

STRENGTH PROPERTIES OF POROUS MEDIA BY MEANS OF A NON-LINEAR ELASTIC APPROACH

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Abstract. *The present paper describes a formulation of macroscopic strength properties of porous media relying on the simulation of the regime o plastic flow of the solid matrix by means of an appropriate fictitious non-linear elastic law. Explicit expression for the latter is derived in the particular case of a frictional solid matrix characterized by a Drucker-Prager yield condition. The approach is thus implemented in a numerical procedure specifically devised for evaluating the yield stresses of a porous medium. The finite element solutions are therefore compared to micromechanics-based analytical expression previously developed for the yield function. The last part of the paper is devoted to the formulation of phenomenological yield function for porous media with frictional solid matrix by combining the micromechanics-based approach and fitting the finite element results.*

Keywords*: porous medium, non-associated plasticity, strength properties, micromechanics, finite element method*

1. INTRODUCTION

Assessment of strength properties or limit states of heterogeneous materials from the properties of their constituents still remains of major concern in the field of material and structural engineering. Unlike the situation of elastic behavior where efficient techniques have been developed, application of micromechanical tools to the modeling of non-linear behavior of composite materials is relatively recent and several issues are still open (Zaoui, 2002).

In this context, the determination of strength properties of porous geomaterials is an important task in geomechanics. The contributions in this domain are dedicated mainly to purely cohesive constituents described by a von Mises failure condition (Suquet, 1997). Few works have dealt with porous media with frictional solid matrix, which is commonly encountered for rocks and soils (see Dormieux et al., 2006). In this respect, the works developed in Barthelémy and Dormieux (2003) or Maghous et al. (2009) can be considered are pioneering contributions in the field.

Trillat et al. (2006) presented finite element solutions for the macroscopic yield criterion of a porous Drucker-Prager material. Idealizing the morphology of an isotropic porous medium by means of the hollow sphere model, these authors implemented numerically the limit analysis methods in the framework of a second order conic programming formulation. Comparison of the numerical solution with the analytical criterion derived from a non-linear homogenization technique by Barthelémy and Dormieux (2003) showed an excellent agreement in the domain of macroscopic tensions together with an improvement for compressive stresses.

The aim of this contribution is to provide an efficient numerical tool to model by F.E procedure the limit states of porous materials defined by a frictional solid matrix. The approach is based on the simulation of the elastoplastic response of the solid matrix to a monotonic loading process by means of a fictitious non-linear elastic behavior. The numerical results will then be used to formulate a phenomenological macroscopic yield function improving the micromechanical approach in the domain of predominating compressive stresses.

2. MICROMECHANICS

Let us consider a representative elementary volume (r.e.v) Ω of a randomly porous material as displayed in Fig. 1. The domais occupied in the r.e.v by te solid matrix and the pore space are respectively denoted by Ω^m and Ω^p. The volume fraction of pores, i.e. the porosity, is defined as the ratio $\phi = |\Omega^p| / |\Omega|$.

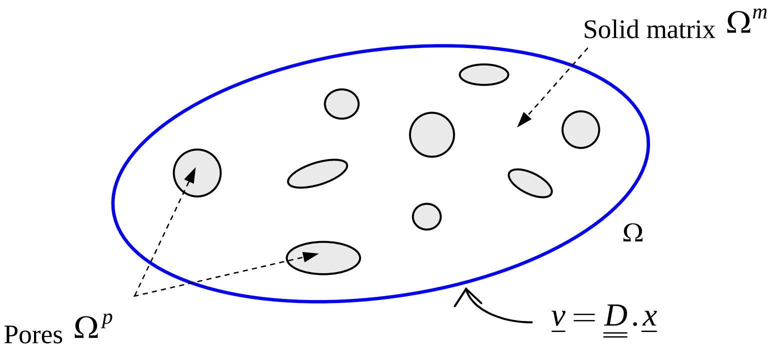


Figure 1. Representative elementary volume of a porous material and loading mode

The strength properties of the solie matrix are defined by a Drucker-Prager yield condition:

$$f^m(\underline{\underline{\sigma}}) = \sigma_d + T(\sigma_m - h) \leq 0 \quad (1)$$

where $\sigma_m = \mathrm{tr}\,\underline{\underline{\sigma}}/3$ is the mean stress and $\sigma_d = \left\|\underline{\underline{\sigma}}_d\right\| = \sqrt{\underline{\underline{\sigma}}_d : \underline{\underline{\sigma}}_d}$ is the norm of the deviatoric stress $\underline{\underline{\sigma}}_d = \underline{\underline{\sigma}} - \sigma_m \underline{\underline{1}}$. The parameters h and T respectively denote the tensile strength and friction coefficient of the material. In addition, the matrix plastic yielding is characterized by the non-associated plastic potential

$$g^m(\underline{\underline{\sigma}}) = \sigma_d + t\,\sigma_m \quad (2)$$

which yields the following flow rule

$$d_v = t\,d_m \quad \text{if } \underline{\underline{\sigma}} \neq h\underline{\underline{1}} \quad (3)$$

$\underline{\underline{d}}$ is the strain rate tensor. $d_v = \mathrm{tr}\,\underline{\underline{d}}/3$ and $d_d = \left\|\underline{\underline{d}} - d_m\underline{\underline{1}}\right\|$ are the volume strain rate and magnitude of deviatoric strain rate. The dilatancy coefficient t is such that $0 \leq t \leq T$. Equality $t = T$ corresponds to associated plasticity (normality rule).
The determination of the macroscopic strength condition, which defines the set of macroscopic limit states of the porous material, requires to previously defining the loading mode of the r.e.v. The latter is prescribed by means of uniform strain rate boundary conditions

$$\underline{v}(\underline{x}) = \underline{\underline{D}} \cdot \underline{x} \quad \forall\, \underline{x} \in \partial\Omega \quad (4)$$

where $\underline{v}$ is the velocity field and $\underline{\underline{D}}$ is the macroscopic strain tensor , i.e. the volume average on of the microscopic strain rate field $\underline{\underline{d}} = (grad\,\underline{v} + {}^T grad\,\underline{v})/2$ associated with the velocity field $\underline{v}$:

$$\underline{\underline{D}} = \langle \underline{\underline{d}} \rangle_\Omega = \frac{1}{|\Omega|}\int_\Omega \underline{\underline{d}}\,\mathrm{d}\Omega \quad (5)$$

2.1. Macroscopic limit states

Extending the definition of limit stress states proposed in Boushine et al. (2001) to the context of homogenization, it comes that the macroscopic yield function F^{hom} can be defined by

$$F^{\text{hom}}(\underline{\underline{\Sigma}}) \le 0 \quad \Leftrightarrow \quad \exists(\underline{\underline{\sigma}},\underline{v}) \;\; such\, that \;\; \begin{cases} \text{div}\,\underline{\underline{\sigma}} = 0 & in\ \Omega \\ \underline{\underline{\sigma}} = 0 & in\ \Omega^p \\ \underline{\underline{\Sigma}} = \langle \underline{\underline{\sigma}} \rangle_\Omega & \\ \underline{v} = \underline{\underline{D}}\,.\,\underline{x} & along\ \partial\Omega \\ f^m(\underline{\underline{\sigma}}) \le 0 & in\ \Omega^m \\ \underline{\underline{d}} = \partial g^m / \partial \underline{\underline{\sigma}} & in\ \Omega^m \end{cases} \tag{6}$$

where $\underline{\underline{\Sigma}}$ and $\underline{\underline{\sigma}}$ respectively refer to the macroscopic and microscopic stress tensors. The determination of F^{hom} requires to solving problem (6), which turns to be an uneasy task for a medium with randomly distributed pores. In addition, the limit analysis theorems and associated approaches do not apply in the present situation since the limit analysis framework is implicitly dedicated to the strength properties of materials with associated flow rule. An original strategy has been proposed and implemented in Maghous et al. (2009). It relies on:

- The rigid-plastic behavior with non-associated flow rule of the constitutive material of matrix is viewed formally as the limit of viscous-behaviors with pre-stress. This makes it possible to substitute problem (6) within a sequence of viscoplastic material stated on the r.e.v.
- The above mentioned sequence of viscoplastic problems is addresses by means of nonlinear homogenization technique based on the concept of effective stress and performed in the framework of theso-caled modified secant method (Suquet, 1997; Dormieux et al. 2002).

It is shown Maghous et al. (2009) that such micromechanical approach yields to the following expression for the strength condition of the porous medium

$$F^{\text{hom}}(\underline{\underline{\Sigma}}) = \frac{1+2\phi/3}{T^2}\,\Sigma_d^2 + \left(\frac{3\phi}{2T^2}-1\right)\Sigma_m^2 + 2\,(1-\phi)\,h\,\Sigma_m - (1-\phi)^2\,h^2 \le 0 \tag{7}$$

which delimits in the plane (Σ_m , Σ_d) an elliptic domain characterizing the set of macroscopic limit states of the porous medium (Fig. 2). It is first observed that condition (7) derived from micromechanics does not depend on the value of matrix dilatancy coefficient t. This is consistent with the results previously established in Barthélémy and Dormieux (2003) for $t = T$ and in Barthélémy (2005) $t = 0$. As far as the limit states of the porous medium are concerned, one may proceed adopting any (fictitious) value for t, not necessary the effective value of the dilatancy coefficient of the matrix. Adopting $t = T$ for instance, the yield condition for the porous medium may actually be derived in the framework of limit analysis. An important feature of the strength properties of randomly porous media is related to the flow rule at the microscopic level. If the geometrical domain of the limit states is independent on the value of dilatancy coefficient t of the solid matrix, the macroscopic flow rule is, in contrast, highly sensitive to this parameter. Indeed, it can be established that that for any limit stress state $\underline{\underline{\Sigma}}$, that is complying with the yield condition $F^{\text{hom}}(\underline{\underline{\Sigma}}) = 0$, the associated macroscopic strain rate $\underline{\underline{D}}$ is defined by

$$\underline{\underline{D}} = \frac{\partial G^{\text{hom}}}{\partial \underline{\underline{\Sigma}}} \qquad with \qquad G^{\text{hom}}(\underline{\underline{\Sigma}}) = \frac{1+2\phi/3}{2\,t\,T}\,\Sigma_d^2 + \left(\frac{3\phi}{4\,t\,T}-1\right)\Sigma_m^2 + 2\,(1-\phi)\,h\,\Sigma_m \tag{8}$$

It is readily seen from Eqs. (7) and (8) that if $t = T$ (associated flow rule at the microscopic level), the normality rule will hold at the macroscopic level, i.e. $\underline{\underline{D}} = \dfrac{\partial F^{\text{hom}}}{\partial \underline{\underline{\Sigma}}}$.

Finally, the case of a von Mises solid matrix with parameter k as shear strength is obtained as the limit of Eq. (7) and Eq. (8) when $T\,h = \sqrt{2}k$, $T \to 0^+$, $t \to 0^+$

$$F^{\text{hom}}(\underline{\underline{\Sigma}}) = \left(1+\frac{2\phi}{3}\right)\Sigma_d^2 + \frac{3\phi}{2}\,\Sigma_m^2 - 2\,(1-\phi)^2\,k^2 \le 0 \quad \text{and} \quad G^{\text{hom}}(\underline{\underline{\Sigma}}) = \left(1+\frac{2\phi}{3}\right)\Sigma_d^2 + \frac{3\phi}{2}\,\Sigma_m^2 \tag{9}$$

which as expected corresponds to an associated behavior. Expressions (9) have already been established in the past by several authors, such as Leblond et al. (1994) or Suquet (1997).

2.2. Case of pressurized pore space

In this section, we examine the question of the macroscopic limit states when the pore space is saturated by a fluid at pressure P. A stress-pressure state $(\underline{\underline{\Sigma}}, P)$ is said to be a limit state for the saturated porous medium if there exits a couple $(\underline{\underline{\sigma}}, \underline{v})$ of microscopic stress and velocity fields satisfying the set of equations (6) in which condition $\underline{\underline{\sigma}} = 0$ is replaced by condition $\underline{\underline{\sigma}} = -P\underline{\underline{1}}$ within the pore space Ω^p. It is assumed that, similarly to the drained situation (i.e. $P = 0$) examined in section 2.1, the domain of limit states is independence on the dilatation coefficient of the solid matrix. Under this assumption, a general result established in de Buhan and Dormieux (1996) indicates that:

"the stress-pressure state $(\underline{\underline{\Sigma}}, P)$ is a limit state for the pressurized porous material if and only if

$$\underline{\underline{\Sigma}}^{eff} = \frac{\underline{\underline{\Sigma}} + P\underline{\underline{1}}}{1 + P/h} \tag{10}$$

is a limit state for the dry (or drained) porous material (i .e. $P = 0$)"

Accordingly, the yield condition reads

$$F^{\text{hom}}(\underline{\underline{\Sigma}}, P) = \frac{1 + 2\phi/3}{T^2}\left(\Sigma_d^{eff}\right)^2 + \left(\frac{3\phi}{2T^2} - 1\right)\left(\Sigma_m^{eff}\right)^2 + 2(1-\phi)\,h\,\Sigma_m^{eff} - (1-\phi)^2\,h^2 \le 0 \tag{11}$$

This means that the macroscopic strength is controlled by the effective stress $\underline{\underline{\Sigma}}^{eff}$. As opposed to the traditional concept of effective stress, it is noted that $\underline{\underline{\Sigma}}^{eff}$ does not linearly depend on the pore pressure P. The value of P being fixed, the set of limit stress-pressure states $(\underline{\underline{\Sigma}}, P)$ is obtained in the plane (Σ_m, Σ_d) by means of a simple geometric transformation of the set of limit states for the dry porous material. This geometric transformation is the combination of a translation of quantity $-P$ along the Σ_m-axis and an isotropic expansion of ratio $1 + P/h$.

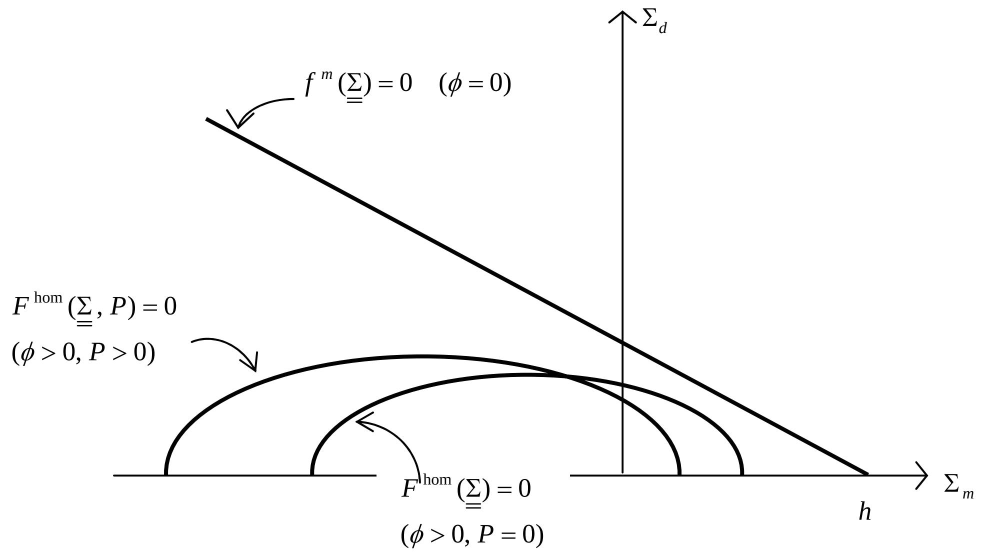


Figure 2. Schematic representation of the set of limit states for the porous material

3. FINITE ELEMENT ANALYSIS

Expression (7) of the macroscopic yield function $F^{\text{hom}}(\underline{\underline{\Sigma}})$ has been derived within the context of non-linear homogenization technique and the concept of effective strain. It represents therefore an approximate solution of problem (6) and as such, it is not the exact expression for the macroscopic yield function of the porous material. The objective her is to check the accuracy of the micromechanical estimate (7) through comparison with finite solutions.

The starting point consists in taking advantage of the local equivalence, under monotonic loading, between a rigid plastic behavior an an appropriate fictitious non-linear elastic behavior. The notations previously introduced for the microscopic and macroscopic strain rates $\underline{\underline{d}}$ and $\underline{\underline{D}}$ are straightforwardly transposed to local and macroscopic linearized strains $\underline{\underline{\varepsilon}}$ and $\underline{\underline{E}}$. The principle of the method is to look for a non-linear elastic material, characterized by its secant stiffness tensor

$$(\underline{\underline{\varepsilon}}) = 3k^m(\varepsilon_v, \varepsilon_d)\ \mathbb{J} + 2\mu^m(\varepsilon_v, \varepsilon_d)\ \mathbb{K} \tag{12}$$

where k^m is the bulk modulus, μ^m is the shear modulus, $\mathbb{J}_{ijkl} = 1/3\delta_{ij}\delta_{kl}$, $\mathbb{K} = \mathbb{I} - \mathbb{J}$ ($\mathbb{I}$ being the fourth-order identity tensor), and such condition $f^m(\underline{\underline{\sigma}})$ =0 is fulfilled asymptotically for large values of the deviatoric strain

$$\lim_{\varepsilon_d/\varepsilon_0 \to \infty} f^m\left(\underline{\underline{\sigma}} = \mathbb{C}(\underline{\underline{\varepsilon}}) : \underline{\underline{\varepsilon}}\right) = 0 \tag{13}$$

where $\varepsilon_0 << 1$ physically represents the order of magnitude of the shear strain at plastic yielding. A simple choice for the fictitious material consists in adopting a constant value for k^m. Eq. (13) therefore implies that

$$\mu^m(\varepsilon_v, \varepsilon_d) \approx \frac{T}{2\varepsilon_d}(h - k^m\varepsilon_v) \qquad if\ \varepsilon_d / \varepsilon_0 >> 1 \tag{14}$$

Figure 3 displays the non-linear elastic representation of the strength behavior obtained from Eqs. (12) and (14). For the F.E numerical implementation, the following expression has been adopted for the shear modulus

$$\mu^m(\varepsilon_v, \varepsilon_d) \approx \frac{T}{2\varepsilon_d}(h - k^m\varepsilon_v) \times \frac{1/\varepsilon_0}{1 + \varepsilon_d/\varepsilon_0} \tag{15}$$

It can be readily seen that $\mu^m \to 0$ in the asymptotic regime, whereas k^m remains constant. This implies that $\mu^m / k^m \to 0$, which in turns implies that asymptotically $\varepsilon_v / \varepsilon_d \to 0$. The fictitious non-linear elastic material behaves for large values of shear strain as incompressible material. This means that non-linear elastic material will asymptotically model a Drucker-Prager solid matrix with a no plastic dilatancy, i.e. $t = 0$. However, this is not a restriction, since it has been seen that the macroscopic yield domain does not depend on the value of $t \in [0, T]$.

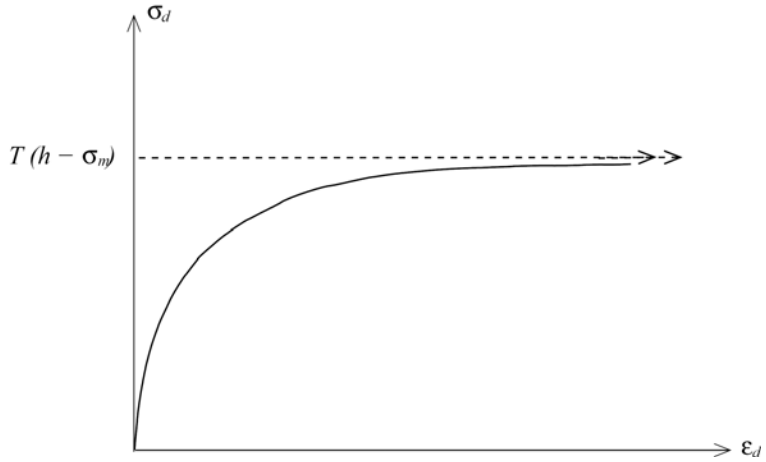


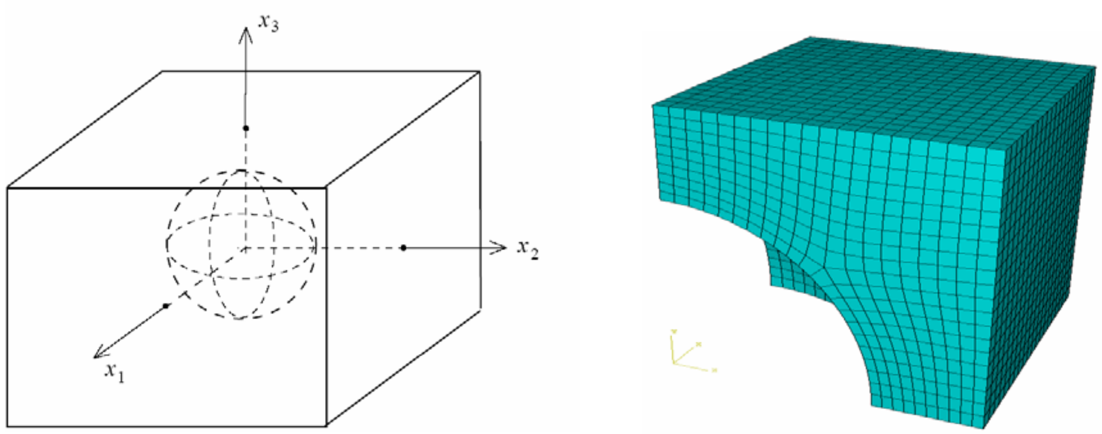
Figure 3. Representation of the non-linear elastic behavior associated with the rigid plastic solid matrix

The above behavior has been implemented in the code Abaqus as UMAT subroutine. A simplified morrphology for the porous material, which refers to a periodic configuration has been considered. The corresponding elementary cell is sketched in Fig. 7a (cubic volume of solid phase surrounding a centered spherical void). The loading of the elementary cell is defined by uniform boundary strain conditions, with macroscopic strains of the form

$$E = E_0 \left(e_3 \otimes e_3 + \alpha \; (e_1 \otimes e_1 + e_2 \otimes e_2) \right) \tag{16}$$

where parameter α stands for the loading path. Owing to periodicity and the material symmetry, only the eighth of the elementary cell is discretized into finite elements (Fig. 4.b).

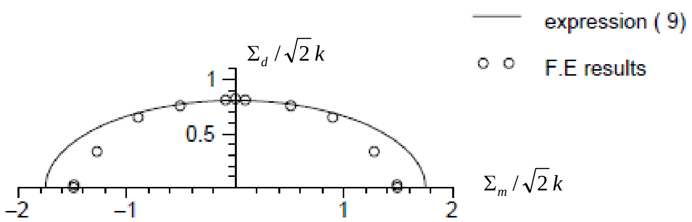
Even the periodic configuration considered for the finite element analysis introduces a slight anisotropy in strength, it is expected that comparison between the micromechanical prediction (7) and the finite element solution would provide useful indication as regards then accuracy of the technique based on solving the limit state problem by means of the concept of an effective strain.



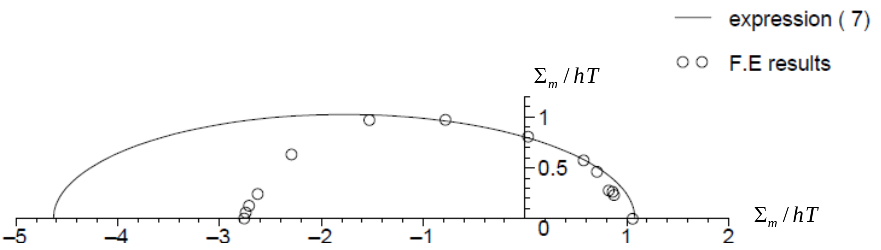
(a) (b)

Figure 4. Elementary cell of the porous material and F.E discretization

Figures 5(a)-(b) show the results obtained numerically as well as the micromechanical predictions provided by Eq. (7) for a porosity $\phi = 15\%$, which is rather moderate for geomaterials. A good agreement is observed except for high macroscopic compressive stresses ($\Sigma_m \leq 0$) for which the proposed micromechanical approach overestimates the set of limit states. This emphasizes the efficiency of the technique based on the concept of effective strain. It also points out that this technique should be improved in the domain of predominating compressive stresses. The shortcoming of the method in the range of predominating compressive stresses is attributed to the implementation of a single effective strain for the whole solid matrix, which is not accurate enough for low porosity. Indeed, highly heterogeneous strain rates are expected to concentrate around the pores in this case, so that an average value over the whole solid matrix fails to capture the local strain rate level.



(a) von Mises solid matrix



(b) Drucker-Ptrager solid matrix with $T = 0.3$

Figure 5. Strength domain for a porous medium $\phi = 15\%$

4. EMPIRICAL MODEL FOR YIELD FUNCTION

In consistence with the results provided in Trillat et al. (2006), the F.E analysis of the previous section gave evidence of the yield surface asymmetry between tension and compression. It clearly indicates that the micromechanics-based yield function given by Eq. (7) can lead to a significant overestimate of the domain of limit sates for predominating compressive stresses. This may particularly occur for frictional matrix (i.e. $T \neq 0$) with relatively low porosities. The purpose herein is to propose a modified yield function $\tilde{F}^{\text{hom}}(\underline{\underline{\Sigma}})$ which would correlates better with the F.E solutions. This will be achieved on the basis on both micromechanical approach and numerical solutions as described below.

Let $\Sigma_m^0 = (1-\phi)h/(1-3\phi/2T^2)$ denotes the abscisse, in plane $\left(\Sigma_m,\ \Sigma_d\right)$, of the center of ellipse $F^{\text{hom}}(\underline{\underline{\Sigma}}) = 0$. We then introduce the macroscopic stress state $\underline{\underline{\tilde{\Sigma}}}^0$ defined by its coordinates $\tilde{\Sigma}_m^0 = \frac{1}{2}\Sigma_m^0$ and $\tilde{\Sigma}_d^0$ such that $F^{\text{hom}}(\underline{\underline{\tilde{\Sigma}}}^0) = 0$. In view to improve the prediction of the isotropic compressive limit state, the value of limit pressure for a hollow sphere under isotropic compression

$$\tilde{\Sigma}_m^c = h\left(1-\phi^{\sqrt{2/3}T/(1+\sqrt{2/3}T)}\right) \tag{17}$$

will naturally provide a reasonable estimate for the value of macroscopic limit in isotropic compression state.

To summarize, the new yield function proposed $\tilde{F}^{\text{hom}}(\underline{\underline{\Sigma}})$ is defined by

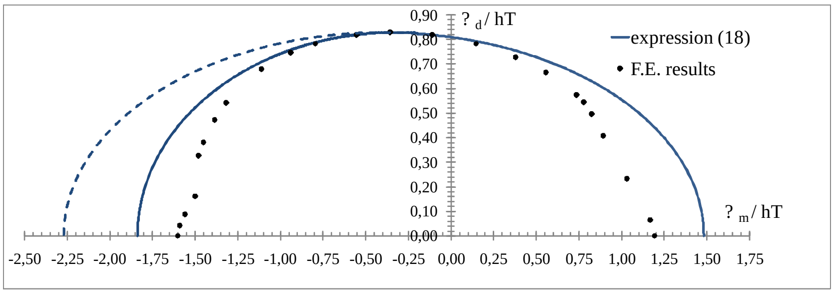
$$\tilde{F}^{\text{hom}}(\underline{\underline{\Sigma}}) = \begin{cases} F^{\text{hom}}(\underline{\underline{\Sigma}}) & if\ \Sigma_m \geq \tilde{\Sigma}_m^0 \\ F_c^{\text{hom}}(\underline{\underline{\Sigma}}) & if\ \Sigma_m \leq \tilde{\Sigma}_m^0 \end{cases} \tag{18}$$

where $F_c^{\text{hom}}(\underline{\underline{\Sigma}}) = 0$ represents the equation of ellipse coinciding with ellipse $\tilde{F}^{\text{hom}}(\underline{\underline{\Sigma}}) = 0$ at stress state $\underline{\underline{\tilde{\Sigma}}}^0$ (f=f=0 e df=df), and intersecting the Σ_m -axis at isotropic compressive stress $\tilde{\Sigma}_m^c$:

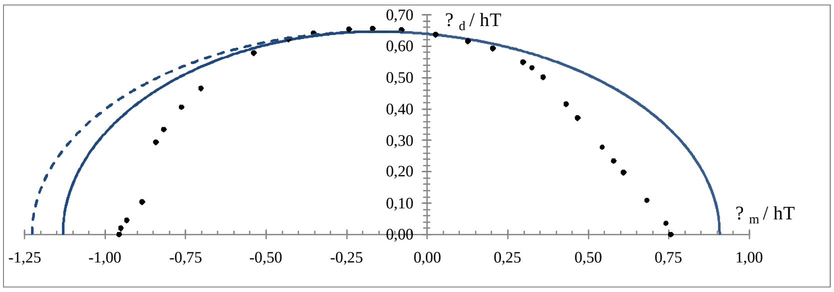
$$F_c^{\text{hom}}(\underline{\underline{\tilde{\Sigma}}}^0) = F^{\text{hom}}(\underline{\underline{\tilde{\Sigma}}}^0)\ ;\ \frac{\partial F_c^{\text{hom}}}{\partial \underline{\underline{\Sigma}}}(\underline{\underline{\tilde{\Sigma}}}^0) \quad and \quad F_c^{\text{hom}}(\Sigma_m = \tilde{\Sigma}_m^c, \Sigma_d = 0) = 0 \tag{19}$$

which means that the micromechanics-based prediction (7) is preserved for tension and moderate compression stress states, while the phenomenological approximation $F_c^{\text{hom}}(\underline{\underline{\Sigma}}) = 0$ is adopted for predominating compressive stresses.

The accuracy of such a yield function is assessed through comparison to F.E solutions. Figures 6(a)-(d) display in the plane $\left(\Sigma_m,\ \Sigma_d\right)$ the yield function $\tilde{F}^{\text{hom}}$ (curves) along with the F.E results (symbols). It is noteworthy that the yield function $\tilde{F}^{\text{hom}}(\underline{\underline{\Sigma}})$ correlates better with the F.E results for wide range of the porosity, the friction coefficient of solid matrix and the compressive hydrostatic stresses. Clearly enough, the proposed yield function $F_c^{\text{hom}}(\underline{\underline{\Sigma}})$ for stresses such that $\Sigma_m \leq \tilde{\Sigma}_m^0$ is only a first attempt toward a formulation of more accurate predictive model for macroscopic limit states of porous media.



(a)



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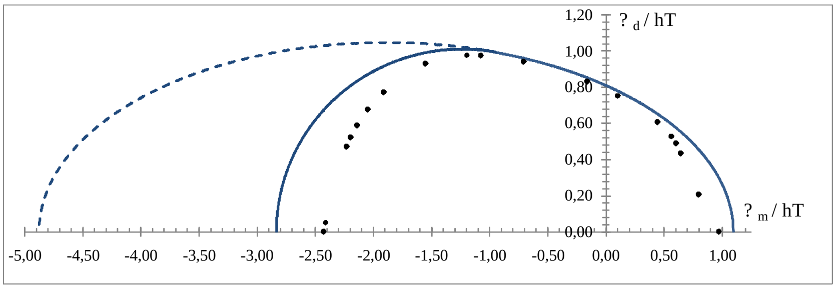
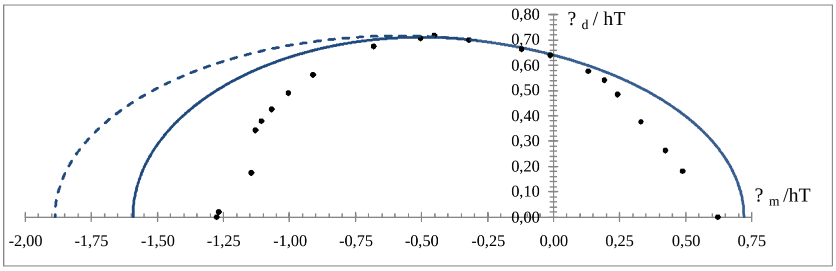


(c)



(d)

Figure 6. The yield function $\tilde{F}^{\text{hom}}$ (curves) and F.E results (symbols) for: (a) $T = 0.1$ and $\phi = 15\%$; (b) $T = 0.1$ and $\phi = 30\%$; (c) $T = 0.3$ and $\phi = 15\%$; (d) $T = 0.3$ and $\phi = 30\%$

5. CONCLUDING COMMENTS

The macroscopic limit stress states of a porous material with frictional solid matrix have been addressed within the framework of a micromechanics-based formulation. First, the micromechanical approach based on a non-linear method and the concept of effective strain rate, as well as the related analytical expression for yield function, have been recalled. In order to assess the accuracy of the micromechanical prediction, a finite element approach to limit stress states have been then proposed. It is based on the formal equivalence under monotonic loading between the response of an elasto-plastic material and that of an appropriate non-linear elastic material.

In this context, a non-linear elastic behavior has been formulated allowing to simulate asymptotically the regime of plastic flow of a Drucker-Prager material. It has therefore been introduced within a F.E procedure aiming at deriving numerically limit stress states for a frictional porous material. Comparison between numerical solutions and micromechanical predictions emphasized the accuracy of the micromechanical yield function for a large range of porosities and macroscopic hydrostatic stresses. It also shows, however, that the micromechanics-based model may significantly overestimate the limit state domain for predominating hydrostatic compressive stresses, more specifically in the situation of small porosities.

On the basis of these observations, a macroscopic yield function for porous material have been proposed by combining the micromechanical approach and by fitting the F.E results of the yield stresses obtained for a simplified geometry of the porous material. The phenomenological model thus obtained prove to correlate well with the F.E solutions in the range of predominating compressive stresses.

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