ON A NONLINEAR DYNAMICS OF A NON-IDEAL OSCILLATOR, WITH A SNAP-THROUGH TRUSS ABSORBER (STTA)

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Abstract. *In this paper, we consider a snap-through truss absorber (STTA), coupled to a nonlinear oscillator under an excitation force of an electric motor, with eccentricity and limited power supply, characterized as non-ideal oscillator (NIO). Special attention it is focused on the passage through resonance region, when the oscillating frequency of the excitation forces it is near the natural frequency of the supporting. We showed by numerical simulations, the attenuation of the Sommerfeld effect.*

Keywords: non-ideal oscillator, vibration absorber, Sommerfeld

1. INTRODUCTION

In this paper, we consider a study of non-ideal vibrating systems, that is, when the excitation is influenced by the response of the system, has been considered a major challenge in theoretical and practical engineering research. When the excitation is not influenced by the response, it is said to be an ideal excitation or an ideal source of energy. On the other hand, when the excitation is influenced by the response of the system, it is said to be non-ideal.

Then, depending of the excitation, one refers to vibrating systems as ideal or non-ideal. The behavior of the ideal vibrating systems is well known in the current literature, but there are few results on non-ideal ones. Generally, non-ideal vibrating systems are that for which the power supplies is limited. The behavior of the vibrating systems departs from the ideal case, as power supply becomes more limited. For non-ideal dynamical systems, one must add an equation that describes how the energy source "supplies the energy to the equations" that governs the corresponding ideal dynamical systems and the response is unknown. Thus, as a first characteristic, the non-ideal vibrating system has one more degree of freedom than its ideal counterpart.

We remarked that in non-ideal oscillators it is present the so-called Sommerfeld effect: steady state frequencies of the DC motor will usually increase as more power (voltage) is given to it in a step-by-step fashion. When a resonance condition with the structure it is reached, the better part of this energy is consumed to generate large amplitude vibrations of the foundation without sensible change of the motor frequency. Eventually, enough power is supplied to the motor to initiate the jump; the operating frequency increases and the foundation amplitude decreases, resulting in lower power consumption by the motor.

Jump phenomena and the increase in power required by a source operating near resonance are manifestations of a non-ideal energy source and are often referred as the SOMMERFELD Effect. Sommerfeld suggested that the structural response provided a sort of energy sink. Thus, we pay to vibrate our structure rather than operate the machinery. One of the problems often faced by designers is how to drive a system through resonance and avoid the "energy sink" described by Sommerfeld. In this paper we will illustrate this problem as an example.

For more details on non-ideal systems theory, please see: (Kononenko, 1969); (Nayfeh and Mook, 1979); (Dimentberg et al, 1994); (Balthazar et al., 2001, 2003); (Bolla et al., 2007); (Dantas and Balthazar, 2007) and a number of others authors.

The implementation of vibration absorbers for non-ideal oscillators it was proposed by (Felix et al; 2005), using a tuned liquid column damper and saturation phenomenon through of the numerical simulations. The exhibition, in details, of the snap-through phenomenon was presented by (Pi, et al., 1971), they investigated the influence of random loadings on a long cylindrical shell panel capable.

In this paper, we investigate the interaction between the non-ideal oscillators and snap-through truss absorber, based on previous the work done by (Avramov and Mikhlin, 2004). They investigated theoretically an ideal oscillator with a snap-through truss.

2 NON-IDEAL OSCILLATOR MODELING

The non-ideal oscillators and snap-through truss absorber, based on previous work done by (Avramov and Mikhlin, 2004), may be defined by Fig1.

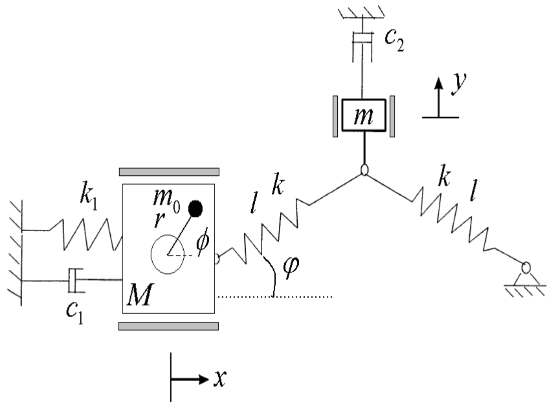

Figure 1 Schematic of a non-ideal structure attachment coupled to snap-through truss absorber

The mathematical model of the non-ideal oscillator (Fig 1), it is represented by the following governing equations of motion:

$$m_t\ddot{x} + f(x,\dot{x}) + \frac{\partial U(x)}{\partial x} = m_0 r(\dot{\phi}^2 \sin\phi - \ddot{\phi}\cos\phi),$$
$$I\ddot{\phi} = L(\dot{\phi}) - H(\dot{\phi}) - m_0 r\ddot{x}\cos\phi \quad (1)$$

where $m_t = M + m_0$ it is the total mass and x it is horizontal displacement of the NIO, ϕ it is rotation angle of the D.C. motor shaft, the parameters r and m_0 are the eccentricity and mass of unbalanced shaft of the D.C. electric motor; I it is the moment of inertia of the rotor, the function. $H(\dot{\phi})$ it is the resistive torque applied to the motor, the function $L(\dot{\phi})$ it is the driving torque of the source of energy (motor). Note that, usually, the inductance is much smaller than the mechanical time constant of the system and then in stationary regime, we can consider $L(\dot{\varphi}) - H(\dot{\varphi})$ as (linear) $L(\dot{\phi}) - H(\dot{\phi}) = u_1 - u_2\dot{\phi}$, where are u_1 related to voltage applied across the armature of the D.C. motor, that is, a possible control parameter of the problem and u_2 is a constant for each model of DC motor considered. $f(x,\dot{x})$ is the nonlinear and non-conservative part of the restoring force, while $\frac{\partial U(x)}{\partial x}$ is its conservative part (U is the potential, or strain energy). The function $f(x,\dot{x})$ and the potential U are defined as:

$$f(x,\dot{x}) = c_1\dot{x},\ U(x) = \frac{1}{2}k_1x^2,\ f(x,\dot{x}) = c_1\dot{x},\ U(x) = \frac{1}{2}k_1x^2 + \frac{1}{4}k_2x^4,\ f(x,\dot{x}) = c_1\dot{x},\ U(x) = \frac{1}{2}k_1x^2 + \frac{1}{4}k_2x^4 + \frac{1}{6}k_3x^6,$$
$$f(x,\dot{x}) = -c_1\dot{x} + c_2\dot{x}^3,\ U(x) = \frac{1}{2}k_1x^2,\ f(x,\dot{x}) = -c_1\dot{x} + c_2x^2\dot{x},\ U(x) = \frac{1}{2}k_1x^2. \quad (2)$$

2.1 Coupling between NIO and STTA

The governing equations of motion of the coupling system between the STTA and the non-ideal oscillator are given by:

$$(M+m_0)\ddot{x} + k_1x + 2k\left[\begin{array}{l} x - l\cos\varphi \\ +\dfrac{l(l\cos\varphi - x)}{\sqrt{l^2 + 2l(y\sin\varphi - x\cos\varphi) + x^2 + y^2}} \end{array}\right] = m_0r(\dot{\phi}^2\sin\phi - \ddot{\phi}\cos\phi) - c_1\dot{x},$$

$$I\ddot{\phi} = \Gamma(\dot{\phi}) - m_0r\ddot{x}\cos\phi,$$

(3)

$$m\ddot{y} + 2k\left[\begin{array}{l} l\sin\varphi + y \\ -\dfrac{l(l\sin\varphi + y)}{\sqrt{l^2 + 2l(y\sin\varphi - x\cos\varphi) + x^2 + y^2}} \end{array}\right] = -c_2\dot{y},$$

where (x, y, ϕ) are the generalized coordinates of the NIO, STTA and rotor respectively; the quantity (m_0, M, m) are the unbalance mass, NIO mass, STTA mass, (k_1, k) are linear spring stiffness and (c_1, c_2) are the linear damping of the NIO and STTA respectively, l is a length of the spring and φ angle defines the stable equilibrium position of the STTA.

Considering the following non-dimensional parameters:

$$\omega_n = \sqrt{\frac{k_1}{M + m_0}},\quad \tau = \omega_n t,\quad u = \frac{x}{l},\quad v = \frac{y}{l},\quad \eta_1 = \frac{m_0 r}{(M + m_0)l},\quad \eta_2 = \frac{m_0 rl}{I},\quad \alpha_1 = \frac{c_1}{(M + m_0)\omega_n},\quad \alpha_2 = \frac{c_2}{m\omega_n},\quad \gamma = \frac{k}{k_1},\quad \mu = \frac{M + m_0}{m},$$
$$\Gamma(\varphi') = a - b\varphi',\ a = \frac{u_1}{I\omega_n^2},\ b = \frac{u_2}{I\omega_n},\ c = \cos\varphi,\ s = \sin\varphi, \tag{4}$$

The governing equation of motion (3), itself reducing to the following dimensionless equations:

$$\begin{aligned} &u'' + u + 2\gamma(u - c)K(u, v) = \eta_1(\phi'^2\sin\phi - \phi''\cos\phi) - \alpha_1 u', \\ &\phi'' = a - b\phi' - \eta_2 u''\cos\phi, \\ &v'' + 2\mu\gamma(s + v)K(u, v) = -\alpha_2 v', \end{aligned} \tag{5}$$

Where

$$K(u, v) = 1 - \frac{1}{\sqrt{1 + 2(vs - uc) + u^2 + v^2}} \tag{6}$$

It is the function of coupling nonlinear stiffness, which produces the energy transfer of the NIO to STTA.

2.2 Equilibrium Point and Stability

Taking new variables, defining by $x_1 = u$, $x_2 = u'$, $x_3 = v$, $x_4 = v'$, $x_5 = \phi$, $x_6 = \phi'$,

We will obtain from (5) that

$$\begin{aligned} &x_1' = x_2, \\ &x_2' = \frac{1}{\Delta}[-x_1 - 2\gamma(x_1 - c)K(x_1, x_3) \\ &\quad + q_1 x_6^2\sin x_5 - \alpha_1 x_2 - q_1(a - bx_6)\cos x_5], \\ &x_3' = x_4, \end{aligned}$$

(7)

$$\begin{aligned} &x_4' = -2\mu\gamma(s + x_3)K(x_1, x_3) - \alpha_2 x_4, \\ &x_5' = x_6, \\ &x_6' = \frac{1}{\Delta}\{(a - bx_6) - q_2[-x_1 - 2\gamma(x_1 - c)K(x_1, x_3) + q_1 x_6^2\sin x_5 - \alpha_1 x_2]\cos x_5\}, \end{aligned}$$

$$\Delta = 1 - q_1 q_2\cos^2 x_5 \neq 0,\ K(x_1, x_3) = 1 - \frac{1}{\sqrt{1 + 2(x_3 s - x_1 c) + x_1^2 + x_3^2}}. \tag{8),(9}$$

First of all, we will investigate the points of equilibrium of equation (7). They may be found by setting $x_1' = x_2' = x_3' = x_4' = x_5' = x_6' = 0$. All of them, are defined as being solutions of non-linear algebraic coupled equations

$$\begin{aligned} &x_1 + 2\gamma(x_1 - c)K(x_1, x_3) + q_1 a\cos x_5 = 0, \\ &(s + x_3)K(x_1, x_3) = 0, \\ &a - q_2[-x_1 - 2\gamma(x_1 - c)K(x_1, x_3)]\cos x_5 = 0,\ x_2 = 0,\ x_4 = 0,\ x_6 = 0, \end{aligned} \tag{10}$$

Considering $a = 0$, due to $\Delta \neq 0$, see Eq. (8), and assuming that $x_5 = 0$, the system (10) will obtain three equilibriums points:

2.2-1 First Case:

$$x_1^4 - 2cx_1^3 + (1-s^2)x_1^2 - \frac{4\gamma^2 c^2}{(1+2\gamma)^2} = 0, \quad x_1 < 0,$$
$$1 - s^2 - 2cx_1 + x_1^2 > 0, \tag{11}$$
$$x_3 = -s, \quad x_2 = 0, \quad x_4 = 0, \quad x_5 = 0, \quad x_6 = 0.$$

2.2-2 Second Case:

$$x_1 = 0, x_2 = 0, x_3 = 0, x_4 = 0, x_5 = 0, \quad x_6 = 0 \tag{12}$$

2.2-3 Third Case:

$$x_1 = 0, \quad x_2 = 0, \quad x_3 = -2s, \quad x_4 = 0, \quad x_5 = 0, x_6 = 0,$$
$$1 + 2x_3 s + x_3^2 > 0 \tag{13}$$

Note that, the Jacobian matrix it was constructed, in order to determine the stability of the fixed points:

$$J = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ J_{21} & J_{22} & J_{23} & 0 & J_{25} & J_{26} \\ 0 & 0 & 0 & 1 & 0 & 0 \\ J_{41} & 0 & J_{43} & -\alpha_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ J_{61} & J_{62} & J_{63} & 0 & J_{65} & J_{66} \end{bmatrix} \tag{14}$$

where

$$J_{21} = \frac{1}{\Delta}\{-1 - 2\gamma K - 2\gamma(x_1 - c)(-c + 2x_1) \times (1 + 2(x_3 s - x_1 c) + x_1^2 + x_3^2)^{-3/2}\},$$

$$J_{22} = -\frac{\alpha_1}{\Delta},$$

$$J_{25} = [q_1 x_6^2 \cos_5 + q_1(a - bx_6)\sin x_5]\Delta^{-1} - [f - q_1(a - bx_6)\cos x_5](q_1 q_2 \sin 2x_5)\Delta^{-2},$$

$$J_{26} = \frac{1}{\Delta}(2q_1 x_6 \sin x_5 + q_1 b \cos x_5),$$

$$J_{41} = -2\mu\gamma(s + x_3)\{(-c + x_1) \times [1 + 2(x_3 s - x_1 c) + x_1^2 + x_3^2]^{-3/2}\},$$

$$J_{43} = -2\mu\gamma K - 2\mu\gamma(s + x_3)\{(s + x_3) \times [1 + 2(x_3 s - x_1 c) + x_1^2 + x_3^2]^{-3/2}\},$$

$$J_{61} = \frac{1}{\Delta}\{-q_2 \cos x_5 [-1 - 2\gamma K - 2\gamma(x_1 - c) \quad \times(-c + x_1)\left(1 + 2(x_3 s - x_1 c) + x_1^2 + x_3^2)^{-3/2}\right)]\},$$

$$J_{62} = \frac{\alpha_1 q_2 \cos x_5}{\Delta}, \quad J_{63} = \frac{1}{\Delta}\{-q_2 \cos x_5 [-2\gamma(x_1 - c)(s + x_3) \times \left(1 + 2(x_3 s - x_1 c) + x_1^2 + x_3^2)^{-3/2}\right)]\},$$

$$J_{65} = (fq_2 \sin x_5 - q_1 q_2 x_6^2 \cos^2 x_5)\Delta^{-1}$$
$$-(a - bx_6 - fq_2 \cos x_5) q_1 q_2 \sin 2x_5 \Delta^{-2}, \quad J_{66} = \frac{1}{\Delta}\left(-b - q_1 q_2 x_6 \sin 2x_5\right),$$

where

$$K = 1 - [1 + 2(x_3 s - x_1 c) + x_1^2 + x_3^2]^{-1/2}, \quad f = -x_1 - 2\gamma(x_1 - c)K + q_1 x_6^2 \sin x_5 - \alpha_1 x_2 . \tag{16}$$

By evaluating the eigenvalues of the Jacobian matrix, the stability it is determined by the equilibrium points. Eigenvalues should not have positive real parts to maintain the stability of the system.

3. NUMERICAL RESULTS

The numerical simulations were carried out by using Matlab™ using as numerical integrator the Runge-Kutta fourth order algorithm with variable time length. The adopted parameters for numerical simulations those will be carried out, are considered of the Table 1: According to Table 1, we have $c = 0.92$, $s = 0.39$ and we considered a as a control parameter, in the range $1.0 \le a \le 3.0$ and the initial conditions $x_1 = -0.5$, $x_2 = 0$, $x_3 = -0.4$, $x_4 = 0$, $x_5 = 0$, $x_6 = 0$.

Table 1. Dimensionless parameters of the system NIO-STTA

Variables	Symbol	Values
Interaction coefficients	η_1, η_2	0.05, 0.35
Damping coefficients	α_1, α_2	0.01, 0
Stiffness coefficient	γ	0.9
Mass coefficient	μ	100
Angle	φ	0.405
Constant	b	1.5

The Fig. 2 shows the passage of the angular velocity ϕ', during three regions as R1: Pre-resonant region in the range $1.0 \le a < 1.4$, with $\phi' < 1$, R2: resonance region in the range $1.4 \le a < 2.0$ with $\phi' \approx 1$, R3: post-resonance in the range $2.0 \le a \le 3.0$ with $\phi' > 1$. Furthermore, we observed the presence of the Sommerfeld effect: in the electric motor during passage through resonance: The angular velocity it is captured in R2, with large vibration (due the influence of the support response) generating large vibration amplitude of the support (Fig. 3a, black points). Eventually, enough power it was supplied to the D.C. motor to cause a jump, in the vicinity of a =2: the operating frequency increasing and the support amplitude decreasing (R3), resulting in lower power consumption by the DC motor.

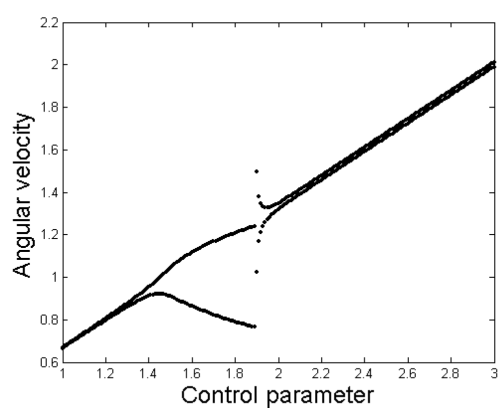

Figure 2. Sommerfeld effect- in D.C. motor

The Fig. 3 represents the resonance curve, in stationary motion corresponding the amplitudes of NIO and STTA versus control parameter, during the passage through resonance region in the range $1.0 \le a \le 3.0$, with variation of increment Δa =0.01 and over the dimensionless time range $0 \leq \tau \leq 3000$. Near of the resonance, note the presence of large oscillation and jump phenomenon in the motion of NIO without STTA (Fig. 3a, black points). When it is activated the STTA, in the range $1.0 \leq a \leq 2.0$, their amplitudes oscillate near of 2 (Fig. 3b), while the amplitudes of the support were drastically reduced to that oscillate near of zero (Fig. 3a, gray points) and jump phenomenon it is eliminated drastically (Sommerfeld effect it is eliminated). On the post-resonance region, range $1.0 \leq a \leq 2.0$, we observed an interaction predominant, between the NIO and the STTA showing large amplitudes in the oscillation. We conclude, that the parameters values γ and μ influence positively on the attenuation of vibration amplitudes of NIO due at the action of STTA during the motion in the resonance region.

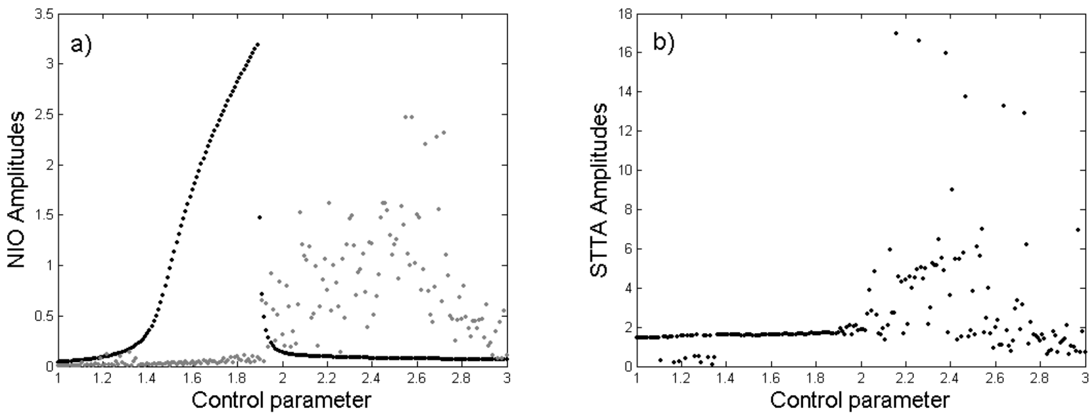

Figure 3 Resonance curve of the amplitudes: a) NIO without STTA (black points), with STTA (gray points); b) STTA (black points)

In the following result of the numerical simulation, we will show a technique of control the amplitudes of oscillation near of zero (Fig. 4), during the motion of pos-resonance, in this, it will be to control the parameters γ and μ . In the practice, it will be possible by modifying the spring and mass of STTA. In Fig. 4, we showed a technique to control the strong interaction between the D.C. Motor and the support during the motion through pos-resonance, in the range $2.01 \leq a \leq 3.0$. When it initiated the pos-resonance motion, it has two proceeding: decoupling the STTA of NIO or modify, immediately, the two physics elements, as are the mass and spring. In this case, it will be to consider the values γ=0.01 and μ=100, then resulting amplitudes of the support near of 0.1 (Fig. 4a).

Fig. 4(b) shows the amplitudes of STTA of small oscillation in comparison with amplitudes of oscillation, as is shown in Fig. 3(b). In following results of the numerical simulation, showed the time histories diagrams, the drastically reduction of the Sommerfeld effect, resonance capture of the angular velocity of the motor and the reduction of the support amplitude.

In Fig. 5 and 6 show the time histories in resonance and pos-resonance, respectively. Fig. 5(a) shows the resonance capture phenomenon of the angular velocity (fluctuating between two rotational frequencies), while the displacement has two oscillations, with large amplitude (Fig. 5b); note that both diagrams are in black line.

When it is activated the STTA whose time solution is shown in Fig. 5(c). We note that the angular velocity driving is in same frequency around of 1.2 (was eliminated the resonance capture), while the vibration amplitude of the foundation it is reduced drastically (Fig. 5a, b; in gray diagrams).

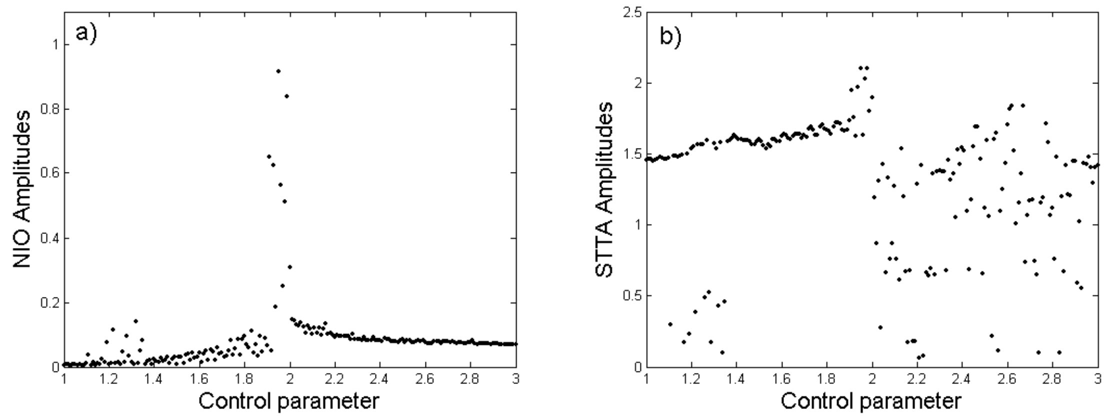

Figure 4 Resonance curve of the amplitudes for γ =0.9 and μ =100 on the range 1 ≤ a ≤ 2.0 and γ =0.01 and μ =100 on the range 2.01 ≤ a ≤ 3.0: a) NIO, b) STTA.

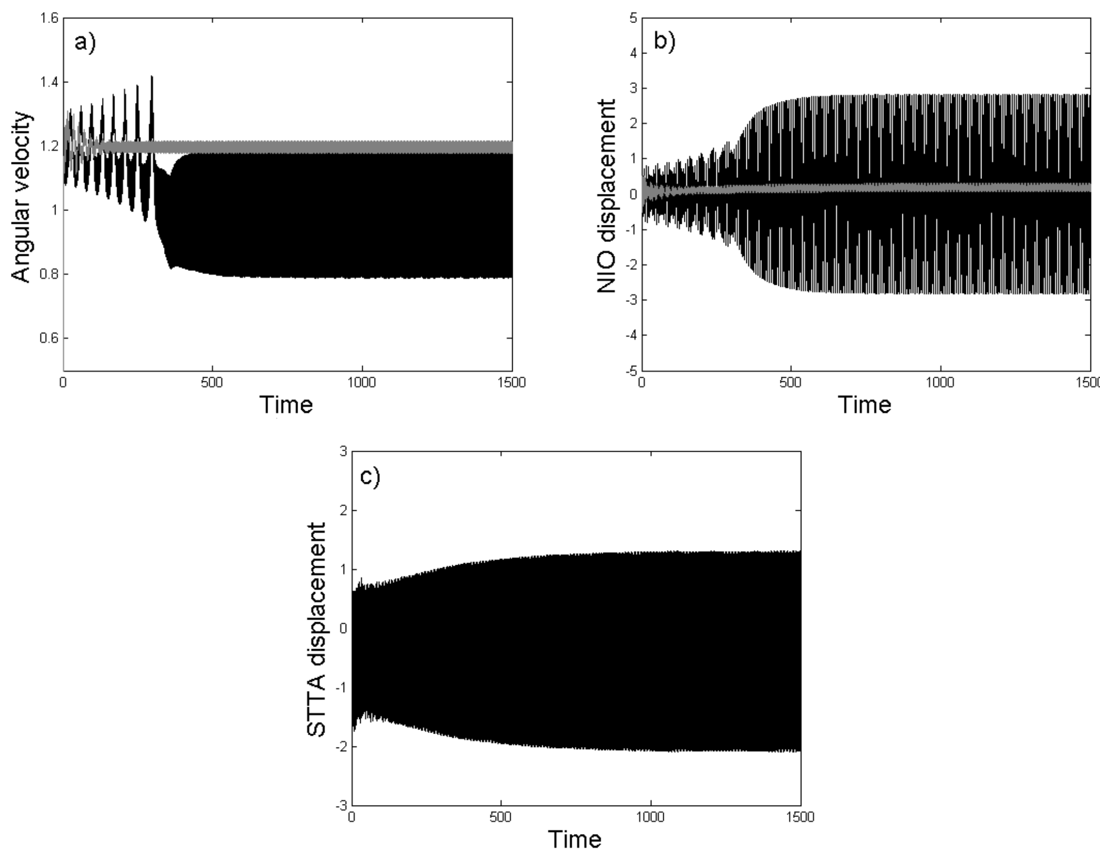

Figure 5 Time histories for a =1.8: a) angular velocity, b) NIO response, c) STTA response. STTA off (black diagram) and STTA on (gray black).

Figure 6 shows in the pos-resonance motion the time histories of the NIO and STTA where the NIO response without STTA is in black diagram and with STTA is in gray diagram. With action of the STTA (Fig. 6c) the angular velocity has predominant oscillation (Fig. 6a) while the oscillation of the support (Fig. 6b) is creasing and decreasing in different time interval. The comparison of the time responses of the NIO and STTA (Fig. 6b, c) shows the energy transfer with the possibility of the presence of internal resonance in the system.

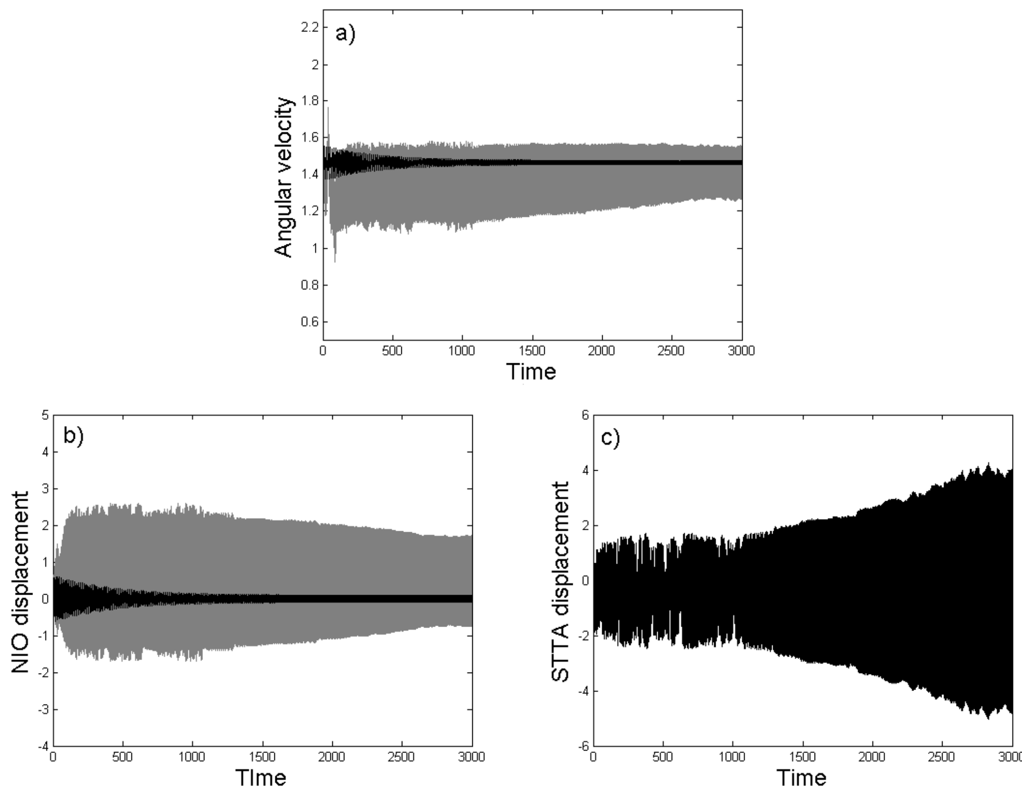

Figure 6 Time histories for a =2.2: a) angular velocity, b) NIO response, c) STTA response. STTA off (black diagram) and STTA on (gray diagram).

Figure 7, shows the snap-through motion of the STTA inside the resonance region corresponding to a =1.8. It describes two kinds of periodic motions (two left and right loop), which are separated by homoclinic trajectories (separatrix). The trajectory that is outside the separatrix, periodic orbit, corresponds to a snap-through motion (Pi, et al., 1971; Avramov and Mikhlin, 2004).

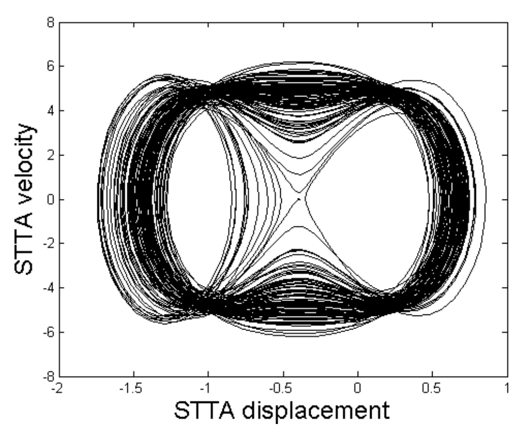

Figure 7. Phase plane (portrait) of snap-through motion

4. Conclusions

This paper has dealt with the numerical simulation of a non-ideal oscillator coupled to a snap-through truss absorber. When the STTA it is inactivated the Sommerfeld effect it is predominant. When was activated the STTA, drastically decreasing the vibration amplitudes of the support and it is drastically reduced the Sommerfeld effect, as the resonance capture of the angular velocity of the motor in the resonance region. We observer that is present the snap-through motion of the STTA during the transient motion. Two points of equilibrium depend of the angle that defines the stable equilibrium position of the STTA.

5. ACKNOWLEDGEMENTS

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