BEARING PARAMETERS ESTIMATION USING CORRELATION ANALISYS AND RANDOM RESPONSE

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Abstract. *This work presents an identification methodology of stiffness and damping parameters of anisotropic rolling bearings in a rotor-bearing system. The proposed method is based on Lyapunov matrix equation and state space representation. Through the definition of correlations used in matrix equation, it is possible to generate an algorithm that relates system physical parameters to the correlation matrices of the measured state variables in time domain without the knowledge of the external input forces. The purpose of this work is to make a theoretical study of the viability of this methodology to be used in real systems. The numerical procedures were performed considering a rotating system with twenty degrees of freedom modeled by finite elements and excited by random forces. The numerical results show the proposed method is feasible and it can be used to identify real machines.*

Keywords*: Parametric Identification, Correlation Analysis, Lyapunov Matrix Equation, Rotordynamics*

1. INTRODUCTION

In many industries, the demand for high power and high speed is increasing in a significant way. In order to achieve such performance, the accurate prediction and control of the dynamic behavior (unbalance response, critical speeds and the instability) is also an important requirement. The bearings are vital components in any rotating machine and the knowledge of their physical properties is pre-requisite to the prediction of the machine´s behavior, as the dynamic behavior of a rotating shaft is significantly influenced by the stiffness and damping characteristics of the bearings.

The influence of bearings on the performance of rotor-bearing systems has been recognized for many years. In a general way, the identification methods of bearing parameters (stiffness and damping) are different concerning the domain, time or frequency, and the kind of excitation used. So, these methods require input signals (forces) and output signals (displacements/velocities/accelerations) of the dynamic system to be measured and the unknown parameters are calculated based on input-output relationships (Tiwari et al. 2004).

Different excitations have been used, such as unbalance (Muszynska *et al*, 1993; Chen and Lee, 1997; Kim *et al*., 2007), impulsive (Tiwari and Chakravarthy, 2006; De Santiago and San Andrés, 2007a), unknown forces (Tiwari and Vyas, 1995; Khan and Vyas, 1999). In practice, each method has its own strong and weak features, which lead to different levels of accuracy in specific applications.

Another way to determine the dynamic bearing properties is using correlation analysis. One advantage of this method is the non-necessity of measuring the input signals. The parameters are estimated through the correlation matrices among the output signals in many time instants. Isermann *et al.* (1974) compared six identification and parameter estimation methods and highlighted some advantages of correlation analysis: short computational time, only little *a priori* knowledge about the structure of the model, initial values for the parameters are not necessary.

Desforges *et al.* (1995) showed that correlation techniques produce good parameters estimation using just the responses to white noise excitation. Cooper *et al.* (1995) used 'colored' input signals to estimate modal parameters with success. Pederiva (1992) used correlation analysis of the state variables measured and Lyapunov matrix equation to identify parameters of a rotor-bearing system with 6 degrees of freedom. Eduardo (2003) used Lyapunov matrix equation and Artificial Neural Network (ANN) to diagnose faults in rotating mechanical systems excited by unbalance and stochastic forces.

This work uses the method proposed by Pederiva (1992), extended by Sanches (2008), to identify physical bearing parameters of a rotating system modeled by finite elements and excited by white noise. Reductions of the number of the measured state variables are also considered.

2. MATHEMATICAL MODELING

Consider a system that can be modeled by a linear and time invariant mechanical system with *n* degrees of freedom is described by the differential matrix equation:

$$\left[M\right]\left\{\ddot{\xi}(t)\right\}+\left[P\right]\left\{\dot{\xi}(t)\right\}+\left[K\right]\left\{\xi(t)\right\}=\left[S\right]\left\{u(t)\right\} \tag{1}$$

where [M] is the mass matrix, [P] is a matrix of speed proportional forces, [K] is a matrix of displacements proportional forces, $\{\xi(t)\}, \{\dot{\xi}(t)\}$ and $\{\ddot{\xi}(t)\}$ are the vectors corresponding to displacements, speeds and accelerations in this order, [S] is the input matrix and {u(t)} is an input vector.

The system described by Eq. (1) can be rewritten in a state-space representation as:

$$\{\dot{x}(t)\} = [A]\{x(t)\} + [B]\{u(t)\} \tag{2}$$

where the state vector is given by:

$$\{x(t)\} = \begin{Bmatrix} \xi(t) \\ \dots \\ \dot{\xi}(t) \end{Bmatrix} \tag{3}$$

The output equation of the system described by Eq. (2) is:

$$\{y(t)\} = [C]\{x(t)\} \tag{4}$$

[C] is the output matrix and {y(t)}is the output vector.

Equation (2) has the solution given by (Melsa and Sage, 1973):

$$\{x(t)\} = \Phi(t,t_0)\{x_0\} + \int_{t_0}^{t} \Phi(t,\tau)[B]\{u(\tau)\}d\tau \tag{5}$$

where Φ(t,τ) is the state transition matrix.

The expression for correlation matrix is defined by:

$$\left[R_{xx}(t,t+\tau)\right] = \left[R_{xx}(\tau)\right] = \varepsilon\left\{\{x(t)\}\{x^T(t+\tau)\}\right\} \tag{6}$$

Performing some procedures detailed by Eduardo (2003), it is possible to obtain an equation that correlates the measured outputs with the physical system parameters. This expression is called *Generalized Lyapunov Matrix Equation*:

$$[A][R_{xx}(\tau)] + [R_{xx}(\tau)][A]^T = -[B]\Psi_{uu}(t)[B]^T e^{[A]^T\tau} \tag{7}$$

where Ψ_{uu} is the intensity matrix of the excitation process {u(t)}.

Equation (7) can be expanded (Pederiva, 1992) as:

$$\begin{bmatrix} 0 & I \\ A_1 & A_2 \end{bmatrix}\begin{bmatrix} R_{\xi\xi}(\tau_i) & R_{\xi\dot{\xi}}(\tau_i) \\ R_{\dot{\xi}\xi}(\tau_i) & R_{\dot{\xi}\dot{\xi}}(\tau_i) \end{bmatrix} + \begin{bmatrix} R_{\xi\xi}(\tau_i) & R_{\xi\dot{\xi}}(\tau_i) \\ R_{\dot{\xi}\xi}(\tau_i) & R_{\dot{\xi}\dot{\xi}}(\tau_i) \end{bmatrix}\begin{bmatrix} 0 & A_1^T \\ I & A_2^T \end{bmatrix} = -\begin{bmatrix} 0 & 0 \\ 0 & B_1\Psi_u B_1^T \end{bmatrix} e^{A^T\tau_i} \tag{8}$$

with

$$[A] = \begin{bmatrix} 0 & I \\ -M^{-1}K & -M^{-1}P \end{bmatrix} = \begin{bmatrix} 0_{(n,n)} & I_{(n,n)} \\ A_{1_{(n,n)}} & A_{2_{(n,n)}} \end{bmatrix} \tag{9}$$

Evaluating Eq. (8), it is possible to obtain an expression that relates the physical parameters with the correlations of the measured output:

$$R_{\dot{\xi}\dot{\xi}}(\tau_i) + R_{\xi\xi}(\tau_i)A_1^T + R_{\xi\dot{\xi}}(\tau_i)A_2^T = 0 \tag{10}$$

All the assumptions considered until now were made considering that the entire state vector can be measured. In practice, this does not happen because some degrees of freedom are not available by reasons such as dangerous conditions and difficulties of accessing some points of the system. By this way, a reduction of the number of the output measurements should be considered in the identification process in order to apply this methodology in real systems.

2.1. Reduction of the output measurements

It is known that displacements and speeds measurements are redundant to the mechanical systems observability. So, only with the displacement measurements it is possible to observe completely the system (Pederiva, 1992). If a reduction of the number of measured state variables is considered, Eq. (10) cannot be used because some variables are not measured. In addition to this, it was mentioned that it is impossible to access all the points in a machine, so there is a reduction of the displacement measurements to be taken in account.

In order to identify the system, additional correlations should be used to replace the correlations of the unmeasured state variables. Pederiva (1992) proposes the utilization of a filter system to generate these additional data to be possible to identify the system. Figure 1 shows how the filter actuates in the mechanical system.

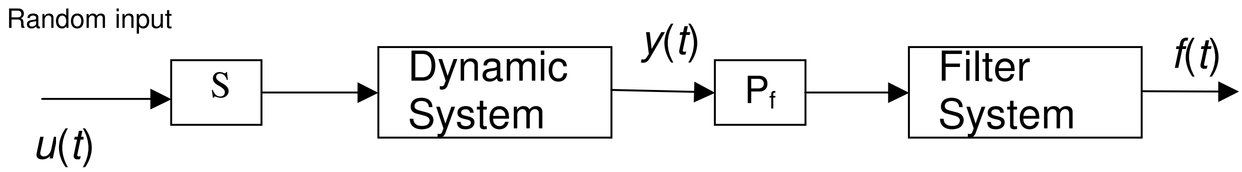

Figure 1. Schematic representation of the filter actuation

The filter system is represented by:

$$\{\dot{f}(t)\} = [N]\{f(t)\} + \left[P_f\right]\{y(t)\} \tag{11}$$

with:

$$\{f^T(t)\} = \{\eta_1 \quad \eta_2 \quad \cdots \quad \eta_q\} \tag{12}$$

$$[N] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 1 \\ n_1 & n_2 & n_3 & n_4 & \cdots & n_q \end{bmatrix} \tag{13}$$

and

$$\left[P_f\right] = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & 0 \\ p_1 & p_2 & \cdots & p_m \end{bmatrix} \tag{14}$$

The filter system has order q and it is stable, controllable and described according to Eq. (11). The values p_1 to p_m are the input filter parameters.

When a reduction of the number of output measurements is done, the system observability should be guaranteed by the available output. Dynamic systems described in the way showed by Eq. (2) and Eq. (4) can be represented by different forms which are known as normal forms of state equation, which have others coordinate systems (Pederiva, 1992). So the system input-output description changes as well as the input and output system matrices.

Through a specific coordinate base it is possible to have matrices [A^*], [B^*] and [C^*] with simple particular internal structures which can be utilized in a parameter estimation algorithm. These special structures are obtained by the choice of specific bases using a coordinate transformation.

Performing a coordinate transformation in the system:

$$\left\{x^*\right\} = [T]\{x\} \tag{15}$$

where [T] is the coordinate transformation matrix. The system can be rewritten as:

$$\left\{\dot{x}^*(t)\right\} = \left[A^*\right]\left\{x^*(t)\right\} + \left[B^*\right]\left\{u(t)\right\} \tag{16}$$

and

$$\left\{y^*(t)\right\} = \left[C^*\right]\left\{x^*(t)\right\}. \tag{17}$$

From Eqs. (15), (16) and (17) the following relations are obtained:

$$\left[A^*\right] = [T][A][T]^{-1} \tag{18}$$

$$\left[B^*\right] = [T][B] \tag{19}$$

$$\left[C^*\right] = [C][T]^{-1} \tag{20}$$

Considering the dynamic system completely observable, if [T] is the observability matrix, so the matrices [A^*] and [C^*] have the internal structures (Shafai and Carroll, 1984):

$$\left[A^*\right] = \begin{bmatrix} 0_{(f-m,m)} & I_{(f-m,f-m)} \\ A^*_{1(m,m)} & A^*_{2(m,f-m)} \end{bmatrix} \tag{21}$$

$$\left[C^*\right] = \begin{bmatrix} I_{(m,m)} & \vdots & 0_{(m,f-m)} \end{bmatrix} \tag{22}$$

$f = 2n$, with n the number of degrees of freedom, and m the number of available measurements. [B^*] has no especial structure (Pederiva, 1992).

As discussed before, when the entire state vector is not available it is necessary to use an auxiliary system in order to become possible the system identification. The new system is formed by the dynamic system to be identified and the filter and they will be designated as *expanded system* and its expanded state equation is given by:

$$\left\{\dot{x}_e(t)\right\} = \left[A_e\right]\left\{x_e(t)\right\} + \left[B_e\right]\left\{u(t)\right\}, \tag{23}$$

with

$$\left\{x_e\right\} = \begin{Bmatrix} x^* \\ \cdots \\ f \end{Bmatrix}, \tag{24}$$

$$\left[A_e\right] = \begin{bmatrix} A^* & 0 \\ P_f C^* & N_{(q,q)} \end{bmatrix}, \tag{25}$$

and

$$[B_e] = \begin{bmatrix} B^*_{(f,s)} \\ 0_{(q,s)} \end{bmatrix}. \tag{26}$$

Equation (7) can be rewritten as:

$$[A'][R'] + [R'][A']^T = [Q'] \tag{27}$$

3. CASE STUDY: ¼ OF STATE VECTOR AVAILABLE

The main purpose of this work is to make a theoretical study of the proposed methodology in real systems. The system analyzed is showed by Fig. 2 and it is composed by one shaft, two discs and three rolling bearings.

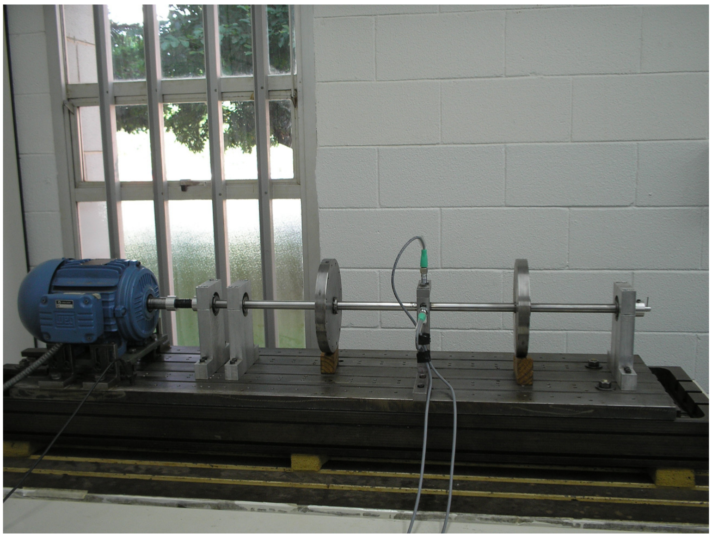

Figure 2. Rotor-bearing system

The system was modeled by finite elements with 5 nodes and 20 degrees of freedom. Computational simulations were performed by using the softwares *Matlab 7.0* and *Mathematica 6*.

Considering that the angular displacements are difficult to be measured, it is proposed an identification method in which only the linear displacements (1/4 of the state vector), which are easily obtained by transducers, are known. In this case, it is necessary to assure that, with only the linear displacements, the system is still observable. So the transformation matrix is based on observability matrix and it is given by:

$$[T] = \begin{bmatrix} C \\ C.A \\ C.A^2 \\ C.A^3 \end{bmatrix} \tag{28}$$

It is proposed a filter of order 6 to identify the system (Sanches, 2008), which has the following form:

$$[N] = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ n_1 & n_2 & n_3 & n_4 & n_5 & n_6 \end{bmatrix} \tag{29}$$

$$\left[P_f\right]=\begin{bmatrix} 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & 0 \\ p_1 & p_2 & \cdots & p_9 & p_{10} \end{bmatrix}_{(6,10)} \tag{30}$$

Equation (27) will generate relations that can be used to estimate the bearing parameters and the matrices involved have the structures (Pederiva, 1992 e Sanches, 2008):

$$\left[A'\right]=\left[\begin{array}{cccccccccc} 0_{(30,10)} & \vdots & I_{(30,30)} & \vdots & & & & & & \\ \cdots & \cdots & \cdots & \vdots & & & & 0_{(40,6)} & & \\ A^*_{01_{(10,10)}} & \vdots & A^*_{02_{(10,30)}} & \vdots & & & & & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & & & \cdots & & \cdots \\ & \vdots & & \vdots & 0 & 1 & 0 & 0 & 0 & 0 \\ 0_{(5,10)} & \vdots & & \vdots & 0 & 0 & 1 & 0 & 0 & 0 \\ & \vdots & 0_{(6,30)} & \vdots & 0 & 0 & 0 & 1 & 0 & 0 \\ & \vdots & & & 0 & 0 & 0 & 0 & 1 & 0 \\ \cdots & \vdots & & \vdots & 0 & 0 & 0 & 0 & 0 & 1 \\ \tilde{c}_{(1,10)} & \vdots & & \vdots & n_1 & n_2 & n_3 & n_4 & n_5 & n_6 \end{array}\right], \tag{31}$$

$$\left[R'\right]=\left[\begin{array}{cccccccccc} & & & & & & & & \vdots & \\ & & & R_{x^*x^*_{(40,40)}}(\tau_i) & & & & & \vdots & R_{x^*f_{(40,6)}}(\tau_i) \\ & & & & & & & & \vdots & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & & \cdots \\ r_{\eta_1 x_m^*} & \vdots & r_{1,1} & \vdots & r_{1,2} & \vdots & r_{1,3} & \vdots & & \\ r_{\eta_2 x_m^*} & \vdots & r_{2,1} & \vdots & r_{2,2} & \vdots & r_{2,3} & \vdots & & R_{ff_{(6,6)}}(\tau_i) \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & & \\ r_{\eta_6 x_m^*} & \vdots & r_{6,1} & \vdots & r_{6,2} & \vdots & r_{6,3} & \vdots & & \end{array}\right], \tag{32}$$

$$\left[A'\right]^T=\left[\begin{array}{cccccccccc} 0_{(10,30)} & \vdots & A^{*T}_{01_{(10,10)}} & \vdots & & & 0_{(10,5)} & & \vdots & \tilde{c}^T_{(10,1)} \\ \cdots & \cdots & \cdots & \vdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ I_{(30,30)} & \vdots & A^{*T}_{02_{(30,10)}} & \vdots & & & 0_{(30,6)} & & & \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ & & & \vdots & 0 & 0 & 0 & 0 & 0 & n_1 \\ & & & \vdots & 1 & 0 & 0 & 0 & 0 & n_2 \\ & 0_{(6,40)} & & \vdots & 0 & 1 & 0 & 0 & 0 & n_3 \\ & & & \vdots & 0 & 0 & 1 & 0 & 0 & n_4 \\ & & & \vdots & 0 & 0 & 0 & 1 & 0 & n_5 \\ & & & \vdots & 0 & 0 & 0 & 0 & 1 & n_6 \end{array}\right], \tag{33}$$

and

$$[Q'] = \begin{bmatrix} & & \vdots & \\ D_{(40,40)} & & \vdots & 0_{(40,6)} \\ & & \vdots & \\ \cdots \quad \cdots & \cdots & \cdots & \cdots \\ 0_{(6,40)} & & \vdots & 0_{(6,6)} \end{bmatrix}. \tag{34}$$

m is the number of state variables measured and x_m^* is the measured state variable (linear displacements).

Working with the lines 41 to 45, two estimation equations can be obtained. These relations contain the physical system parameters and the correlations among the filter outputs and the measured linear displacements (Sanches, 2008):

$$-r_{\eta_5 x_m^*}(\tau) + \left\{ r_{\eta_1 x_m^*}(\tau) \;\vdots\; -r_{\eta_2 x_m^*}(\tau) \;\vdots\; r_{\eta_3 x_m^*}(\tau) \;\vdots\; -r_{\eta_4 x_m^*}(\tau) \right\} \left[\Theta^* \right] = \{0\} \tag{35}$$

$$-r_{\eta_6 x_m^*}(\tau) + \left\{ r_{\eta_2 x_m^*}(\tau) \;\vdots\; -r_{\eta_3 x_m^*}(\tau) \;\vdots\; r_{\eta_4 x_m^*}(\tau) \;\vdots\; -r_{\eta_5 x_m^*}(\tau) \right\} \left[\Theta^* \right] = \{0\} \tag{36}$$

with

$$\left[\Theta^* \right] = \begin{bmatrix} A_{01}^{*T} \\ A_{02}^{*T} \end{bmatrix} \tag{37}$$

3.1. Numerical results

The filter parameters were chosen based on the system stability (Sanches, 2008):

- n_1 = -2,5469.10^{19}, n_2 = -3,3293.10^{14}, n_3 = -2,9132.10^{13}, n_4 = -2,244.10^{8}, n_5 = -1,02.10^{7}, n_6 = -30, p_1=p_2=…=p_{10}=1.

The three rolling bearings were assumed to be equal and anisotropic in which the damping and stiffness are different in the vertical (z) and horizontal (x) directions, the cross terms were not considered. The estimation procedure was done by least squares applied to Eqs. (35) and (36) with 2 and 3 time delay instants (τ). Two simulated cases were performed to test the robustness of this methodology:

Case 1:

Simulations were performed considering an experimental FRF. The parameter values for stiffness and damping are: K_{xx} = 30.10^{7}N/m, K_{zz} = 2,5.10^7 N/m, C_{xx} = 70.000 Ns/m and C_{zz} = 10.000 Ns/m.
The identified parameters are shown in Tab. 1 and Tab. 2:

Table 1. Identified parameters by Eq. (35)

Time delay	K_{xx}	K_{zz}	C_{xx}	C_{zz}
2 τ's Error	29,973x10^7 (0,09%)	2,4814x10^7 (0,74%)	69260 (1,057%)	10838 (8,38%)
3 τ's Error	29,972x10^7 (0,093%)	2,4876x10^7 (0,49%)	68416 (2,26%)	10991 (9,91%)

Table 2. Identified parameters by Eq. (36)

Time delay	K_{xx}	K_{zz}	C_{xx}	C_{zz}
2 τ's Error	29,973x10^7 (0,09%)	2,4983x10^7 (0,068%)	69445 (0,79%)	10900 (9,00%)
3 τ's Error	29,971x10^7 (0,097%)	2,4988x10^7 (0,048%)	68439 (2,23%)	11023 (10,23%)

Case 2:

In this case, the bearing parameters have different physical properties. The aim here is to test the algorithm robustness taking into account these variations. The values adopted are: K_{xx} = 30.10^{6}N/m, K_{zz} = 4,0.10^7 N/m, C_{xx} = 60.000 Ns/m and C_{zz} = 15.000 Ns/m.

The identified parameters are shown by Tab. 3 and Tab. 4.

Table 3. Identified parameters by Eq. (35)

Time delay	K_{xx}	K_{zz}	C_{xx}	C_{zz}
2 τ's Error	29,987x10^6 (0,043%)	3,9975x10^7 (0,063%)	60038 (0,063%)	15463 (3,08%)
3 τ's Error	29,982x10^6 (0,06%)	3,9973x10^7 (0,068%)	60133 (0,22%)	15339 (2,26%)

Table 2. Identified parameters by Eq. (36)

Time delay	K_{xx}	K_{zz}	C_{xx}	C_{zz}
2 τ's Error	29,529x10^6 (1,57%)	3,8953x10^7 (2,62%)	60035 (0,058%)	15967 (6,45%)
3 τ's Error	29,478x10^6 (1,74%)	3,8904x10^7 (2,74%)	60000 (0%)	15980 (6,53%)

4. CONCLUSIONS

An identification method for the bearing parameters estimation is presented for *mdof* flexible rotor-bearing systems. The Lyapunov matrix equation and correlation analysis were used to perform the estimations and the identification algorithm was illustrated through a numerical rotor-bearing model.

The estimation results are considered satisfactory, however the smaller the number of outputs the more difficult the estimation procedure. This results from the transformation matrix that leads to a concentration of parameters.

The filters are auxiliary systems and must be chosen in such a way that the system outputs are not smoothed.

The unbalance effect is the next step to be taken into account in the proposed procedure so that by improving this identification methodology it will be possible to apply it to real machines in the field.

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