolutions  $dx=dy=2.8\text{mm}$ .

Generating a mesh of a three-dimensional surface from curves representing slices of the object needs to construct transversal curves intersecting the previous ones, from corresponding points that defines the object shape. As each transversal curve must intersect only one point of each slice it is necessary that all cross-section curves have the same number of points.

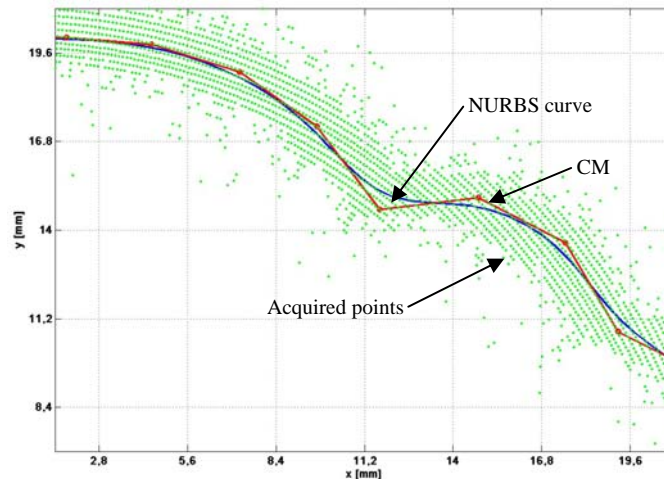


Figure 4. Example of a NURBS curve obtained from initial CM.

However, the number of points at each cross-section is not the same, mainly in a conical surface, in which as near is the vertices less is the necessary number of points to define it. To overpass this problem an approach had been proposed to connect the control points of consecutive cross-section.

The first step of this approach consists in defining a transversal line to be used as reference for numbering the CM. In this work it had been chosen a line which pass by the axis  $x=0$  as shown in Fig. 5, and then, at each cross-section, the first CM is the nearest of this reference line, no matter if the CM is before or after the reference line, represented in Fig. 5 by black points. Then, the knots and control points are numbered from these first CM, which numbering had been adopted counterclockwise, but can be done another way. From the numbered CM a first curve of the cross-section can be draw and then a novel set of knots and control points can be defined.

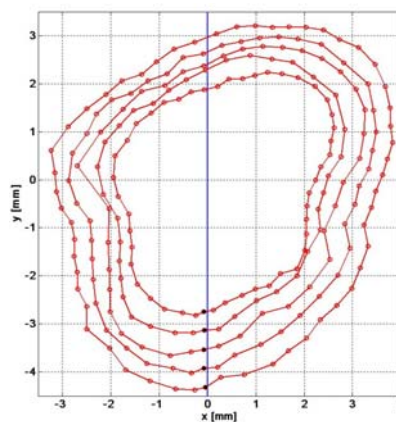


Figure 5. Selecting process for numbering the CM in order to draw the NURBS curve.

At each cross-section the novel set of knots are equidistant and are in same quantity of the CM defining the cross-section, and are used to obtain a new set of control points (CP) that will be use in the surface meshing.

When the consecutive cross-section is reduced, a smaller number of points is necessary to define it. From numerical simulations had been defined a procedure to connect two control points of consecutive cross-section to simplify the meshing process without compromise the surface shape. The procedure is as follows.

- The number of control points from two consecutive cross-sections must be compared. The curve in which has a bigger number of control points (in general the bigger curve) should connect the difference in pairs of control points with the same quantity of control point of the smaller curve. For example: the curve 1 has 15 CP (the bigger curve); the curve 2 has 10 CP (the smaller curve); the difference of CP is 5; then, 5 pairs of CP of curve 1 should be connected to 5 CP of curve 2.
- To select which CP should be connected, the distance between CP of the bigger curve are computed. The CP whose the distance are smaller are chosen to be connected.
- The connecting sequence is the same numbering sequence of CP. This is justified because the CP whose are closed in the bigger curve should be closed in the smaller one. It is important to note that two consecutive cross-section must be closed for well represent the surface shape.

In order to obtain a regular mesh with a reduced double connections, it has been used the following algorithm.

- a) Select a curve that will be used as a base curve. At the start of the surface reconstruction, the first curve is the first cross-section of the object.
- b) If the next curve is till 25% smaller than the length of the base curve, it is imposed to the smaller curve the same number of CP of the base curve, which is bigger. For that the knot vector of the base curve is repeated to the smaller curve, recalculating a new set of knot points for the smaller curve.
- c) The procedure is repeated for the subsequent curves up to find a curve where its length is smaller then 25% of the base curve. In this case, the precedent procedure for connecting the CP in pairs is applied.
- d) The smaller curve becomes the new base curve and the procedure is repeated till the last curve.

The described procedure can be used for smaller or bigger curve. In Figure 6 is represented the connecting procedure in pairs.

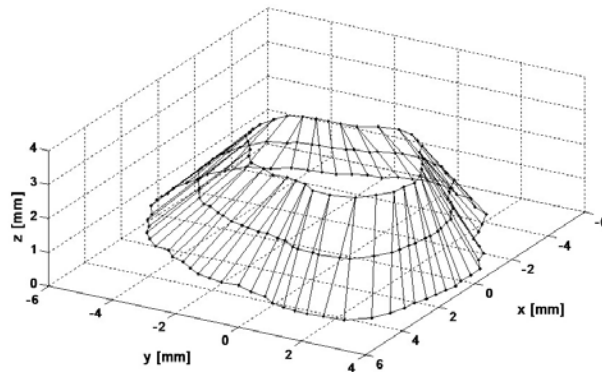


Figure 6. Procedure for connecting control points in pairs.

To obtain a softy surface, one can create intermediary cross-sections between the acquired cross-sections that is based on the knots and control points of then. More details can be seen in Piegl and Tiller (1996).

### 3. NUMERICAL SIMULATIONS AND RESULTS

#### 3.1. Applying to a cylindrical surface

In order to verify the validity of the proposed method it had been used a set of unorganized points and badly acquired resulting in many empty regions (without points) as represented in Fig. 7. It was taken five slices to represent the object and the CM of each cross-section have been organized in a same plane by using the least square method.

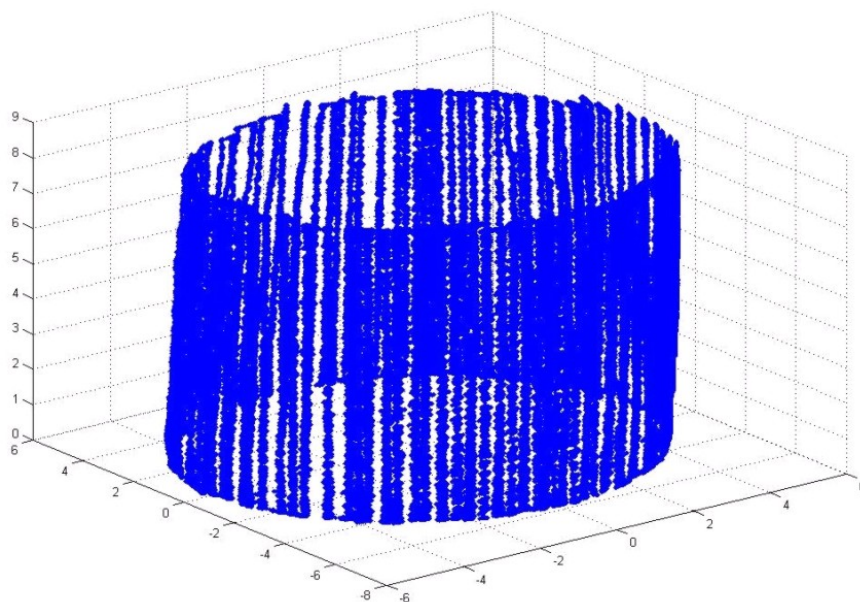


Figure 7. Acquired points representing a cylindrical surface.

As the cross-sections have approximately the same diameter, it was not necessary to change the knot points. By using commercial software for rendering the surface, after the mesh had been concluded, the obtained surface is as represented in Fig. 8.

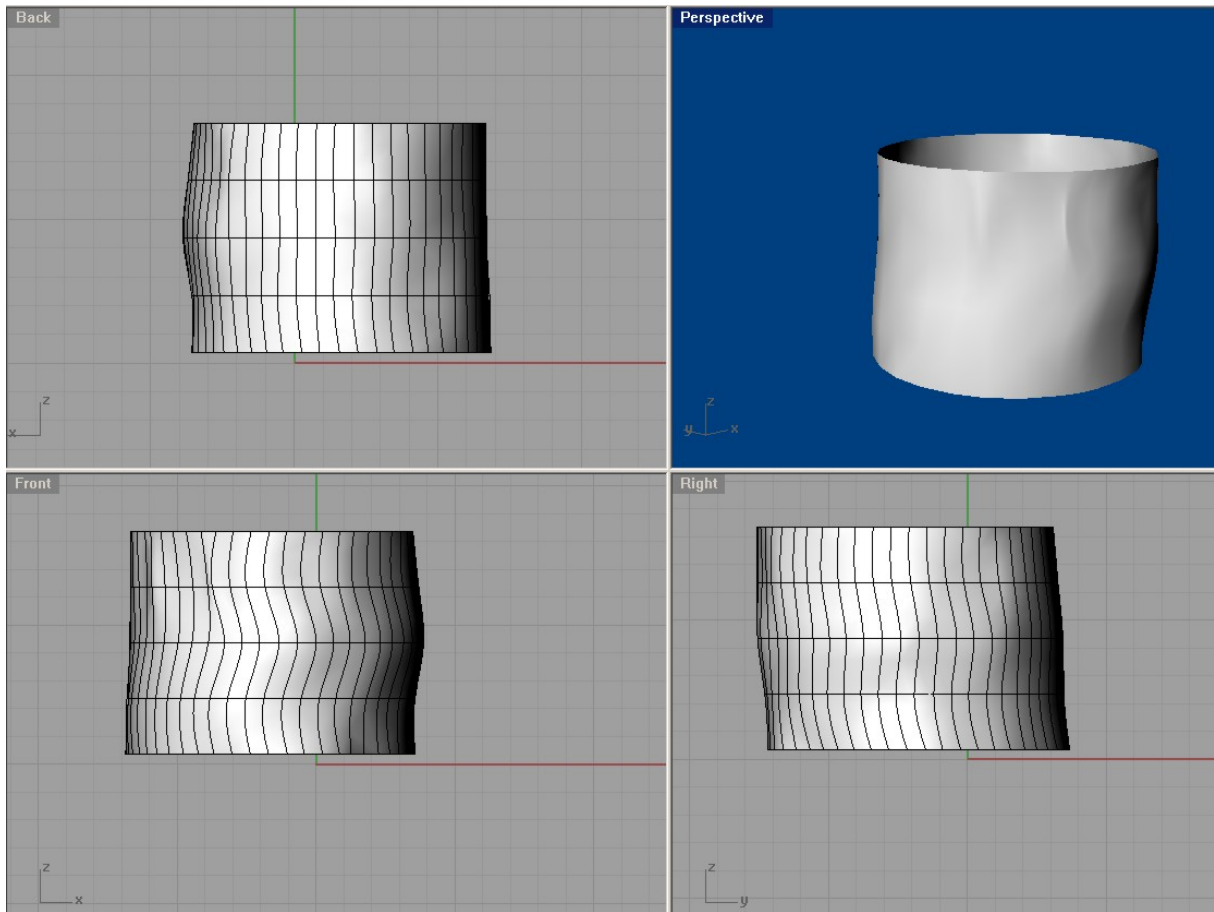


Figure 8. A cylindrical surface rendered from five slices using the proposed methodology.

### 3.2. Applying to a conical surface

The next example it had been used a conical surface where the points are represented in Fig. 9. As the previous example, the points are unorganized and not regular. In the figure each represented point is composed by a small cloud of points.

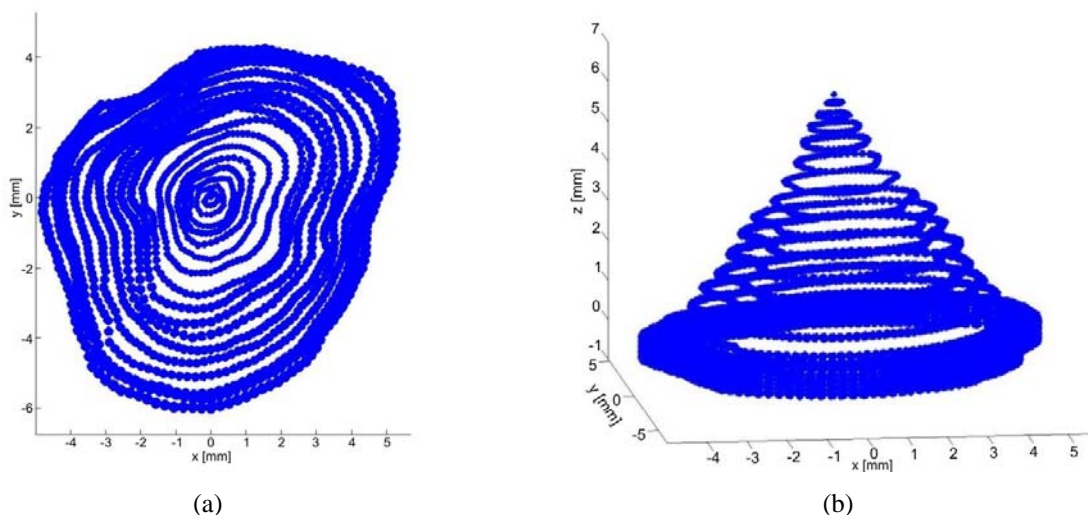


Figure 9. Cloud of points representing a conical surface.  
a) Top view; b) Three-dimensional view.

Applying the described methodology it had been obtained the NURBS curves as represented in Fig. 10a and the mesh as in Fig. 10b.

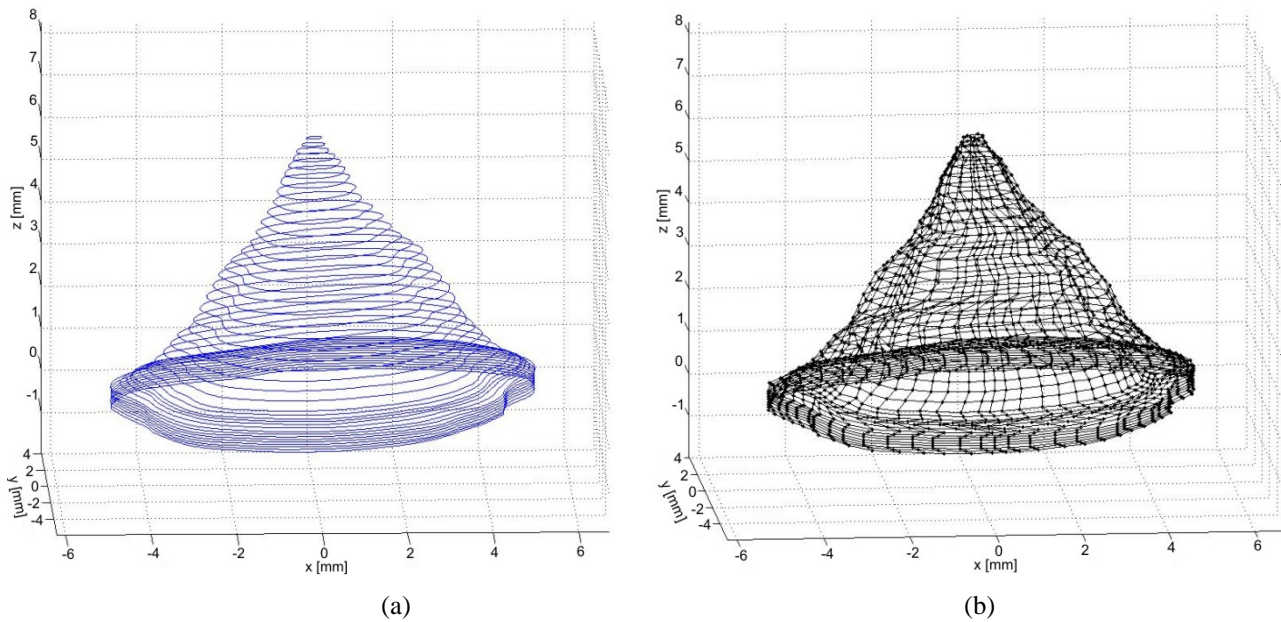


Figure 10. Applying the proposed methodology to a conical surface. a) The obtained NURBS curves; b) The obtained mesh.

Again, using commercial software for rendering the surface, after the mesh had been concluded, the obtained surface is as represented in Fig. 11.

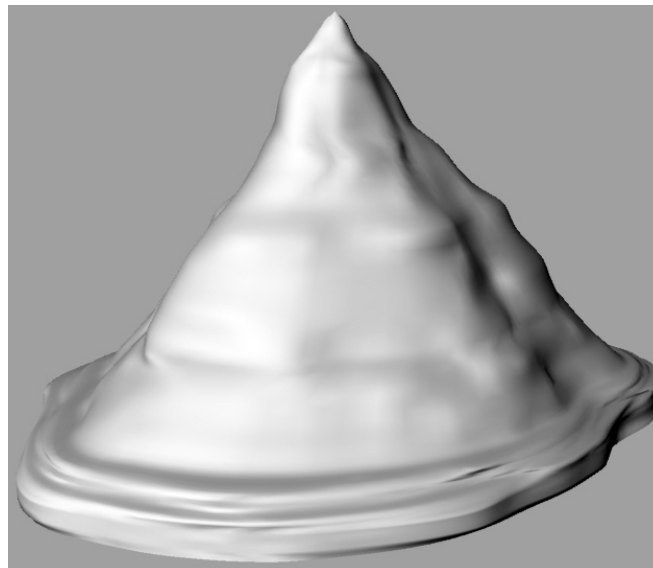


Figure 11. A conical surface rendered using the proposed methodology.

#### 4. CONCLUSIONS

As stated by Remondino (2003) “the mesh generation is the core part of almost all reconstruction programs”. But it is possible after a pre-processing to remove a certain amount of errors introduced by the scanning device, to reduce the input redundancy and the noise and gaps in the point clouds.

Thus, in this paper it had been presented a new methodology for 3D surface reconstruction from unorganized cloud of points. In the first step of the proposed methodology the number of acquired points is reduced by substituting a set of points closed in a defined region by one at its center of mass, considering a point as a unitary mass. From the new set of points the contour of the object cross-section is defined by using NURBS. The analysis of consecutive cross-sections



enables to define a new number of knots and control points when the object cross-section is bigger or smaller than the previous ones, in order to simplify the data processing when the cross-section presents a reduction or an augmentation (jagged surfaces or protrusions).

From all defined points the mesh can be done and then, using commercial software the object rendering can be made for its visualization. For the presented examples it was used the Rhinoceros® software for rendering process ([www.rhino3d.com](http://www.rhino3d.com)).

Simulations results had been shown that the proposed method is effective for clustering unorganized point clouds for meshing surfaces.

We can depict some important advantages of this proposed method: a) the gaps of the cloud of points must not be filled as others methods; b) the cloud of points must not be well organized, and c) it is not necessary an algorithm to reduce noises (they are eliminated by applying the reduction approach). These advantages had been shown in the presented examples. In a future a methodology to consider surfaces having edges will be included.

## 5. ACKNOWLEDGMENTS

The authors are thankful to CNPq and FAPEMIG - Fundação de Amparo à Pesquisa do Estado de Minas Gerais for financing support of this research work from projects TEC 759/01, TEC 712/04 and TEC 2166/07.

## 6. REFERENCES

- Alexa, M., Behr, J., Cohen-Or, D., Fleishman, S., Levin, D., Silva, C.T., 2003, "Computing and Rendering Point Set Surfaces", *Trans. Vis. Comp. Graph.*, Vol. 9, No. 1, pp. 3-15.
- Amenta N., Bern M., Kamvyselis M., 1998, "A New Voronoi-Based Surface Reconstruction Algorithm". *SIGGRAPH'98*, pp. 415-421.
- Amenta, N. and Kill, Y. J., 2004, "Defining Point-Set Surfaces", *ACM Trans. on Graphics*, Vol. 23, No. 3, pp.264-270, Special Issue: Proc. of SIGGRAPH 2004.
- Bertalmo, M., Li-Tien Cheng, Osher, S., Sapiro, G., 2001, "Variational Problems and Partial Differential Equations on Implicit Surfaces", Vol. 174, No. 2.
- Curless, B.L., 1997, "New Methods for Surface Reconstruction from Range Images", PhD Thesis, Stanford University, USA.
- Dey, T.K., Sun, J., 2005a, "An Adaptive MLS Surface for Reconstruction with Guarantees", Tech. Rep. OSU-CISRC-4-05-TR26.
- Dey, T.K., Sun, J., 2005b, "Extremal Surface Based Projections Converge and Reconstruct with Isotopy", Tech. Rep. OSUCISRC-05-TR25.
- Edelsbrunner, H., Mucke, E.P., 1994, "Three-Dimensional Alpha Shapes," *ACM Trans. Graphics*, Vol. 13, No. 10, pp. 43-72.
- Fritzke B., 1993a, "Kohonen Feature Maps and Growing Cell Structures - A Performance Comparison". In: Giles CL, Hanson SJ, Cowan JD (eds) *Advances in Neural Information Processing Systems-5-(NIPS-92)*.
- Fritzke B., 1993b, "Growing Self-Organizing Network for Unsupervised and Supervised Learning". Technical Report ICSTR-93-026. Int. Computer Science Institute, Berkeley.
- Fritzke B., 1996, "Growing Self-Organizing Networks - Why?". *ESANN'96: European Symp. on Artificial Neural Networks*, pp. 61-72.
- Gopi, M., Krishnan, S., Silva, C. T., 2000, "Surface Reconstruction Based on Lower Dimensional Localized Delaunay Triangulation", *EUROGRAPHICS 2000, Computer Graphics Forum*, Vol. 19, No. 3.
- Hoppe H., Deroose T., Duchamp T., McDonald J., Stuetzle W., 1992, "Surface Reconstruction from Unorganized Points". *SIGGRAPH'92 Proc. Conference*, pp. 71-78.
- Ivrissimtzis I., Jeong W., Seidel H., 2003, "Using Growing Cell Structures for Surface Reconstruction". *Proc. of the Shape Modeling International*, pp. 78-86.
- Kamberov, G., Kamberova, G., 2007. "Geometric Integrability and Consistency of 3D Point Clouds". *11th IEEE Int. Conf. on Computer Vision - ICCV 2007, Rio de Janeiro*.
- Kawahara S., Saito T., 1996, "On a Novel Adaptive Selforganizing Network". *CNNA'96, 4th IEEE Int. Workshop on Cellular Neural Networks and Their Applications*, pp. 41-46.
- Mangiameli P., Chen S., West D., 1996, "A Comparison of SOM Neural Network and Hierarchical Clustering Methods". *Eur. J. Oper. Res.*, Vol. 93, pp. 402-417.
- Mencl, R., 2001, "Reconstruction of Surfaces from Unorganized 3D Points Clouds". PhD Thesis, Dortmund University, Germany.
- Patias, P., 2001, "Photogrammetry and Visualization". Technical Report, ETH Zurich, Available at: <http://www.photogrammetry.ethz.ch/research/quest.html>.
- Piegl, L.; Tiller, W., 1996, "The NURBS Book". 2<sup>nd</sup> Edition, Springer, 646p.

- Remondino, F., 2003, "From Point Cloud to Surface: The Modeling and Visualization Problem", The Int. Workshop on Visualization and Animation of Reality-based 3D Models, Switzerland.
- Xie, H., McDonnell, K., and Qin, H., 2004, "Surface Reconstruction of Noisy and Defective Data Sets", Proc. Visualization 04, pp. 259-266.
- Yu Y., 1999, "Surface Reconstruction from Unorganized Points Using Self-Organizing Neural Networks". IEEE Visualization 99, pp. 61-64.

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