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AN OPTIMIZATION APPROACH TO DESIGN FEEDBACK CONTROLLERS FOR FLIGHT CONTROL SYSTEMS

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Abstract. *The role of feedback in the design of aircraft stability and control is becoming increasingly important. The basic reason driving this trend is the necessity to cope with a growing number of more stringent requirements, which are sometimes conflicting. This work presents a design methodology applicable to flight control laws, which can be either a control and stability augmentation system (CSAS) or an automatic flight control system (AFCS). A key feature of the proposed method is the definition of a simple cost function, which incorporates both performance and robustness requirements. The optimization of this cost function is employed to tune some of the control law parameters. For illustration purposes, two design examples are presented, namely a pitch attitude rate regulator and a pressure altitude regulator. The aircraft model under consideration was obtained from Boeing 747 data. Sensors and actuators dynamics, as well as transport delays due to digital control implementation, are considered. Finally, aspects related to data pre-processing, including the employment of complementary filters, are also discussed. The results show that the proposed methodology is able to satisfactorily fulfill design requirements normally applicable to flight control laws.*

Keywords: *Feedback Design, Robust Control, AFCS, CSAS, Flight Controls*

1. INTRODUCTION

This work presents a design methodology applicable to feedback control laws for flight controls applications. Two essential characteristics are prominent on the proposed design methodology. The first characteristic is the employment of an optimization scheme to compute some of the control law parameters. The second one is the usage of the two degree of freedom control law structure.

The first question to be answered is why the proposed design process has these features defining its foundation. This is discussed in the sequence.

1 - Optimization Scheme Rationale: Nowadays, the increasing performance of off-the-shelf computers, as well as the availability of efficient and reliable optimization algorithms (Fletcher, 2000), makes the use of optimization schemes especially attractive to be employed in many different engineering problems. This also holds true in the control engineering area (Looye and Joos, 2006). The idea applied in this work is to let the activities of defining the feedback requirements, along with the control law structure, to the flight control law designer. The designer is also responsible to map the requirements into a cost function, which shall then have its value minimized. The cost function format was formulated in order to permit the quoted mapping to be done as straightforwardly as possible. The optimization routine is used to achieve the control law requirements with minimal effort from the designer in the tuning of parameter values for the controllers.

2 - Two Degree of Freedom Control Structure Rationale: This control structure is proposed because it enables the designer to classify the requirements into two separate parts (Kreisselmeier, 1999). The first part consists of the requirements for the feedback portion (disturbance rejection capability and robustness with respect to model uncertainties) while the second one relates to the desired input-output transmission. This separation facilitates the task of defining the overall control law requirements. This, for instance, enabled the formulation of the proposed unified framework for the design of the control law feedback portion.

The proposed design steps are presented below. The detailed discussion of these is presented in the subsequent Sections.

1. Proper modeling of the aircraft dynamics, as well as the main flight control system elements (sensors, actuators and flight controls computer);

2. Determination of the flight control law range of operation;

3. Determination of the flight control law requirements;
4. Choosing the basic feedback element structure;
5. Setting the data pre-processing, based on the available sensors characteristics and on the decided basic feedback structure;
6. Choosing the set-point generator structure (Pre-filter and Trajectory generator);
7. Computing the values of the control law parameters (scheduled parameters);

Two design examples are given, with the final objective of showing how to apply the methodology presented in this work. The aircraft dynamics explored in these examples are representative of the longitudinal dynamics of a large commercial aircraft. The actuation dynamics, as well as the sensor dynamics, are representative of systems currently available in industry.

This text is organized as follows. Section 2 provides the adopted modeling arrangement, along with a short description of the particular models used in the design examples of Section 6. Section 3 presents the relevant topics concerning the control law range of operation. Section 4 presents the discussion that takes place during the control law requirements definition phase. Section 5 presents one general flight control law structure, along with the overall description of its components. It also provides the details of a proposed technique to automatically determine the values of the control law parameters. Section 6 provides two design examples following the described approach. Final remarks are given in Section 7.

2. MODEL SET UP

The model arrangement depicted in Fig.1 shows the links encountered in a general flight control law application (Aircraft, Sensors, Actuators, flight control computer, Pilot, etc). Aircraft dynamics is represented by the multiple inputs multiple outputs (MIMO) system, having as inputs U_1, U_2, $\ldots$, U_p and as outputs the signals In_1, In_2, In_3, $\ldots$, In_n. The sensors dynamics are located inside the block *Sensor Models*.

The flight control law takes the sensed signals, S_n, and the reference signals, R_t, which are provided either by the pilot or by the navigation computer. The control algorithm provides the demands for the aircraft inputs, $ComU_p$. Pure time delays, representing either the data transportation on digital buses or the computer calculation lag, are also represented inside the block *Flight Control Law*.

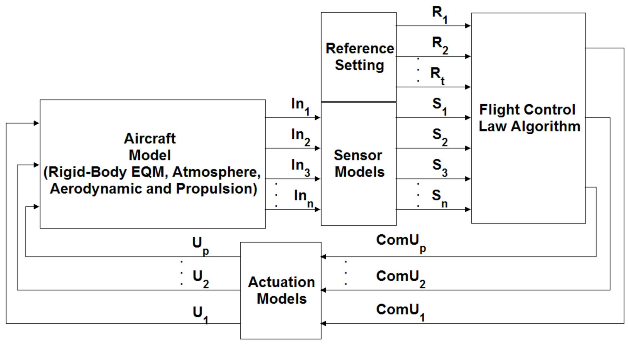


Figure 1. Model Arrangement - Overview

2.1 Aircraft Dynamics

Based on the information presented in (Hanke, 1971; Hanke and Nordwall, 1970; Heffley and Jewell, 1972), a non-linear model of the Boeing 747 aircraft was assembled using Simulink/Matlab.

The analysis in Section 6 is based on two aircraft linear longitudinal models. These represent the two extreme mass configurations available in one flight condition. Table 1 presents the flight condition, together with the description of the mass configurations.

The aircraft linear models have elevator position as input and pitch attitude rate ($\dot{\theta}$), pitch angle (θ), longitudinal acceleration at the center of gravity location (A_{XCG}), normal acceleration at the center of gravity location (A_{ZCG}) and pressure altitude (h_P) as outputs.

Table 1. Linear Models - Flight Condition and Mass Configurations

Case	**Mass** [kg]	$\mathbf{CG}_{POS}$ (1) [%]	$\mathbf{V}_{CAS}$ (2) $[m/s]$	$\mathbf{h}_p$ [m]
Model 1	181440	25	133.7	9100
Model 2	317510	25	133.7	9100

(1): Aircraft center of gravity position, in the longitudinal axis
(2): Calibrated airspeed

2.2 Sensor Models

The output of each sensor carries not only the aircraft specific parameter dynamics, but also has some spurious dynamics due to the sensing device itself. The lesser the contamination of the data with the sensor dynamics, as well as with sensor noise, the better is the specific sensor signal.

In this study, the sensors are modeled as a second order lag plus one pure time delay. For linear analysis purpose, the pure time delays are replaced by second order Padé approximation (Stevens and Lewis, 1992). Table 2 presents the chosen values of the second order lag natural frequency (ω_n), the associated poles damping ratio (ζ) and the pure time delay value.

Table 2. Sensor dynamics Values

Sensed Signal	$\dot{\theta}$	θ	A_{XCG}	A_{ZCG}	h_P
Time Delay [ms]	10	20	10	10	20
ω_n $[rad/s]$	75.4	94.2	50.3	50.3	2.5
ζ	1.0	1.0	1.0	1.0	0.7

2.3 Actuation Models

A practical flight control law design must also take into account the limitations imposed by the actuation system, which can be mathematically described by an input-output relationship between the intended and actual positions.

The elevator actuator is modeled as a second order lag having natural frequency (ω_n) of 31.4 rad/s, damping ratio (ζ) of 0.7 and steady state gain (K) of 1.

2.4 Flight Controls Computer

The control law is considered to be digitally implemented. Table 3 presents the chosen transport delays from sensor to the flight control computer (FCC). In real application, these values are dictated by the priority assigned for each signal, given the limitation imposed by the capacity of the particular digital bus used.

Table 3. Transport delay from Sensor to FCC

Sensed Signal	$\dot{\theta}$	θ	A_{XCG}	A_{ZCG}	h_P
Transport Delay [ms]	10	20	10	10	50

The computation lag is represented by a 25 ms delay, added at the output of the control law algorithm. The effects of the sampler plus the zero order hold (ZOH) is also accounted for.

For linear analysis purpose, the pure time delay is replaced by a second order Padé approximation, while the ZOH is replaced by a first order approximation. These approximations can be found in (Stevens and Lewis, 1992).

3. CONTROL LAW RANGE OF OPERATION

One of the first definitions necessary for a flight control law design is the determination of its range of operation. This includes the description of the required flight envelope, aircraft mass envelope and the expected limit disturbances.

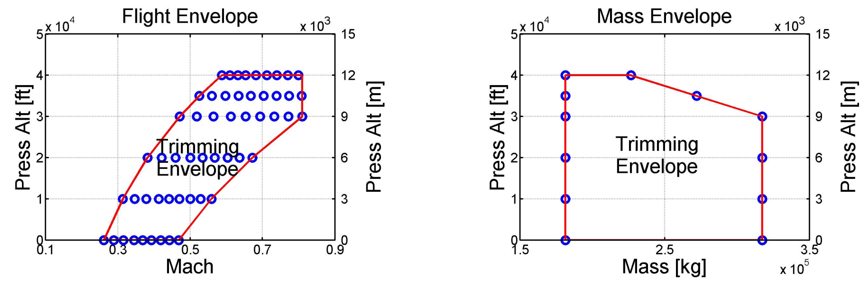
Flight envelope setting consists of determining the expected range of flight conditions, i.e. the range of speed, dynamic pressure, Mach number, pressure altitude, etc, in which the control law is supposed to operate.

Normally, the aircraft mass and its center of gravity (CG) position are not reliably available during the control law operation. This is the case considered in this work. It means that the control law shall be robust enough in keeping the required performance in all possible combinations of aircraft mass and CG positions applicable to a given flight condition.

Part of the control law requirements depends on the knowledge of the expected disturbances that can act on the aircraft during its operation. For instance, it is not appropriate to specify a maximum tracking error for a control system without specifying the expected limit disturbances.

The approach taken in this work consists of defining discrete conditions of the flight envelope and, for each of these, it is determined the aircraft mass and CG envelope in which the control law is expected to operate. Figure 2 (a) presents one typical example of flight envelope applicable to flight control laws (red curve). The blue circles represent the chosen set of discrete flight conditions. Figure 2 (b) provide the mass envelope as function of pressure altitude. The blue circles represent again the selected discrete cases.

Once this done, the next step is to generate aircraft linear models for all the combinations of aircraft mass, aircraft CG and flight conditions, like the ones presented in Table 1. The design work is performed in a set of these models, while the final analysis is performed in each of the defined linear model conditions.



(a) (b)

Figure 2. Flight and mass envelope

4. CONTROL LAW REQUIREMENT DEFINITION

Broadly speaking, it is possible to state that there are two main sources of requirements for a flight control law. These stem from the following two questions:

1. What is the desired behavior of the set formed by the control law plus the aircraft?
2. What are the constraints that might prevent the designer in achieving the desired behavior?

The desired behavior is extracted from Federal Aviation Regulation (FAR) requirements (or the equivalent regulations from another country), advisory circular, Military specifications, industry standards and perhaps, most importantly, from the particular company experience (Gangsaas et al., 2008).

It is of vital importance to define clearly what are the objectives of a given flight control law. Without this knowledge, it becomes difficult to establish an orderly design process.

The plant dynamics, which encompasses aircraft, actuators, sensors dynamics, as well as transport delays, might place limitation in achieving the design targets. This type of limitations is described in (Skogestad and Postlethwaite, 2001) as the output of the *Plant input-output controllability* analysis.

Another source of constraints is linked with the controls input activity. It is necessary to place limitation in this in order to avoid problems linked with reaching either its position or its speed limit as well as issues linked with loads, fatigue, etc. Input activity higher than the necessary can also create issues linked with instability of high frequency modes, i.e. modes located outside of the control law operational range of frequency.

The fact that one particular aircraft can be well modeled as a rigid body does not mean that its flexible modes can be completely disregarded. Design considerations on the flexible modes location and its resonance amplification must be factored in during the design in order to avoid Aeroservoelasticity (ASE) instability.

The design process presented herein tackles the requirements definition by dividing the control law structure into two major sections, the *feedback* portion and the *feedforward* one. This is conventionally known as *two degree of freedom control structure*. An interesting characterization of this type of control structure can be found in (Kreisselmeier, 1999).

The Feedback component is responsible for the disturbance rejection and robustness of the solution. Feedforward (also known as pre-filter) is responsible for shaping the input-output closed loop transfer function matrix to the desired format.

The AFCS functions, like the auto pilots, require one additional element as part of its structure. This is called here as *Trajectory generator* and, as implied by the name, its task consists of generating the desired set-point to be followed by the feedback system.

5. A GENERAL FLIGHT CONTROL LAW ALGORITHM

Figure 3 (a) shows a general layout for a flight control law. In simple terms, the *Data Pre-Processing* portion is responsible for calculating the aircraft variables estimates, which are used by the control algorithm. The *Set-Point Generator* computes the feedback set-points, based on either the pilot or navigation computer inputs and, finally, the *feedback element* computes the aircraft input demands based on the error between the estimated variable and the respective set-point.

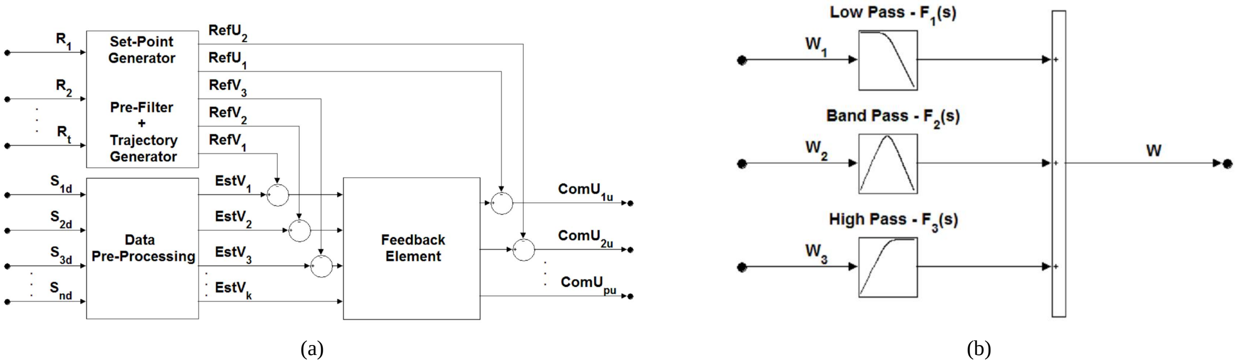


Figure 3. Flight control law general structure and a complementary filter scheme

The following Sub-Sections provide the details of the adopted design process for the *Data Pre-Processing* as well as for the *feedback element*. These two elements make up the first degree of freedom, i.e. they are the ones responsible in guaranteeing the required disturbance rejection and the solution robustness.

5.1 Data Pre-Processing

The signal pre-processing block computes the signals estimate to be used on the control law. It takes the available measurement from the aircraft sensors and performs data computation and filtering.

Examples on data computation are:

- Compute estimates of either linear acceleration, velocity or position in a point other than the sensor particular location, for instance, calculating the longitudinal acceleration at the aircraft center of rotation (or CG), given the inertial measurements at the inertial unit location;
- Compute raw estimate of either $\dot{\alpha}$ or $\dot{\beta}$ based on the inertial measurements.

The filtering is performed on either the raw measurements or on the computed signals. Three broad class of filtering are explored. These are the low pass, the high pass and the complementary filters.

Low pass filters are used when it is desired to remove high frequency content of a given signal. Depending on the particular application, it can be decided to employ a first order, a second order or even a third order filter. Higher order filters can be utilized also, but are usually not required.

High pass is used when the intention is to remove low frequency content from the input signal. The process of setting this type of filters follows a similar path as the process of setting the low pass one.

Both the low pass and the high pass filters incur in deliberately removing frequency content from the original signal. This fact can become a problem in a closed loop application. Complementary filters can be a possible solution when this is the case.

Complementary filters take the low frequency content from one source and the high frequency content from another source. More than two signals can also be used, in a set of low pass, high pass and band pass filters, as illustrated in Fig. 3 (b), where the final estimate of W is obtained out of three raw estimates of W (W_1, W_2 and W_3). The filter is termed complementary filter if $F_1(s) + F_2(s) + F_3(s) = 1$ (Yu and Choe, 2007).

Additional examples exploring complementary filters can be found in (Park, 2004; Looye and Joos, 2001; Yu and Choe, 2007; Looye et al., 2001).

5.2 Feedback Element

As already discussed, two major responsibilities shall be addressed by the set formed by the *data pre-processing* and the *Feedback element*, namely the disturbance rejection and the assurance of overall system robustness.

In order to explain in a concise fashion how these two objectives are tackled, the following definitions, valid in the context of this work, are introduced:

Open Loop Transfer Function($OLTF$): It is the single input single output (SISO) system obtained after the closed loop system is opened at one control law designated location. The particular point where this procedure is carried out can be one of the following: 1 - Any sensor input used by the control law. 2 - Any of the aircraft inputs used by the control law. 3 - Any internal point of the control law block diagram. The abbreviation $OLTF_{In_2}$ denotes open loop transfer function at the signal In_2.

Regulated Variable(RV): It is one aircraft variable whose steady state value shall be controlled by the flight control law. One example is the pressure altitude, in a altitude hold control law.

Performance plot: It is the bode plot of the $OLTF$ relative to either one of the aircraft inputs, used by the control law, or by the summation of all the components relative to the control of a specific regulated variable.

As an illustrative example, consider a case having two regulated variables, namely X_1 and X_3, and two aircraft inputs used by the control law, U_1 and U_2. Figure 4 presents a general layout for the given example. The same discussion as presented in the sequence can be extended for cases having either more or less than two regulated variables.

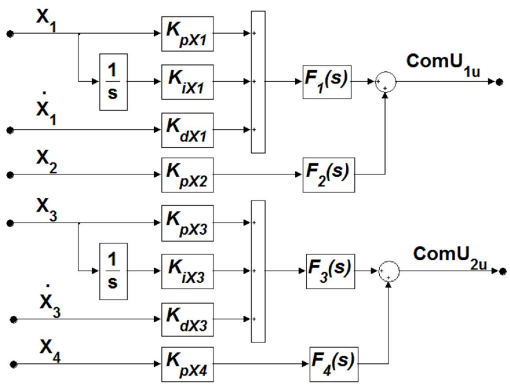


Figure 4. Feedback Layout - Two Regulated Variables

Some important points about Fig. 4 are presented below:

- The chosen control law pairing is X_1 to U_1 and X_3 to U_2;
- The filters $F_1(s)$, $F_2(s)$, $F_3(s)$ and $F_4(s)$ may have the following purposes:
  - Performing phase shift in a desired frequency region (lead or lead-lag);
  - Performing either gain attenuation or increase in a frequency region (lag, low pass, high pass);
  - Filter out the command in specific frequency regions (Notch Filter);
  - Any combination of the above sub-items.
- The first approach is to keep $F_1(s)$, $F_2(s)$, $F_3(s)$ and $F_4(s)$ fixed throughout the entire control law flight envelope, but it can be decided to adapt it, in accordance to the flight condition, if the analysis shows this is necessary;
- All the remaining gains are set in order to fulfill the desired *feedback requirements*;
- These *remaining gains* are the parameters which are *scheduled* from one flight condition to another.

The process of finding the quoted gains (K_{pX1}, K_{iX1}, K_{dX1},K_{pX2}, etc) is worked out by creating one cost function which shall be minimized. In the present work, this cost function has the general format described by the Equation 1:

$$\begin{aligned} &J(K_{PV1}, K_{IV1}, K_{dV1}, K_{pV2}, ...) = G_1(\min\xi - \xi_o)^2 + G_2(Gain_{f_1}^{X_1} - Gain_{f_1}^{X_1 O})^2 + \\ &+G_3(Gain_{f_2}^{X_1} - Gain_{f_2}^{X_1 O})^2 + G_4(Gain_{f_3}^{X_2} - Gain_{f_3}^{X_2 O})^2 + G_5(Gain_{f_4}^{X_2} - Gain_{f_4}^{X_2 O})^2 + \\ &+G_6(Gain_{f_5}^{U_1} - Gain_{f_5}^{U_1 O})^2 + G_7(Gain_{f_6}^{U_2} - Gain_{f_6}^{U_2 O})^2 \end{aligned} \tag{1}$$

where G_n are constants, chosen in a way that 1 is the threshold between good and bad for J, $\min\xi$ is the minimum damping of all closed loop poles, lying in a chosen frequency range, ξ_o is the target damping for $\min\xi$, $Gain_{f_z}^{X_p}$ is the magnitude of the performance plot, for the variable X_p, at the frequency f_z and $Gain_{f_z}^{X_p O}$ is the target magnitude of the performance plot, for the variable X_p at the frequency f_z. The same logic can be applied for the remaining symbols of this equation.

This cost function format efficiently encompasses the design requirements applicable to the feedback element of flight control laws. Its first term tackles the requirements on closed loop poles location. The experience shows that this sort of requirements is almost invariably placed in terms of a minimum acceptable damping for the closed loop poles.

The remaining terms are linked with the shape of the control law performance plots. Figure 5 presents a general shape for a performance plot. It can be noted an inferior boundary (blue region), which is linked with the required performance of the closed loop system, such us disturbance rejection. The upper boundary (green region) represents the inputs activity constraints.

The blue curve in Fig. 5 exemplifies one performance plot which respects both the inferior and the upper limit. The discrete points of the performance plots, chosen to be interpolated, shall be located in between the two defined boundaries.

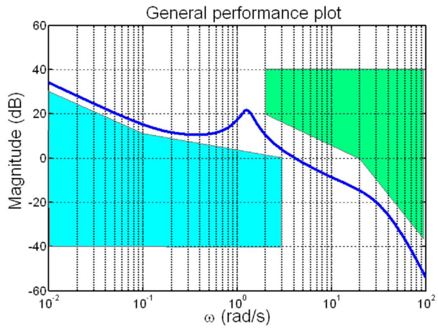


Figure 5. General shape of a performance plot

The process described in this Sub-Section can be classified as a particular example of multi-objective optimization. The special feature of it lies in the chosen format for the cost function. As already mentioned, this format was proven, in many different designs, to be very effective both in terms of easily allowing to capture the design requirement and also in terms of how easy it can be minimized. Other examples of multi-objective optimization can be found in (Looye and Joos, 2001; Looye et al., 2001; Looye and Joos, 2006).

Two different search algorithms for finding the minimum of J were tested. The first one consisted of a grid search, i.e. a number of parameter sets are evaluated and the one which produces the minimum value for the cost function J is selected.

This algorithm showed up to be inefficient for two reasons. The number of sets needed to be evaluated increases rapidly to an extremely high number, as the number of parameters to be determined increases. Also, depending on the refinement applied on the search grid, the number of cost function evaluations becomes prohibitive. These two characteristics made this search algorithm not practical.

The second tested algorithm is the Nelder-Mead simplex algorithm (Lagarias et al., 1998). Promising results in various design examples were obtained using this numerical routine. Besides the examples presented in this work, it was explored the method capability in the design of angle of attack controller, normal acceleration controller, speed regulator using elevator as the control variable, speed regulator using throttle as the control variable, side slip and roll rate controller, heading controller, bank angle controller, cross track error regulator, etc.

The minimization procedure described herein shall be repeated in a number of trimming points (see Fig. 2 (a)). The subsequent step is the computation of fitting curves for each of the control law scheduled parameters.

6. DESIGN EXAMPLES

Two design examples are presented. The first design is a pitch attitude rate regulator. This type of control law can be used, for example, as part of a CSAS, more precisely, the feedback portion of a longitudinal CSAS. A second possibility is to use it as the *inner loop* of a longitudinal axis auto pilot.

The second design is an altitude regulator. This is accomplished by using the *inner loop* previously designed. In other words, only the *outer loop* is designed in this stage.

It is emphasized that the design methodology presented here allows, in a straightforward manner, the design of the altitude hold in one go, i.e. without using a previously designed *inner loop*. In this case, it is necessary to create one cost function, which takes the performance plots from the $OLTF_{Elev}$ and $OLTF_{htot}$ simultaneously. This cost function shall have as its argument not only the outer loop parameters (gain on altitude, altitude rate and altitude double derivative), but also the inner loop ones (the proportional and integral gains on pitch attitude rate).

6.1 Pitch attitude rate control law

Figure 6 shows the adopted structure for the pitch attitude rate regulator. The reference for the pitch attitude rate signal, $\dot{\theta}_{Cmd}$, is generated by either an auto pilot *outer loop* or a CSAS pre-filter element.

For this particular case, there is one RV, $\dot{\theta}$, and one aircraft input used by the control law, elevator. It can also be noted that it is not applied any type of filtering either on the control law inputs or on its output. Nevertheless, in a real application, it would be possibly necessary to use a notch filter in order to avoid ASE issues. These filters has an impact on the gain calculation, although its effect should be of a second order.

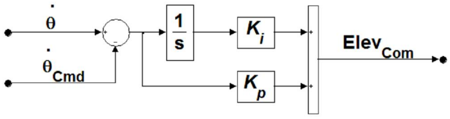


Figure 6. Pitch Attitude Rate Regulator

Suppose that the requirement on the feedback element is to have the performance plot of the $OLTF_{\dot{\theta}}$ passing through the vicinity of 18 db at 0.3 rad/s and 0 db at 3.0 rad/s. It is also required to have a minimum closed loop damping of 0.7 for all closed loop poles which lie in the frequency range of 0.5 rad/s to 20 rad/s. These requirements are applicable to all possible mass configurations on a given flight condition. Based on these requirements, it is assembled the cost function of Eq. (2):

$$J(K_i, K_p) = 400(\min_{0.5 \leq \omega \leq 20} \xi - 0.7)^2 + \frac{1}{50}(Gain_{0.3}^{\dot{\theta}} - 18db)^2 + \frac{1}{50}(Gain_{3.0}^{\dot{\theta}} - 0db)^2 \tag{2}$$

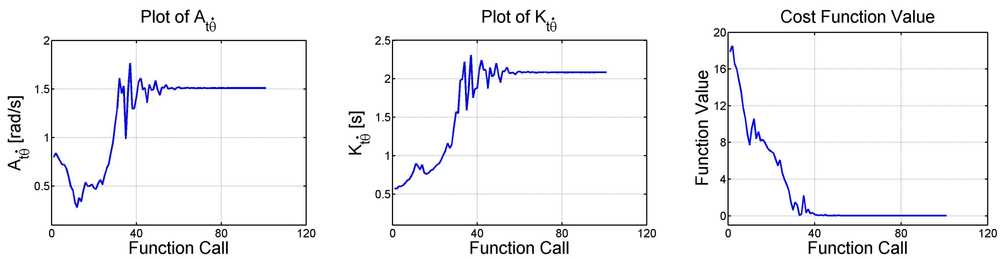
where $Gain_{0.3}^{\dot{\theta}}$ is the average magnitude of the performance plot, for the variable $\dot{\theta}$, at 0.3 rad/s, considering the two linear models presented in Table 1. The same logic is valid for $Gain_{3.0}^{\dot{\theta}}$.

Before submitting the cost function to the simplex minimization routine, the change of variable presented in Eq. (3) was carried out.

$$K_i = K_{t\dot{\theta}} \times A_{\dot{\theta}}, \quad K_p = K_{t\dot{\theta}} \tag{3}$$

It can be shown that the new argument $A_{\dot{\theta}}$ can be thought as a *target zero* for closed loop poles and the argument $K_{t\dot{\theta}}$ is the factor controlling control law bandwidth. This change of variable allows the selection of initial condition closer to the final value.

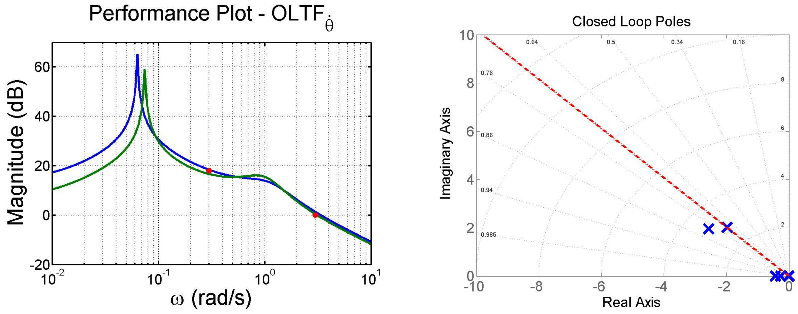
Figure 7 (a) shows the $A_{\dot{\theta}}$ value as a function of the cost function evaluation number. Figure 7 (b) shows the $K_{t\dot{\theta}}$ as a function of the cost function evaluation number, while Fig. 7 (c) presents the evolution of the cost function value.



(a) (b) (c)

Figure 7. Cost function arguments and cost function value

Figure 8 presents the final results obtained after applying the $A_{\dot{\theta}}$ and $K_{t\dot{\theta}}$ determined by the search algorithm. The red dotted line in Fig. 8 (b) marks the 0.7 damping line.



(a) (b)

Figure 8. Final Results

6.2 Altitude hold control law

Figure 9 presents the adopted structure for the altitude hold outer loop. The *Altitude Double Derivative Computation* block computes an approximation of the aircraft center of gravity acceleration, perpendicular to the horizontal line. This computation is based on the rigid body equations of motion. Since the used accelerometer measurements (θ, A_{XCG}, A_{ZCG}) have bias, the estimated signal is not good below a certain frequency. For this reason, this signal is blended with the h_P measurement into the *Third Order Complementary Filter*.

The complementary filter takes the low frequency content from h_P signal and the high frequency content from the output of the *Altitude Double Derivative Computation* block and provides the estimates of altitude, altitude rate and altitude double derivative to be used by the control law. This filter was conveniently set to extract out the bias coming from the accelerometer measurements and to attenuate the high frequency turbulence content, which is commonly present on the pressure altitude measurement, h_P. The elements inside these two blocks represent the data pre-processing of the altitude hold control law.

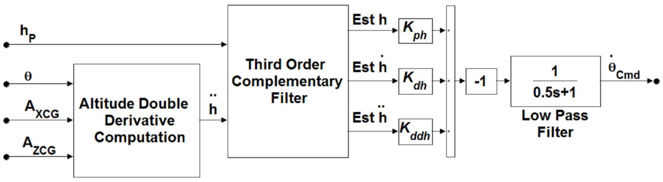


Figure 9. Altitude Hold Control Law - Outer Loop

Suppose that the feedback requirements for this outer loop is to have the performance plot of the $OLTF_{htot}$ passing through the vicinity of 40 db at 0.01 rad/s and 0 db at 0.6 rad/s. It is also required to have a minimum closed loop damping of 0.65 for all closed loop poles which lie in the frequency range of 0.1 rad/s to 20 rad/s. These requirements are again applicable to all possible mass configurations on a given flight condition. Based on these requirements, it is assembled the cost function of Eq. (4):

$$J(K_{ph}, K_{dh}, K_{ddh}) = 400(\min_{0.1 \leq \omega \leq 20} \xi - 0.65)^2 + \frac{1}{50}(Gain_{0.01}^{htot} - 40db)^2 + \frac{1}{50}(Gain_{0.6}^{htot} - 0db)^2 \tag{4}$$

The $OLTF_{htot}$ is generated by opening the loop just after the summing junction of the proportional, derivative and double derivative components of the $\dot{\theta}_{Cmd}$ signal. The performance plot is based on $OLTF_{htot}$ since altitude is the RV of this control law.

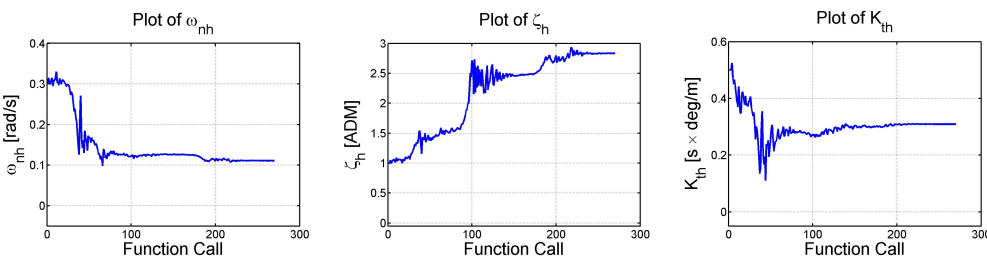
Following the same argument as before, the change of variables described in Eq. (5) was carried out.

$$K_{ph} = K_{th} \times \omega_{nh}^2, \quad K_{dh} = K_{th} \times 2 \times \zeta_h \times \omega_{nh}, \quad K_{ddh} = K_{th} \tag{5}$$

The new argument ω_{nh} can be thought as the natural frequency of a complex pair of *target zeros* for closed loop poles. The ζ_h is the damping ratio for this pair of complex zeros. The argument K_{th} is the the factor controlling bandwidth of the $OLTF_{htot}$.

Figure 10 (a) shows the ω_{nh} value as a function of the cost function evaluation number. Figure 10 (b) shows the ζ_h as a function of the cost function evaluation number, while Figure 10 (c) presents the K_{th} evolution.

In this case, the cost function had a value of 36.06 in the beginning of the minimum search routine and was minimized to 0.09. Since this value is less than 1, it means that the design is ready to have its robustness tested. This procedure was carried out following the methods normally used in the aircraft industry, such as the ones presented in (Gangsaas et al., 2008), as well as additional tests based on the concept of structured singular value. The final design showed up to be robust in accordance to any of the applied tests.



(a) (b) (c)

Figure 10. Cost function arguments

7. CONCLUSIONS

The design process described herein presents one practical route to deal with many of the issues normally encountered during a flight control law design. The starting point consists of assembling models which are able to capture the essential dynamics for the problem at hand.

The signal estimation problem is tackled through the application of data computation and filtering. This work presented a suggested approach of combining the available sensors data in order to compute the signals estimates used by the control law.

This article also proposed a method for tuning the feedback element parameters. This is done through the minimization of a defined cost function, which is composed of two parts: 1 - Minimum damping ratio, considering all closed loop poles. 2 - Location of the performance plots.

The proposed process of computing the feedback parameters showed up to be efficient. This can be credited to a number of reasons, namely: 1 - The designer is free to choose the feedback structure that is more convenient for the problem at hand. 2 - During the minimization process, it can be decided to use as many mass configurations as desired, instead of using only one nominal mass configuration. 3 - The dynamics added by the *Data Pre-Processing* element is considered during the minimization process. 4 - The proposed cost function format is able of encapsulating the feedback requirements in a straightforward manner.

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