ENERGY FLOW IN A CRACKED BEAM SPECTRAL ELEMENT

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Abstract. *Damage detection and structural integrity monitoring are important subjects to automotive, naval and aerospace industries. These topics have received great attention in the last decades. To investigate damage a new method based on mechanical energy flow and spectral element method is proposed. Spectral element method consists in a frequency domain exact analytical solution of the wave equation, using displacement formulation, tailored with the finite element method matrix ideas. In this new method a cracked beam spectral element is evaluated based on its mechanical energy flow propagation. The beam element is excited by a pulse force to evaluate the effect of signal nature to the wave propagation process. Changes in energy density and energy flow related to crack depth and position are observed for different cases of healthy and cracked beams. Simulated results are evaluated and discussed.*

Keywords: *Wave propagation, Damage detection, Spectral element method, Beam.*

1. INTRODUCTION

Currently, in the development of structural projects, investigation to control excessive noise and vibration levels has been done aiming to provide a safe and comfortable environment. The modern industry continually searches for new engineering tools in order to increase human life safety and quality. Better tools to predict noise and vibration are needed to control these phenomena and provide ideal work conditions. A difficult issue in this area relates with the energy vibration path identification and how it propagates through the structure. Identifying correctly these paths can give manners to control the vibration.

In the past, damage detection was done mainly by visual inspection; most recently new technologies have been developed in order to facilitate the damaged structure diagnostic without visual perception. A crack presence in a structure introduces local flexibility changes that affect its vibration response (Dimarogonas, 1996). Then, vibrational energy can be used to investigate the healthy condition of a structure. Recently damage detection researches are concentrating on methods that use elastic wave propagation at medium and high frequencies (Pereira *at al.*, 2008; Palacz and Krawczuk, 2002; Krawczuk *at al.*, 2002; Krawczuk, 2002).

Finite Element Method (FEM) became a well-established numeric prediction tool on structural vibration. It is effective to low frequency analysis, but at medium and high frequencies it produces an inherent problem to generate huge computational models. Statistic Energy Analysis (SEA) is an energy method, which has been applied successfully to solve problems at high frequency. Proposed by Lyon (1995) in the 60's it consists in dividing the structure in a set of subsystems that interacts between them through the energy exchange. However, the spatial variations within each subsystem cannot be obtained because it provides only one energy level to each subsystem.

Spectral Element Method (SEM) was proposed by Doyle (1997) and consists in the exact displacement wave equation analytical solution at frequency domain. Since the spectral element is the exact analytical solution it is equivalent to an infinite number of finite elements. This characteristic and the spectral domain make the spectral element more flexible to present the spatial variation of vibrational energy. In this work a damage detection method using the cracked beam spectral element presented by Krawczuk (2002) is extended to obtain the mechanical energy density and flow instead of displacement, as considered in its original formulation. At high frequency the displacement is relatively small not revealing as the best alternative for crack detection. Then, this work proposes an alternative evaluation for damage detection through the mechanic energy density and flow in a cracked beam spectral element.

The simulated examples use a spectral element for the flexural vibration problem based on the Bernoulli-Euler beam theory. A cracked beam spectral element was implemented and analyzed in terms of mechanical energy propagation throughout its length. The energy change is evaluated in terms of its density and flow as a function of the crack depth and length position. Different cases of the damaged beam are investigated. The results are compared with those obtained for a healthy beam. The beam element is excited by a pulse force in order to determine the nature of the signal process for the wave propagation. The simulated results are presented and the main divergences are discussed.

2. BASIC THEORY

2.1. Healthy beam spectral element

Bernoulli-Euler theory considers the beam as a long, narrow structure with loads applied transverse to the centerline. It is assumed also that transverse displacement and rotation are small. Although the theory assumes transverse shear force, the shear deformations are neglected. On the basis of these preamble the differential equation of movement in its spectral form can be obtain as (Doyle, 1997):

$$\frac{d^4\hat{v}}{dx^4} - k^4\hat{v} = F \,, \tag{1}$$

where ^ indicates the function Fourier transform, v is the transversal displacement, F is the external force and k is the wave number given by:

$$k^2 \equiv \sqrt{\omega^2 \rho S / EI} \,, \tag{2}$$

where ω is the circular frequency, E is the Young's modulus, S is the cross-section area, ρ is the density and I is the inertia moment. The homogeneous solution of Eq. (1) can be written as:

$$\hat{v}(x) = A e^{-ik_1 x} + B e^{-k_2 x} + C e^{-ik_1(L-x)} + D e^{-k_2(L-x)} \,, \tag{3}$$

where A, B, C and D are arbitrary constants determined by boundary conditions, L is the beam length, and the wave numbers are given by $k_1 = \pm k$ and $k_2 = \pm ik$.

Figure 1 shows a two-node elastic beam spectral element subject to dynamic forces in both nodes. A damping term is introduced into formulation by using a complex Young's modulus, $E_c = E(1+i\eta)$, where η is the hysteretic loss factor structural damping.

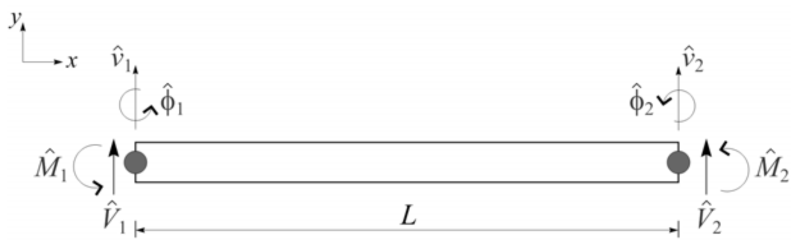


Figure 1. Two-node healthy beam spectral element.

From Figure 1, the nodal displacements and rotations in the beam ends can be written as:

$$\hat{v}(0) \equiv \hat{v}_1 ,\ \hat{\phi}(0) \equiv \hat{\phi}_1 ,\ \hat{v}(L) \equiv \hat{v}_2 \ \text{ and } \ \hat{\phi}(L) \equiv \hat{\phi}_2 \,. \tag{4}$$

From the Eq. (4) and Eq. (3) the coefficients, A, B, C and D can be calculated. Substituting again in the Eq. (3) the expressions founded to calculate the displacements and the rotations in any written arbitrary point of beam are written in the following form:

$$\hat{v}(x) = \hat{g}_1(x)\,\hat{v}_1 + \hat{g}_2(x)\,\hat{\phi}_1 + \hat{g}_3(x)\,\hat{v}_2 + \hat{g}_4(x)\,\hat{\phi}_2 \,, \tag{5}$$

where $\hat{g}_i(x)$ are form functions, which are omitted here for conciseness, but can be found in Khaled, 2001. The nodal loading and dof's are related by the following expressions:

$$\hat{V}(x) = -EI\frac{d^2\hat{\phi}}{dx^2} \,, \quad \hat{M}(x) = EI\frac{d\hat{\phi}}{dx} \,. \tag{6}$$

Applying the boundary conditions on the beam element (Fig. 1) the following matrix equation is obtained:

$$\begin{Bmatrix} \hat{V}_1 \\ \hat{M}_1 \\ \hat{V}_2 \\ \hat{M}_2 \end{Bmatrix} = \underbrace{\frac{EI}{L^3}\left[\hat{K}\right]}_{[\Omega]} \begin{Bmatrix} \hat{v}_1 \\ \hat{\phi}_1 \\ \hat{v}_2 \\ \hat{\phi}_2 \end{Bmatrix}, \tag{7}$$

where **[Ω]** is the healthy beam Bernoulli-Euler element dynamic stiffness matrix, which is a symmetrical matrix (4x4) and in general is complex.

For a harmonic excitation the time averaged flexural energy density in a beam is the sum of the time averaged potential energy density and the time averaged kinetic energy density, given by:

$$\langle e \rangle = \frac{1}{4} EI \left\{ \frac{d^2\hat{v}}{dx^2} \frac{d^2\hat{v}^*}{dx^2} \right\} + \frac{1}{4}\omega^2 \rho S \left\{ \hat{v}\,\hat{v}^* \right\}, \tag{8}$$

where $\langle\ \rangle$ and * denotes the time averaged quantity and complex conjugate, respectively. The time averaged energy flow consists of two terms: one term due to bending moment and the other due to shear force, given by:

$$\langle q \rangle = \frac{1}{2} \Re\left\{ EI \frac{d^3\hat{v}}{dx^3}\hat{v}^* \right\} + \frac{1}{2}\Re\left\{ -i\omega EI \frac{d^2\hat{v}}{dx^2}\frac{d\hat{v}^*}{dx} \right\}, \tag{9}$$

where $\Re$ is the real part of a complex number.

2.2. Cracked beam spectral element

Figure 2 shows a spectral element beam with a transverse open and non-propagating crack, presented for Krawczuk (2002). The crack is modeled by a dimensionless local flexibility, θ, which is calculated by using Castigliano's theorem and the laws of fracture mechanics (Tada, *at al*, 1973).

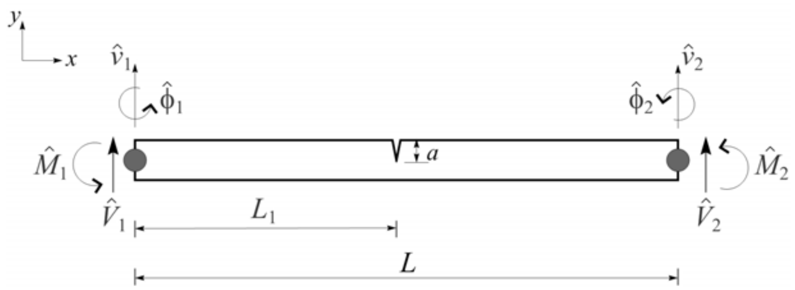


Figure 2. Two-node cracked beam spectral element.

The general solution to the Eq. (1) applied for this element can be written in two parts:

$$\hat{v}^{(l)} = A_1 e^{-ik_1x} + B_1 e^{-k_2x} + C_1 e^{-ik_1(L_1-x)} + D_1 e^{-k_2(L_1-x)} \qquad [0 \le x \le L_1], \tag{10}$$

$$\hat{v}^{(r)} = A_2 e^{-ik_1(L_1+x)} + B_2 e^{-k_2(L_1+x)} + C_2 e^{-ik_1(L-(L_1+x))} + D_2 e^{-k_2(L-(L_1+x))} \qquad [0 \le x \le L-L_1], \tag{11}$$

where $\hat{v}^{(l)}$ and $\hat{v}^{(r)}$ are the displacement element at left and right hand of the crack, respectively. The constants, A_1, B_1, C_1, D_1 and A_2, B_2, C_2, D_2 are determined by: a) the displacement boundary conditions:

$$\hat{v}^{(l)}(0) = \hat{v}_1, \quad \hat{\phi}^{(l)}(0) = \hat{\phi}_1, \quad \hat{v}^{(r)}(L-L_1) = \hat{v}_2, \text{ and } \hat{\phi}^{(r)}(L-L_1) = \hat{\phi}_2; \tag{12}$$

b) the displacement compatibility conditions at the crack position:

$$\hat{v}^{(l)}(L_1)=\hat{v}^{(r)}(0), \quad \hat{\phi}^{(r)}(0)-\hat{\phi}^{(l)}(L_1)=\theta\frac{d^2\hat{v}^{(r)}}{dx^2}; \tag{13}$$

and c) the moment and the shear force equilibrium relationships at the crack:

$$\frac{d^2\hat{v}^{(l)}(L_1)}{dx^2}=\frac{d^2\hat{v}^{(r)}(0)}{dx^2}, \quad \frac{d^3\hat{v}^{(l)}(L_1)}{dx^3}=\frac{d^3\hat{v}^{(r)}(0)}{dx^3}, \tag{14}$$

By applying displacement boundary conditions to the element ends, and the displacement compatibility and force equilibrium to the crack position, the matrix equation is obtained:

$$\begin{Bmatrix}\hat{V}_1\\ \hat{M}_1\\ \hat{V}_2\\ \hat{M}_2\end{Bmatrix}=\underbrace{\frac{EI}{L^3}\left[\hat{K}_c\right]}_{[\Omega_c]}\begin{Bmatrix}\hat{v}_1\\ \hat{\phi}_1\\ \hat{v}_2\\ \hat{\phi}_2\end{Bmatrix}, \tag{15}$$

where $[\boldsymbol{\Omega}_c]$ is the dynamic stiffness matrix for the cracked beam spectral element.

From Castigliano's theorem, the flexibility at the crack location for the one-dimensional beam spectral element can be obtained by (Krawczuk, 2002):

$$c=\frac{\partial^2 U}{\partial Q^2}. \tag{16}$$

where U is the elastic strain energy due the crack and Q is the nodal force on the element. By considering that only crack mode I is present in the beam element, the elastic strain energy can be expressed as:

$$U=\frac{1-\nu^2}{E}\int_{S_c} K_I^2 dS_c, \tag{17}$$

where ν is the Poisson's ratio, S_c is the cracked area and K_I is a stress intensity factor corresponding to the crack mode I, which can be represented by:

$$K_I=\frac{6\hat{M}}{bh^2}\sqrt{\pi\alpha}\, f(\alpha/h), \tag{18}$$

where $\hat{M}$ is bending moment at cracked position, α is the crack depth variation (Fig. 3), h and b are the cross section dimensions, and $f(\alpha/h)$ is a correction function given by

$$f(\alpha/h)=\sqrt{\frac{\tan(\pi\alpha/2h)}{\pi\alpha/2h}}\,\frac{0.752+2.02(\alpha/h)+0.37[1-\sin(\pi\alpha/2h)]^3}{\cos(\pi\alpha/2h)}. \tag{19}$$

From Figure 2 and with some simple transformations, the local flexibility coefficient of the cracked spectral element can be rewritten as:

$$c=\frac{72\pi}{Ebh^2}\int_0^{\overline{a}}\overline{\alpha}\, f^2(\overline{\alpha})d\overline{\alpha}, \tag{20}$$

where $\overline{a}=a/h$, $\overline{\alpha}=\alpha/h$. The dimensionless local flexibility is obtained from Eq.(20) as, $\theta=c\,EI/L$.

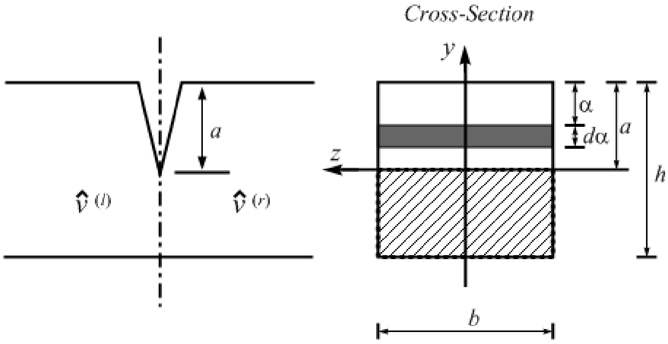
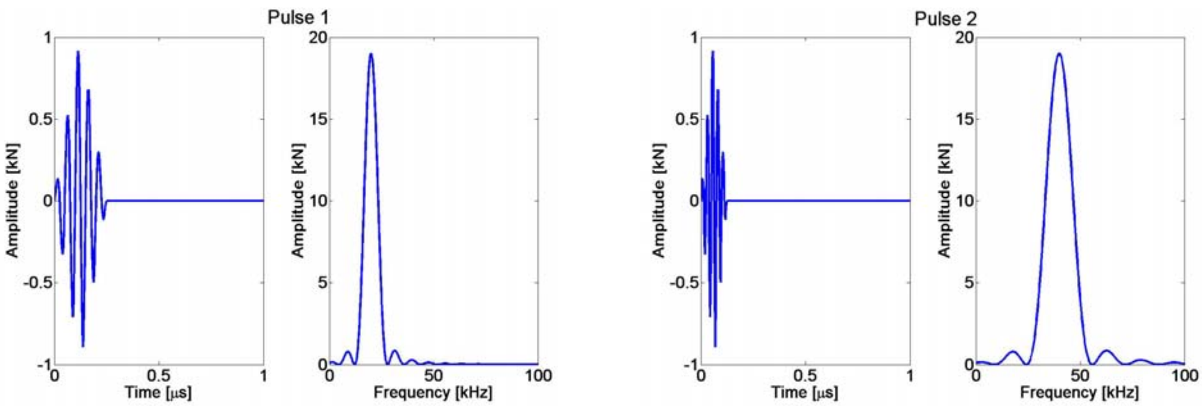


Figure 3. Cracked beam cross-section at crack position.

3. NUMERICAL RESULTS

In order to observe the vibration energy behavior of healthy and cracked beams, some numerical tests were done. For all tests an aluminum beam (E=71.0 GPa, ρ = 2700.0 kg/m^3 and η = 0.03) with length L = 4.0 m, rectangular cross-section (b = 0.02 m e h =0.02 m), and free-free boundary conditions was used. The beam was excited in the left end with a pulse force. Two types of pulse were generated to show the influence of signal in the wave propagation and the possibility to localize the crack. Pulses are obtained with a sinusoidal force modulated by a triangular window. Pulse 1 and 2 were obtained by sine functions with amplitude of 1.0 kN and frequency of 20.0 and 40.0 kHz, respectively. Sine functions were generated with 200 points and the triangular window includes 5 periods. Figure 4 shows the two pulses generated in time and frequency domain.

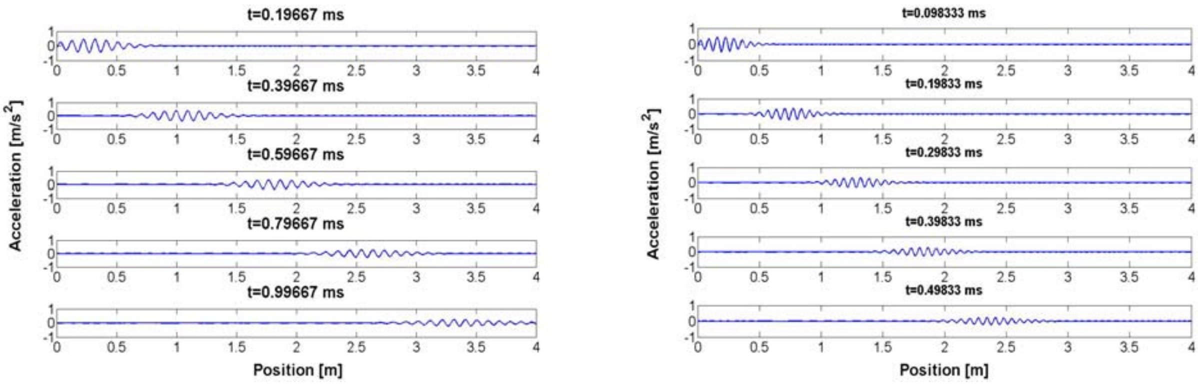


(a) (b)

Figure 4. Excitation pulse forces: (a) Pulse 1 (f_c = 20 kHz) and (b) Pulse 2 (f_c = 40 kHz).

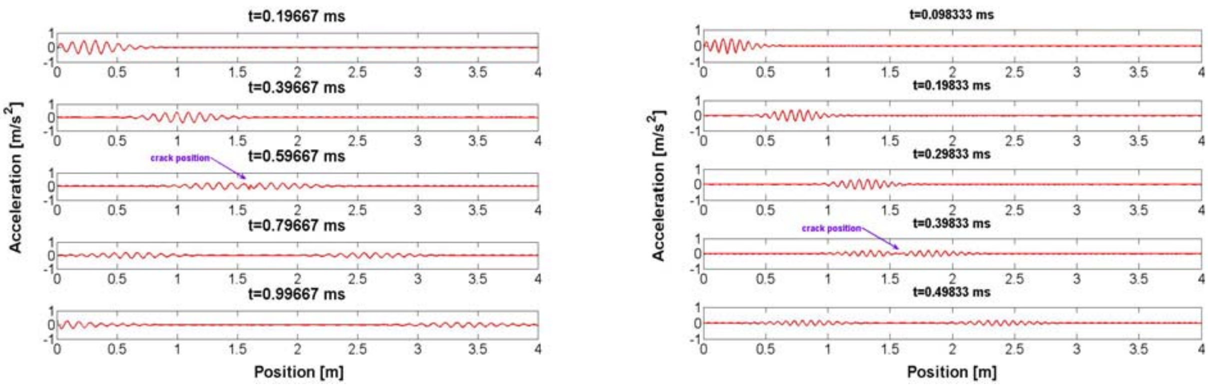
For the damage localization it is important to observe changes between damaged and healthy responses. Figure 5 shows a healthy beam acceleration response in different instant times, excited by Pulse 1 and 2. It can be observed in both results the characteristic flexural wave propagation behavior, which goes reducing the amplitude throughout the beam length due to the internal damping, with reflection waves at ends due free-free boundary conditions. Also, it can be seen that Pulse 1 generates a wave phase velocity greater than Pulse 2. Figure 6 present similar results for a cracked beam, with the crack position at L_1 = 0.4L and dimensionless crack depth a/h = 0.05, excited by Pulse 1 and 2. It is possible to observe that in the crack position the incident wave is dividing in two others, one propagative and another reflective. These results agree with ones presented by Palacz and Krawczuk (2002) and it confirms the great potential of wave propagation methods for damage detection.

To verify the accuracy of cracked beam spectral element a comparison between healthy and cracked elements was conducted for an almost negligible dimensionless crack depth. Figure 7 shows the energy density and flow for a healthy and cracked beam elements with the crack position, x = 0.4L, and the dimensionless crack depth, a/h = 0.0001. The beam is excited in the left end with Pulse 1. The analyses are performed with two-node spectral elements and the energy density and flow are obtained by interpolating along element length with the discretization of 0.01 m. Results of energy density and flow presents a very good convergence of cracked model to the healthy model.



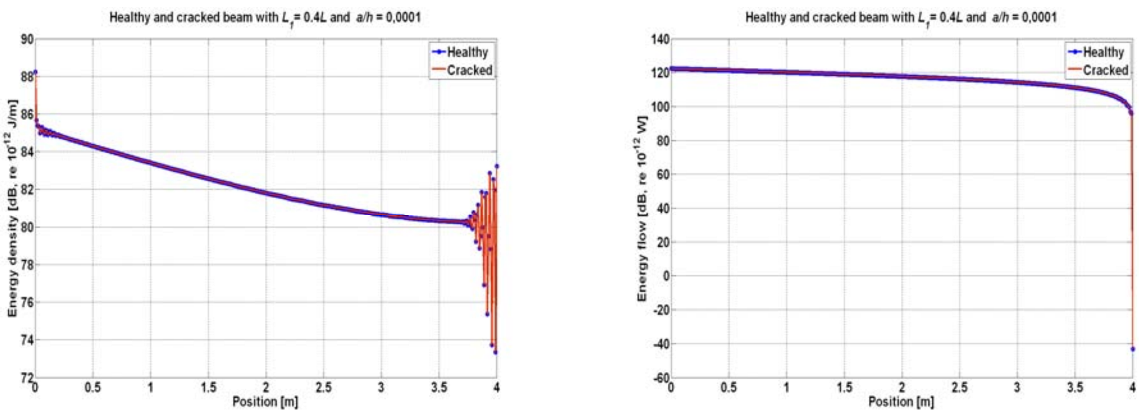
(a) (b)

Figure 5. Healthy beam acceleration response excited by: (a) Pulse 1 (f_c = 20 kHz) and (b) Pulse 2 (f_c = 40 kHz).



(a) (b)

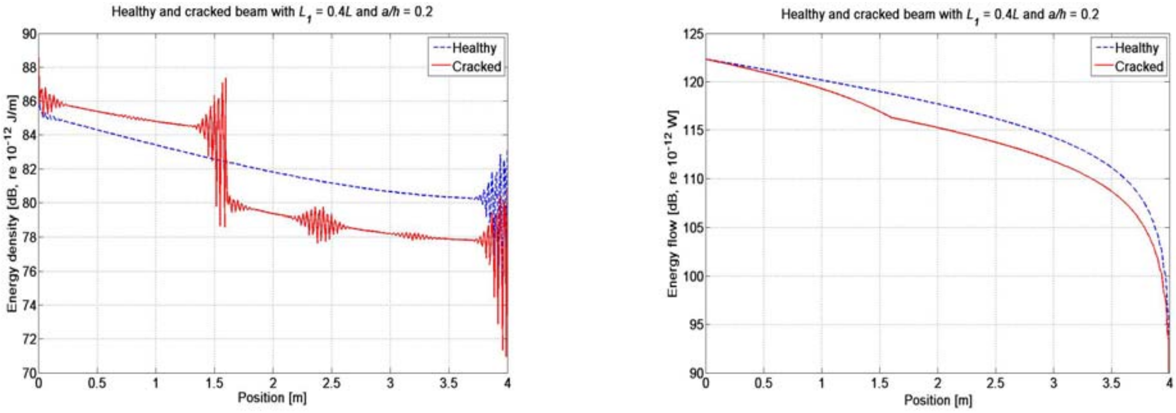
Figure 6. Cracked beam acceleration response, with $L_1 = 0.4L$ and $a/h = 0.05$, excited by: (a) Pulse 1 (f_c = 20 kHz) and (b) Pulse 2 (f_c = 40 kHz).



(a) (b)

Figure 7. Healthy and cracked beams with a negligible crack depth (a/h = 0.0001 and $L_1 = 0.4L$) excited by Pulse 1: (a) Energy density and (b) Energy flow.

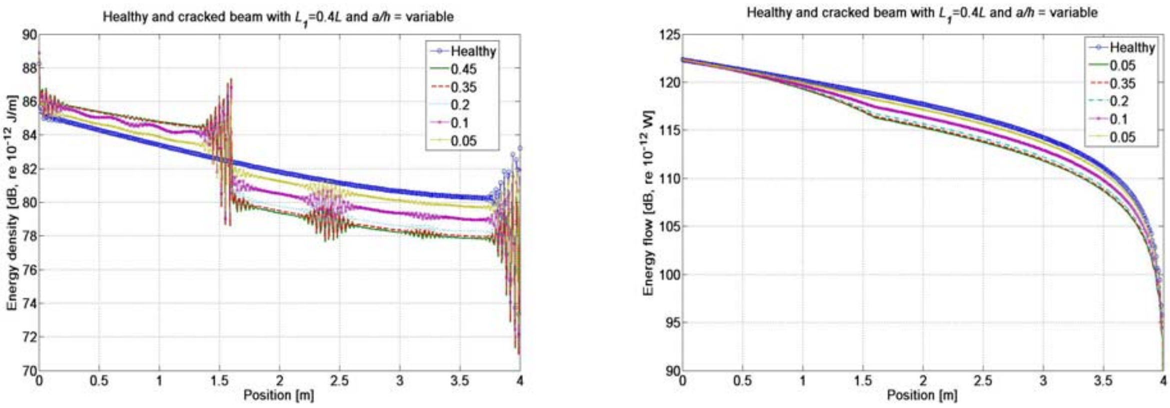
Figure 8 shows the energy density and energy flow for a healthy and cracked beam with the crack position $x = 0.4L$ and dimensionless crack depth $a/h = 0.2$. The beam is excited in the left end with Pulse 2, and the results are obtained by interpolating along element length with the discretization of 0.01 m. Results show that energy density (Fig. 8a) presents a characteristic oscillatory behavior of flexural waves, while the energy flow is smooth (Fig. 8b). Also, the energy density and flow decrease throughout the beam length due to internal damping. Furthermore, in the cracked beam, there is a clear indication of the crack position and crack depth amount.



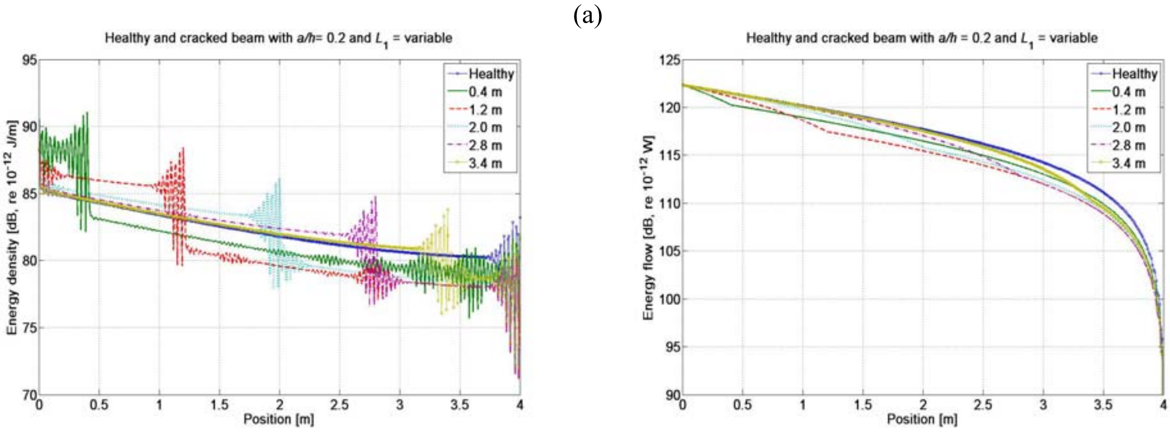
(a) (b)

Figure 8. Healthy and cracked (L_1 = 0.4L and a/h = 0.2) beams excited by Pulse 2: (a) Energy density and (b) Energy flow.

Figure 9a shows the energy density and flow changes for healthy and cracked beam with L_1 = 0.4L and a/h = 0.05; 0.10; 0.2; 0.35 and 0.45. It is observed that energy density presents good sensitivity to localize the crack to all crack depth values, while energy flow starts to become sensitive to values greater then a/h = 0.05. Figure 9b shows the variations of energy density and flow for healthy and cracked beam with L_1 = 0.4; 1.2; 2.0; 2.8; and 3.4 m and dimensionless crack depth a/h = 0.2. The general behavior of energy density and flow are similar to the case before. Since that crack depth is a sensitive value ($a/h \geq 0.05$) the variation of the energy density and flow as function of the crack position always locates the crack.



(a)



(b)

Figure 9. Energy density and flow for healthy and cracked beam with: (a) L_1 = 0.4L and a/h = 0.05; 0.10; 0.2; 0.35 and 0.45 and (b) L_1 = 0.4; 1.2; 2.0; 2.8; and 3.4 m and a/h = 0.2.

4. CONCLUSION

In this work the Spectral Element Method was used for damage detection in beam type structures and was extended to obtain a solution in terms of energy density and energy flow instead of displacement. A beam cracked spectral element was implemented and analyzed in terms of mechanical energy propagation along its length. The beam element is excited by two pulse forces to determine their influences on the nature of the signal process of wave propagation and its capacity to localize a crack. The simulated results of energy density demonstrate that the model presents good sensitivity to crack localization for all analyzed values of crack depth, while energy flow is less sensitive. However, since dimensionless crack depth is superior to 0.05, the energy density and energy flow always will localize the crack. This alternative form to present the results as energy density and energy flow is of bigger easiness for the results interpretation.

5. ACKNOWLEDGEMENTS

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