DEVELOPMENT AND APPLICATION OF A METHODOLOGY FOR PREDICTION OF USEFUL LIFE OF MANDREL SHAFTS FROM COLD ROLLING COILERS SUBMITTED TO CUMULATIVE FATIGUE DAMAGE

Sérgio Rolla Guimarães, sergio.rolla@arcelormittal.com.br
ArcelorMittal Stainless and Electric Steels Brazil, 1º de maio Square, 09, CEP 35180.018, Timóteo, MG, Brazil

Gustavo Luiz Chagas Manhães de Abreu, gumanhaes@yahoo.com.br
Paulista State University Júlio de Mesquita Filho, Brasil Avenue, 56, CEP 15385-000, Ilha Solteira, SP, Brazil.

Antônio Eustáquio de Melo Pertence, pertence@demec.ufmg.br
Federal University of the Minas Gerais State, Department of Mechanical Engineering 6627 Antônio Carlos Avenue, CEP 31270-901, Belo Horizonte, MG, Brazil.

Abstract. *The mandrels promote winding and unwinding of coils in cold rolling processing lines. Mandrel's shafts are subjected to combined bending and torsion loading during coiling process. In order to increase productivity, some steel companies have been working with heavier coils than the original project. Due these overload spectra, mandrel's shafts had become vulnerable to fatigue fractures. This paper presents the development of a methodology to predict the useful life of mandrel shafts from cold rolling coilers, submitted to cumulative fatigue damage. The proposed methodology is based on cumulative fatigue damage of the piece. The load sequence history and the mandrel shaft's physical and geometric characteristics are introduced in the life prediction model. Miner's rule is used to quantify the fatigue damage on the mandrel shaft caused by each processed coil. The main purpose of this work is to serve as a maintenance tool that estimates the remaining fatigue lives of mandrel shafts subjected to cumulative fatigue damage. The methodology was carried out on two experimental cases in order to adjust and assess the applicability of the model. The obtained results have shown that the type and load sequence have a significant influence on the accumulated final damage value and on the degree of adjustment to the proposed linear model. The load cases, with stress amplitudes closer to the endurance limit of the shaft's critical section, were found to be in very good agreement to the linear model. The validity of the model was proven in a third case, where the shaft was removed for inspection during its useful life*

Keywords: *mandrel shaft, coiler, cumulative damage, fatigue, Miner's rule.*

1. INTRODUCTION

The investigations of the variable-amplitude loading effects on the structures have begun in the first decades of XX century. A practical problem associated with VA-loading was the question of how long old structures could still be used without running into fatigue problems. In the second half of the previous century, this question was raised for old bridges, quite often bridges built in the 19th century. The question was whether fatigue problems should be anticipated or whether the bridge should be replaced by a new one. Bridges were often more intensively loaded by heavy traffic than previously expected in the design process long ago (Schijve, 2003).

In this context, during the 90's the steel companies in Brazil have invested in modernization projects in order to increase productivity. ArcelorMittal Stainless and Electric Steels Brazil, a traditional stainless and silicon flat steel, located in Timóteo, Minas Gerais, has adopted heavier coils than the original project as an option for productivity increment. Consequently, some mandrel shafts have been submitted to cumulative fatigue damage and became vulnerable to fatigue fractures in service.

The mandrel shaft's functions are torque transmission and load support. They are strategic for maintenance due to their long time of supplying. Two cases of mandrel shaft's fractures were recorded in the last eight years. Those accidents became a threat to the maintenance team.

Then a technical approach was initiated in order to predict remaining mandrel shafts lives. This methodology is based on the cumulative fatigue damage of the piece. The load sequence history and the mandrel shaft's physical and geometric characteristics are introduced in the life prediction model. Miner's rule is used to quantify the fatigue damage on the mandrel shaft caused by each processed coil. Miner's rule is expressed in Eq. (1).

$$\sum \frac{n_i}{N_i} = 1{,}0 \tag{1}$$

where n_i is the number of cycles at the stress level i, and N_i is the number of cycles to failure at that stress. Miner's rule simply states that fatigue failure is expected when such life fractions sum to unity, i.e., when 100% of the life is exhausted (Dowling, 1999).

In Miner's rule experiments, normally two or more variable amplitude load blocks are used. The adaptation of Miner's rule to coiling process take into account each coil as one single load block that causes partial damage to the shaft. The methodology was carried out on two experimental cases, which shafts had broken in operation, in order to adjust and assess the applicability of the model. According to Socha (2004), a period of stable damage progress, during which defects of the material structure are introduced and growth of micro cracks following its nucleation takes place occupies most of the fatigue life, about 80%. Then, it was assumed that the major part of the time spent in the fatigue process until fracture, corresponds to the crack initiation period.

Due to the large amount of historical data, it was developed a macro in Excel® to calculate the partial damage provoked for each coil and to promote the cumulative fatigue damage summation.

It is a common knowledge that Miner's rule has been criticized for decades due its shortcomings. Then, in order to improve Miner's rule assessment for mandrel shafts application an additional analysis has been carried out. This analysis is based on Halford's criterion (Halford, 1997). Finally, a third case, where the shaft was removed for inspection during its useful life was used to validate the proposed methodology.

The present work is motivated by the possibility of avoiding new fractures of mandrel's shafts in operation. Thus, the main purposes are:

- To serve as a maintenance tool that estimates the remaining fatigue lives of mandrel shafts subjected to cumulative fatigue damage.
- To verify the applicability of Miner's rule for mandrel shafts submitted to cumulative fatigue damage.
- To avoid new fractures of mandrel shafts in operation.

2. CUMULATIVE FATIGUE DAMAGE IN MANDREL SHAFTS

2.1. Coilers

Coilers are electro mechanical equipments installed in the entry and exit sides of cold strip processing lines with the purpose of wind a steel strip around an expansible mandrel shaft at an adequate tension. Figure (1) shows a coiler assembly which is composed by a cantilever type mandrel and a gear reducer. Mandrel's shaft is submitted to alternate stress caused by coil weight and torsion from the motor.

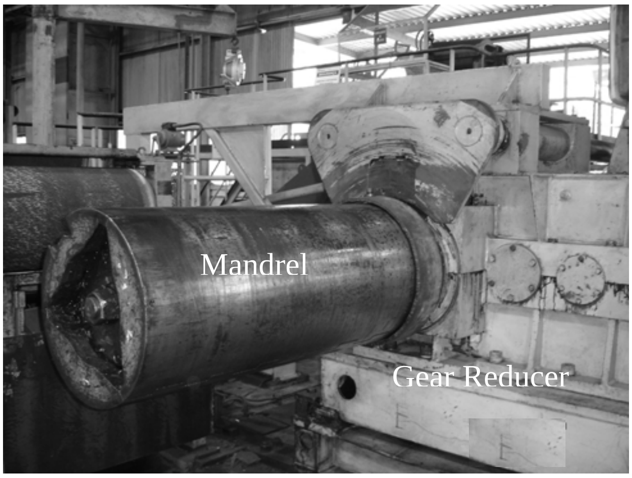


Figure 1. Coiler assembly

Mandrel shafts are manufactured in forged heat treated steel. Like other shafts, they are normally designed with stress raisers such as keyways, shoulders, holes, screw threads and shrink fits, which may cause susceptible conditions to fatigue failures for them.

2.2. Methodology for Mandrel's Shaft Prediction Life

The coiling process generates alternate growing stresses proportional to the coil's weight augmentation and related to the coil wraps. This variable amplitude spectrum load onto the mandrel shaft is repeatedly applied as long as a new coil is introduced. The fatigue process of the mandrel shaft begins when stress cycle with amplitude immediately higher than the shaft endurance strength (S_n) is applied. Then, as coil wraps are increasing, the stress amplitude is continuously compared with S_n and starting from the k-nth wrap, where the stress amplitude σ_k equalizes to the shaft endurance strength, starts the cumulative fatigue damage process until the last coil wrap. Figure 2 shows this fatigue process of the mandrel shaft.

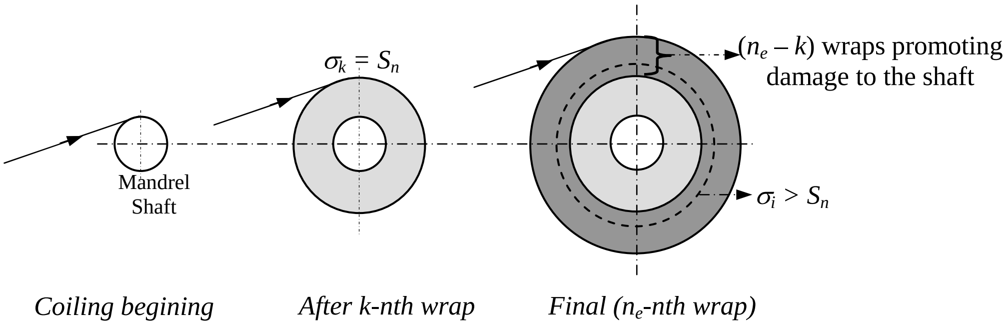


Figure 2. Fatigue process of a mandrel shaft during coiling

The i-nth stress amplitude (σ_i) represents one applied cycle depending on the coil geometric characteristics, as width and thickness. Therefore, at the coiling ending (n_e wraps), each coil has been contributed with (n_e- k) damage cycles applied onto the shaft. This cumulative fatigue damage process may be retaken, conditioned to the applied stresses magnitude of the next loading.

In order to simplify the calculation process of the applied stresses of each coil, in this work, it will be defined for the j-nth coil, the mean resultant stress amplitude of the wraps set (n_e - k) that promotes damage to the shaft critical section as indicated in Eq. (2).

$$\overline{\sigma}_j = \frac{\sum_{i=k_j}^{n_{ej}} \sigma_{i,j}}{n_{ej} - k_j} \text{, for } i = k_j \text{ to } n_{ej} \text{ wraps, and } j = 1 \text{ to } n_b \text{ coils.} \quad (2)$$

The stress amplitude $\sigma_{i,j}$ is assessed at the mandrel shaft critical section as showed in Fig. 3.

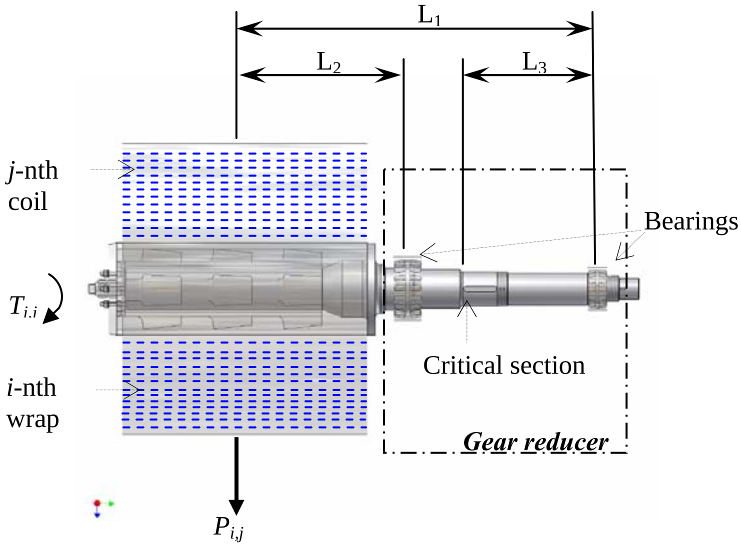


Figure 3. General arrangement and critical section location

Using von Mises criterion, stress amplitude $\sigma_{i,j}$ is calculated in Eq. (3) as the function of the i-nth wrap on the j-nth coil. (Shigley and Mischke, 1989).

$$\sigma_{i,j} = \sqrt{\left(\frac{K_f \times M_{i,j}}{J} \times \frac{d}{2}\right)^2 + 3 \times \left(\frac{T_{i,j}}{W}\right)^2}\ , \tag{3}$$

where d is the shaft diameter at the critical section, K_f is a function of the stress concentration factor K_t and from the notch sensitivity q as indicated in Eq. (4).

$$K_f = 1 + q(K_t - 1), \tag{4}$$

$M_{i,j}$ is the bending moment calculated by Eq. (5).

$$M_{i,j} = P_{i,j} \frac{L_2 L_3}{L_1 - L_2}\ , \tag{5}$$

where, the lengths L_1, L_2 and L_3 are shown in Fig. 3. $P_{i,j}$ is the concentrated coil load, which is a function of the i-nth wrap's weight, given by Eq. (6).

$$P_{i,j} = \frac{(d_m + 2e_j i)^2 - d_m^2}{4} \pi\, l_j \rho\ , \tag{6}$$

where d_m is the mandrel diameter, e_j and l_j are, respectively, the thickness and width of the strip in the j-nth coil and ρ is the coil density. $T_{i,j}$ is the torque applied onto the shaft given by Eq. (7).

$$T_{i,j} = T \frac{d_m + 2e_j i}{2}\ , \tag{7}$$

where, T is the strip tension. J is the moment of inertia of the shaft and W is the polar section modulus described, respectively in Eq. (8) and Eq. (9).

$$J = \frac{\pi}{64}\left(d^4 - d_f^4\right) \tag{8}$$

$$W = \frac{\pi}{16}\left(d^3 - d_f^3\right), \tag{9}$$

where d_f is the shaft internal diameter. In this work, the shaft endurance strength (S_n) is determined by the relation (10). (Shigley and Mischke, 1989).

$$S_n = 0.5 C_{tam} C_{\sup} S_u\ , \tag{10}$$

where S_u is the material ultimate strength and C_{tam} and C_{sup} are the size and material surface factors, respectively.

Then, the mechanical properties of the shaft material (ultimate strength) and the modifying factors of the S-N curve define the critical section endurance strength.

From the mean resultant stress amplitude for each coil $\bar{\sigma_j}$, it is possible to calculate the number of cycles N from S-N curve that corresponds to the stress amplitude $\bar{\sigma_j}$ starting from the relation indicated in Eq. (11). (Shigley and Mischke, 1989).

$$N_j = \left(\frac{\overline{\sigma}_j}{a}\right)^{\frac{1}{b}}, \tag{11}$$

where *a* and *b* are obtained from the expressions indicated in Eq. (12) and Eq. (13).

$$a = \frac{(0.9S_u)^2}{S_n} \tag{12}$$

$$b = -\frac{1}{3}\log\left(\frac{0.9S_u}{S_n}\right) \tag{13}$$

In this work, the partial damage caused by the *j*-nth coil is calculated in Eq. (14).

$$D_j = \frac{n_j}{N_j}, \tag{14}$$

where, n_j is the number of cycles from the mean resultant stress amplitude $\overline{\sigma_j}$ showed in Eq. (15).

$$n_j = n_{ej} - k_j \tag{15}$$

According to Miner's rule (1945), the final cumulative damage is defined by the sum of the partial damages from all the n_b processed coils. Starting from the historical sequence of the coils processed, i.e., from the width (l_j), thickness (e_j) and weight of each coil ($P_{i,j}$), the cumulative damage can be computed according Eq. (16), which states that fatigue failure is expected when such life fractions sum or exceed to unity.

$$\sum_{j=1}^{n_b} D_j \geq 1 \tag{16}$$

The adaptation of Miner's rule to mandrel shafts was done according explanation showed in Fig. 4, where each coil represents one load block.

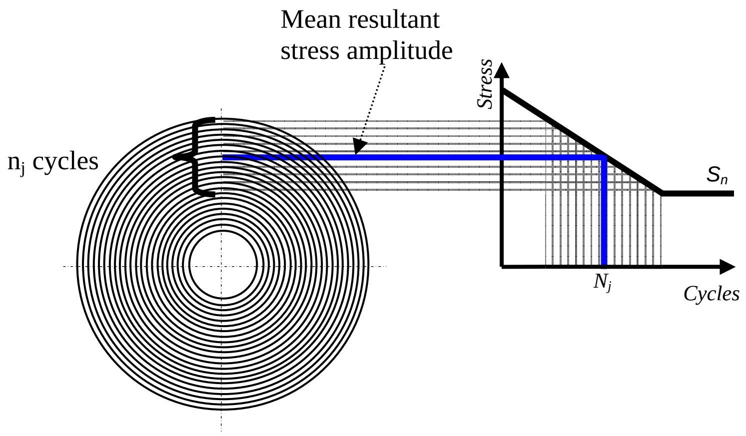


Figure 4. Partial damage caused by one single coil

3. EXPERIMENTAL CASES

The methodology was carried out on two experimental cases, which shafts had broken in operation, in order to adjust and assess the applicability of the model. In both cases, it was assumed that the major part of the fatigue process corresponds to the crack initiation period. The validity of the method was proven in a third case, where the shaft was removed for inspection during its useful life.

3.1. Experimental cases I and II

The main geometric characteristics of the cases I and II are presented in Tab. 1.

Table 1. Main geometric characteristics of the cases I and II

Description	Case I dimensions (mm)	Case II dimensions (mm)
Critical section diameter (d)	282	216
Shaft internal diameter (d_f)	100	82.55
Mandrel diameter (d_m)	610	610
Lengths: L_1; L_2 and L_3	2143; 1000 and 930	1975; 959 and 597

The main data for endurance strength calculation of the cases I and II are presented in Tab. 2.

Table 2. Main data for endurance strength calculation of the cases I and II

Description	Case I	Case II
Ultimate strength (S_u)	981 MPa	951 MPa
Notch sensitivity (q)	0.890	0.830
Size factor (C_{tam})	0.688	0.710
Surface factor (C_{sup})	0.880	0.730

Following Pilkey, 1997, the stress concentration factor for the two cases was calculated by Eq. (17).

$$K_t = 1.426 + 0.1643 \times \left(\frac{0.1}{\frac{r}{d}} \right) - 0.0019 \times \left(\frac{0.1}{\frac{r}{d}} \right)^2 , \tag{17}$$

where r is the keyway fillet radius and d is the critical section diameter.

Table 3 presents the endurance strength (S_n), line's nominal capacity and shaft fatigue capacity (corresponding to S_n), for the cases I and II.

Table 3. Line nominal capacity, endurance strength and shaft fatigue capacity (S_n)

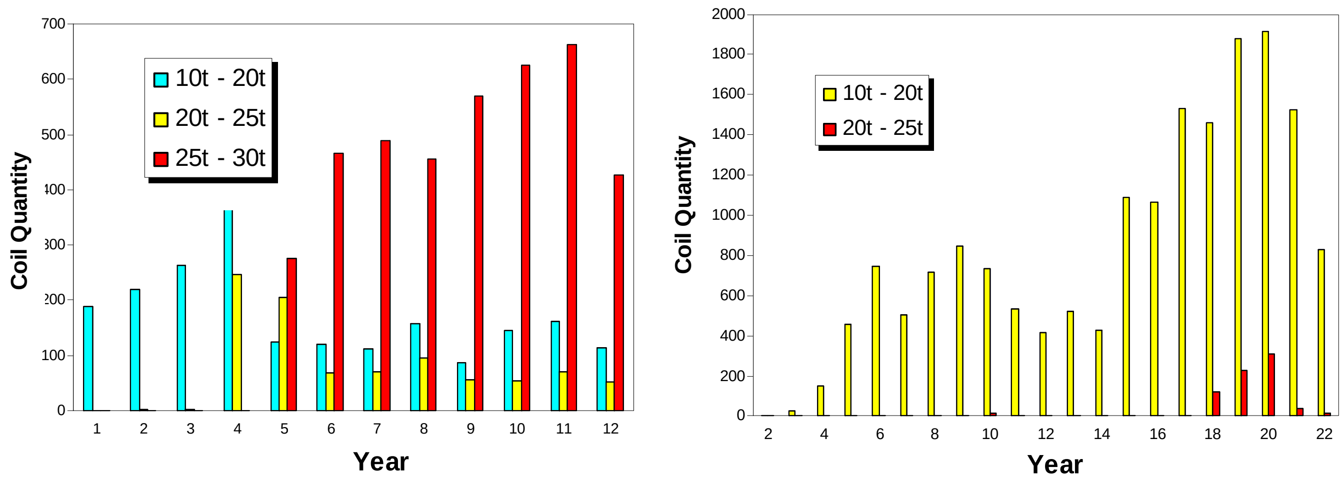
Description	Case I	Case II
Endurance strength (S_n)	297 MPa	246 MPa
Line nominal capacity	22.5 t	15.0 t
Shaft fatigue capacity = (S_n)	24.0 t	12.0 t

The historical sequential loads applied during the operational periods of the cases I and II are presented in Fig. 5. Comparing the historical coil weights applied with the line's nominal capacity, the overload distribution started in the 5^{th} year in the case I according to Fig. 5 (a) and in the 18^{th} year of operation in the case II, as shown in Fig. 5 (b).

Regarding overloads applied in the cases I and II, it is important to remark the overload ratio for each case. In the case I the overload ratio reaches a maximum value of 1.25 x shaft fatigue capacity. However, for the case II, this overload ratio is much more pronounced and reaches a maximum value of 2.08 x shaft fatigue capacity.

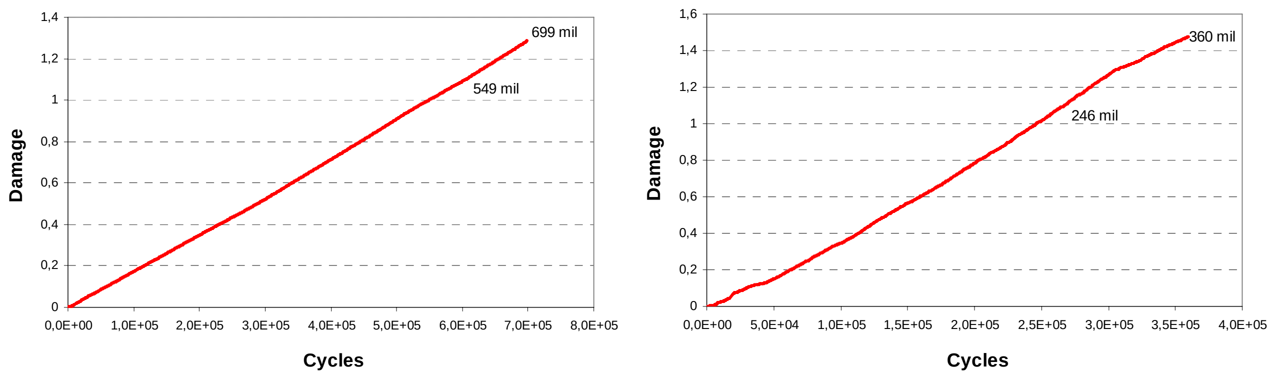
Equation 16 was used to determine the final cumulative fatigue damage, keeping the chronological sequential load preserved. The results of accumulated damage as a function of the number of cycles to failure and the number of cycles foreseen by the method are presented in Fig. 6.

The obtained results for cumulative fatigue damage are 1.28 for case I and 1.47 for case II. Both results indicate that the accumulated damage is a linear function of the number of cycles to failure. Although case II presents a much more pronounced overload, the results are to very close to each other and higher than unity, demonstrating good agreement with Miner's rule. Also, the results are inside a reasonable error bandwidth, i.e. the deviation from real fatigue life is no more than a factor of two, which means that the real life is between 50 and 200% of the prediction, according to Schijve (2001).



(a) (b)

Figure 5. Historical sequential loads for case I (a) and for case II (b)



(a) (b)

Figure 6. Accumulated damage as a function of the applied number of cycles for case I (a) and case II (b).

3.2. Halford criterion

Over the decades since Miner's work, the linear damage rule has been demonstrated, under certain critical circumstances, to suffer limitations in accuracy. According to the experiments after Miner's work the discrepancies can be more severe when there is an appropriate mixture of low-cycle fatigue (LCF) and high-cycle fatigue (HCF) loading. The further apart the lives N in LCF and HCF, the greater the deviation from linearity of damage accumulation. Then, for these severe loading conditions it was observed that complex models are more accurate.

This subject has been extensively discussed by Halford (1997), who has investigated the question: *"When should we apply a more complex, and slightly more costly, nonlinear damage analysis, and when can we get by with the simple linear damage rule?"* To answer this question, Halford propose some criteria for judging when to ignore the linear damage rule, in favor of a nonlinear rule.

One of the criteria is associated with the assessment of the minimum and maximum life levels to be considered in the cumulative fatigue damage analysis. For this purpose, it is necessary to identify the lowest life level, N_1, and the highest, N_2. If N_1 and N_2 are less than two orders of magnitude apart, continue with linear damage rule, because the maximum deviation between nonlinear model and linear damage rule under these conditions will be less than a factor of two on the total predicted life, which is considered inside a reasonable range of life's prediction, as mentioned before.

The definition of order of magnitude between N_1 and N_2 life levels is showed in Fig. 7, which represents in the *S-N* curve for the sequential amplitude stresses σ_1 and σ_2, the order of magnitude equal to two.

Additionally, in order to confirm the applicability of Miner's rule to mandrel shafts, the criterion of order of magnitude of life levels was adapted to the cases I and II. Table 4 presents the cycle life ranges from *S-N* curves and the obtained order of magnitude (N_2 / N_1) results for each case.

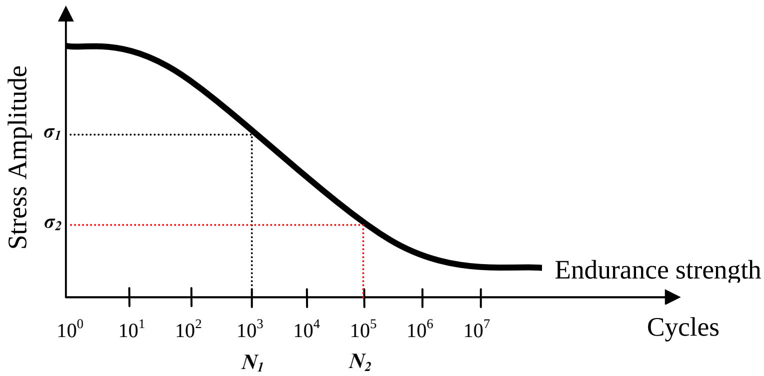


Figure 7. Order of magnitude between life levels

Table 4. N_1 and N_2 cycle levels from *S-N* curve and N_2 / N_1 results

Cases	N_1 (cycle)	N_2 (cycle)	N_2/N_1
I	3.85×10^5	$\pm 10^6$	1.0
II	7.94×10^4	$\pm 10^6$	2.0

The results from Tab. 4 were obtained from the application of maximum and minimum alternate stress of the most critical coil from each case. The result for the order of magnitude was equal to 1.0 for the case I and for the case II was equal to 2.0. According Halford's criterion, the case II is in the limit of linear damage rule application and the case I shows more adequacy to linear damage rule application. The explanation to this adequacy level is related with the distinct overload level presented in each case.

3.3. Experimental case III

After adequacy model approval in cases I and II, the method is carried out in a third case with the purpose of validation to mandrel shafts application. In this particular case, the mandrel shaft was removed during its useful life. The main geometric characteristics of the case III is presented in Tab. 5.

Table 5. Main geometric characteristics of the case III

Description	Dimensions (mm)
Critical section diameter (d)	310
Shaft internal diameter (d_f)	100
Mandrel diameter (d_m)	610
Lengths: L_1; L_2 and L_3	1766; 960 and 720

The main data for endurance strength calculation of the case III is presented in Tab. 6.

Table 6. Main data for endurance strength calculation of the case III

Description	Value
Ultimate strength (S_u)	1095 MPa
Notch sensitivity (q)	0.890
Size factor (C_{tam})	0.680
Surface factor (C_{sup})	0.710

Table 7 presents the endurance strength (S_n), line's nominal capacity and shaft fatigue capacity (corresponding to S_n), for the case III.

The historical sequential loads applied during the operational period of the case III is presented in Fig. 8. Comparing the historical coil weights applied with the line's nominal capacity, the overload distribution started in the 6th year. The maximum overload ratio observed in this case is only 1.12 times shaft fatigue capacity.

Table 7. Line nominal capacity, endurance strength and shaft fatigue capacity (S_n)

Description	Value
Endurance strength (S_n)	263 MPa
Line nominal capacity	22.5 t
Shaft fatigue capacity = (S_n)	26.7 t

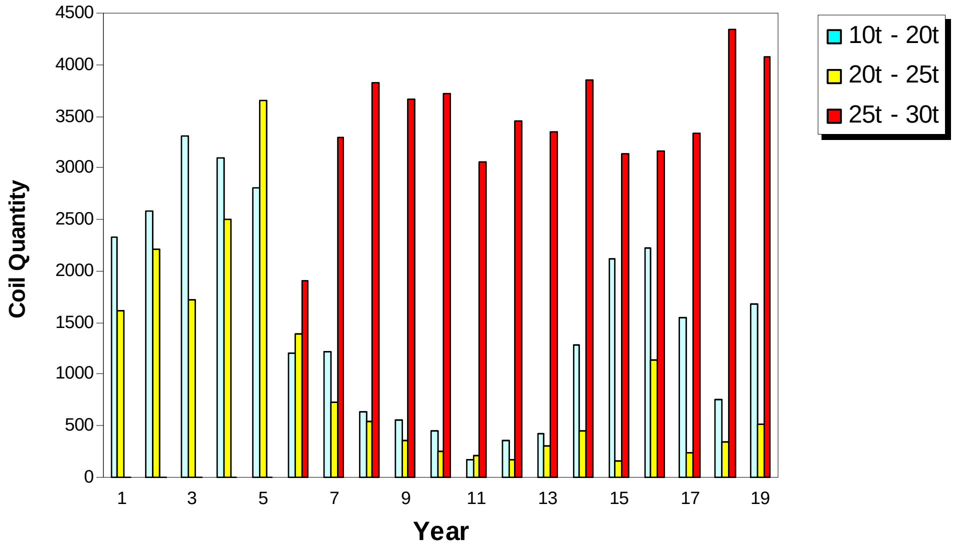


Figure 8. Historical sequential loads for case III

Equation 16 was used to determine the final cumulative fatigue damage, keeping the chronological sequential load preserved. The result is presented in Fig. 9.

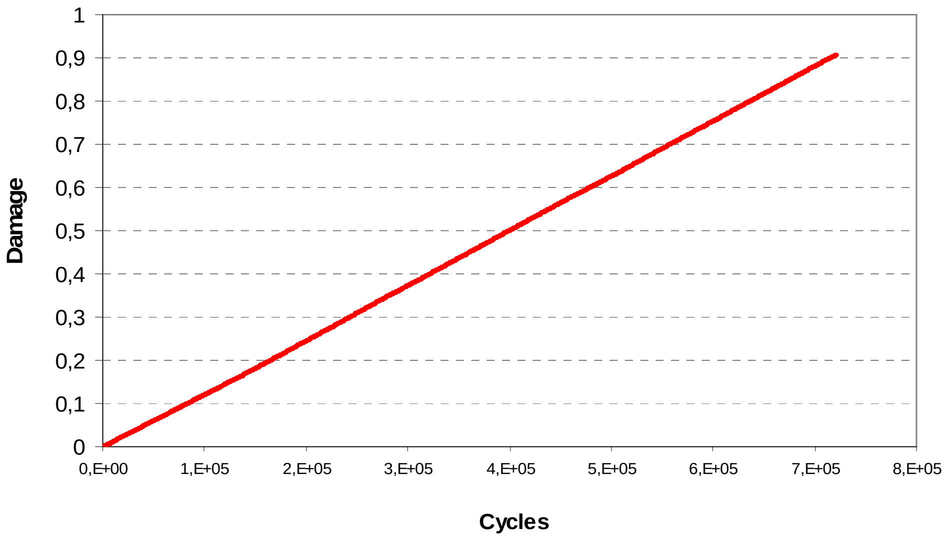


Figure 9. Cumulative damage result as a function of the applied number of cycles - Case III

The final result for cumulative fatigue damage is 0.9, which is lower than unity. The mandrel shaft was inspected using penetrated liquid essay and no discontinuity was revealed, as proven in Fig. 10. Therefore, the result obtained is in accordance with Miner's rule prediction, validating the applicability for mandrel shafts.

Figure 10. Shaft inspection using the penetrated liquid test

4. CONCLUSIONS

The obtained results have shown that the type and load sequence have a significant influence on the accumulated final damage value and on the degree of adjustment to the proposed linear model. When the stress amplitudes are closer to the endurance limit of the shaft's critical section, the obtained results are in very good agreement to the linear model. Finally, the linear model was tested in a third case, where the shaft was removed for inspection during its useful life. It was demonstrated that Miner's rule can be applicable as a strategic maintenance tool for estimate the remaining fatigue lives of mandrel shafts subjected to cumulative fatigue damage.

5. ACKNOWLEDGEMENTS

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