PROBABILISTIC MODEL FOR THE BIT-ROCK INTERACTION MODEL OF A DRILL-STRING SYSTEM

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Abstract. *In this paper a probabilistic model is proposed for the bit-rock interaction model of a drill-string system. A new strategy to take into account uncertainties in a local constitutive nonlinear equation using the nonparametric probabilistic approach is developed. The deterministic model considers the main forces that are applied to the column such as bit-rock interaction, fluid-structure interaction and impact forces. The nonlinear Timoshenko beam theory is applied and the system is discretized by means of the Finite Element Method.*

Keywords: *drill-string dynamics, nonlinear dynamics, bit-rock probabilistic model, local nonlinearity*

1. INTRODUCTION

We are interested in the analysis of flexible dynamical systems taken into account uncertainties [Sampaio and Ritto, 2008]. In the present work we analyze a drill-string system with uncertainties in the bit-rock interaction model.

The drill-string is composed by thin tubes called drill-pipes that together measure some kilometers and some thicker tubes called drill-collars that together have some hundred meters. The region composed by the thicker tubes is called Bottom-Hole-Assembly (BHA). Figure 1 shows the general scheme of the system analyzed. The forces taken into account are the motor torque (as a constant rotational speed at the top Ω_x), the supporting force f_{hook}, the torque t_{bit} and force f_{bit} at the bit, the weight of the column, the fluid forces, the impact and rubbing between the column and the borehole, the forces due to the stabilizer, and also the elastic and kinetic forces due to the deformation and to the motion of the structure.
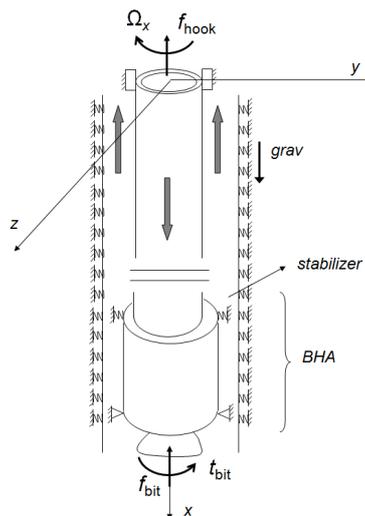
Figure 1. General scheme of the drill-string system.

There are some ways to model the nonlinear dynamics of a drill-string, *e.g.* [Tucker and Wang, 2003], [Khulief, 2007], [Christoforou and Yigit, 1997], [Sampaio et al., 2007], [Ritto et al., 2009a]. These models are able to quantify some effects that occur in a drilling operation (such as the stick-slip oscillations) but they cannot predict correctly the dynamical response of a real system. This is explained since, first, the above models are too simple compared to the real

system and, second, uncertainties are not taken into account. Each author uses a different approach to the problem: [Christoforou and Yigit, 1997] use a one-mode approximation to analyze the problem, [Khulief, 2007, Sampaio et al., 2007] use the Euler-Bernoulli beam model with the Finite Element Method, while [Tucker and Wang, 2003] use the Cosserat theory. We consider the model developed in [Ritto et al., 2009a], which used the Timoshenko beam model with the Finite Element Method.

In a drilling operation there are many sources of uncertainties such as material properties (column and drilling fluid), dimensions of the system (especially the borehole), fluid-structure interaction and bit-rock interaction. The uncertainty analysis of the present paper is focused on the bit-rock interaction because it seems to be one of the most important sources of uncertainties in this problem. There are few articles treating the stochastic problem of the drill-string dynamics, in especial we may cite [Spanos and Chevallier, 2000, Kotsonis and Spanos, 1997, Ritto et al., 2009b]. In [Spanos and Chevallier, 2000], stochastic lateral forces are analyzed at the bit, in [Kotsonis and Spanos, 1997], a random weight-on-bit is analyzed using a simple two degrees of freedom drill-string model and in [Ritto et al., 2009b], the weight-on-hook is taken as random.

The bit-rock interaction model used in the analysis is the one developed in [Tucker and Wang, 2003]. This model is able to reproduce the main phenomena and describes the penetration of the bit into the rock. Thus, it allows the analysis of the rate of penetration (ROP). Usually, the bit is assumed to be fixed [Khulief, 2007, Sampaio et al., 2007] or an average rate of penetration is assumed [Spanos et al., 1995, Christoforou and Yigit, 2003].

The nonparametric probabilistic approach [Soize, 2000, Soize, 2005] is used to model the uncertainties in the bit-rock interaction, which is represented by a nonlinear operator. It should be noticed that a new strategy is developed to take into account uncertainties for a local nonlinear operator.

The paper is organized as follows. In Section 2 the mean model is presented then, in Section 3, the probabilistic model of the bit-rock interaction model is developed. The results are shown in Section 4 and the concluding remarks are made in Section 5.

2. MEAN MODEL

In this Section the equations used to model the problem are presented. To derive the equations of motion, the extended Hamilton Principle is applied. Defining the potential Π by $\Pi = \int_{t_1}^{t_2}(U - T - W)dt$, where U is the potential strain energy, T is the kinetic energy and W is the work done by the nonconservative forces and any force not accounted in the potential energy. The first variation of Π must vanish:

$$\delta\Pi = \int_{t_1}^{t_2}(\delta U - \delta T - \delta W)dt = 0. \tag{1}$$

For short, the expressions of the kinetic energy, strain energy and impact forces are omitted, but they can be found in [Ritto et al., 2009a].

2.1 Finite element discretization

In the discretization by means of the Finite Element Method a two-node approximation with six degrees of freedom per node is chosen. The nodal displacement is written as

$$u_e = \mathbf{N}_u\mathbf{u}_e \quad , \quad v_e = \mathbf{N}_v\mathbf{u}_e \quad , \quad w_e = \mathbf{N}_w\mathbf{u}_e, \tag{2}$$

$$\theta_{xe} = \mathbf{N}_{\theta_x}\mathbf{u}_e \quad , \quad \theta_{ye} = \mathbf{N}_{\theta_y}\mathbf{u}_e \quad , \quad \theta_{ze} = \mathbf{N}_{\theta_z}\mathbf{u}_e, \tag{3}$$

where $\mathbf{N}$ is the shape function, u_e, v_e and w_e are the displacements in x, y and z directions, θ_{xe}, θ_{ye} and θ_{ze} are the rotations about the x, y and z-axis. Linear shape functions are used for the axial and torsional displacements and the shape functions for the lateral displacements are derived by calculating the static response of the beam [Bazoune and Khulief, 2002]. The element coordinate is $\xi = x/l_e$ and $\mathbf{u}_e = \begin{pmatrix} u_1 & v_1 & \theta_{z1} & w_1 & \theta_{y1} & \theta_{x1} & u_2 & v_2 & \theta_{z2} & w_2 & \theta_{y2} & \theta_{x2} \end{pmatrix}^T$, where $(\cdot)^T$ means transpose.

2.2 Fluid-structure interaction model

The drilling fluid (mud) is responsible for transporting the cuttings (drilled solids) from the bottom to the top to avoid clogging of the hole. There is no doubt that the drilling fluid influences the dynamics of a drill-string, but solving the complete problem would be extremely expensive computationally. There are some works that are strictly concerned with the drilling fluid flow, *e.g* [Escudier et al., 2000, Pina and Carvalho, 2006]. We use a linear fluid-structure coupling model similar to [Paidoussis et al., 2007]. In this simplified model there are the following hypotheses: (1) for the inside flow the fluid is taken as inviscid, for the outside as viscous, (2) the flow induced by the rotational speed about the x-axis is not

considered in this first analysis and (3) The pressure varies linearly with x. The element matrices are presented in Eq. (4). These equations are an extension and an adaptation of the model developed in [Paidoussis et al., 2007].

$$[M_\mathrm{f}]^{(e)} = \int_0^1 (M_\mathrm{f} + \chi\rho_\mathrm{f} A_o)\,(\mathbf{N}_w^T\mathbf{N}_w + \mathbf{N}_v^T\mathbf{N}_v) l_e d\xi\,,$$

$$[K_\mathrm{f}]^{(e)} = \int_0^1 \left(-M_\mathrm{f} U_i^2 - A_i p_i + A_o p_o - \chi\rho_\mathrm{f} A_o U_o^2\right)(\mathbf{N'}_w^T\mathbf{N'}_w + \mathbf{N'}_v^T\mathbf{N'}_v)\frac{1}{l_e} d\xi +$$

$$+ \int_0^1 \left(-A_i\frac{\partial p_i}{\partial x} + A_o\frac{\partial p_o}{\partial x}\right)(\mathbf{N}_{\theta_y}^T\mathbf{N}_{\theta_y} + \mathbf{N}_{\theta_z}^T\mathbf{N}_{\theta_z}) l_e d\xi\,,$$

$$[C_\mathrm{f}]^{(e)} = \int_0^1 (-2M_\mathrm{f} U_i + 2\chi\rho_\mathrm{f} A_o U_o)(\mathbf{N}_{\theta_y}^T\mathbf{N}_{\theta_y} + \mathbf{N}_{\theta_z}^T\mathbf{N}_{\theta_z}) l_e d\xi + \qquad (4)$$

$$+ \int_0^1 \left(\frac{1}{2}C_\mathrm{f}\rho_\mathrm{f} D_o U_o + k\right)(\mathbf{N}_w^T\mathbf{N}_w + \mathbf{N}_v^T\mathbf{N}_v) l_e d\xi\,,$$

$$\mathbf{f}_\mathrm{f}^{(e)} = \int_0^1 \left(M_\mathrm{f} g - A_i\frac{\partial p_i}{\partial x} - \frac{1}{2}C_\mathrm{f}\rho_\mathrm{f} D_o U_o^2\right)\mathbf{N}_u^T l_e d\xi\,.$$

in which, M_f is the fluid mass per unit length, ρ_f is the density of the fluid, $\chi = \dfrac{(D_\mathrm{ch}/D_o)^2 + 1}{(D_\mathrm{ch}/D_o)^2 - 1}$ (> 1), D_ch is the borehole (channel) diameter, D_i, D_o are the inside and outside diameters of the column, U_i, U_o are the inlet and outlet flow velocities, p_i, p_o are the pressures inside and outside the drill-string, A_i, A_o are the inside and outside cross sectional area of the column, C_f, k are the fluid viscous damping coefficients.

As pointed out before, it is assumed that the inner and the outer pressures (p_i and p_o) vary linearly with x

$$p_i = (\rho_\mathrm{f} g)\,x + p_\mathrm{cte}\,, \quad p_o = \left(\rho_\mathrm{f} g + \frac{F_{fo}}{A_o}\right)x\,, \quad F_{fo} = \frac{1}{2}C_\mathrm{f}\rho_\mathrm{f}\frac{D_o^2 U_o^2}{D_\mathrm{h}}\,, \qquad (5)$$

where p_cte is a constant pressure and F_{fo} is the frictional force due to the external.

In the above equation, D_h is the hydraulic diameter ($4A_\mathrm{ch}/S_\mathrm{tot}$) and S_tot is the total wetted area per unit length ($\pi D_\mathrm{ch} + \pi D_o$). Note that the reference pressure is $p_o|_{x=0} = 0$. Another assumption is that there is no head loss when the fluid passes from the drill-pipe to the drill-collar (and vice-versa). The head loss due to the change in velocity of the fluid at the bottom (it goes down and then up) is given by

$$h = \frac{1}{2g}(U_i - U_o)^2\,. \qquad (6)$$

If the geometry and the fluid characteristics are given, only the inlet flow at $x = 0$ can be controlled as the fluid speed is calculated using the continuity equation and the pressures are calculated using the Bernoulli equation.

Examining Eq. (4), it can be noticed that the fluid mass matrix is the usual added mass that, in our case, represents a significant value. For example, using representative values, the added mass is approximately 50% of the original mass, what changes the natural frequencies in about 20%.

The fluid stiffness matrix depends on the speed of the inside and outside flow as well as on the pressure and on the pressure derivatives. Analyzing the signs in the equation (Eq. 4) it can be noticed that the outside pressure tends to stabilize the system while the inside pressure and the flow tends to destabilize the system. The term $(-p_i A_i + p_o A_0)$ plays a major role on the stiffness of the system because, even though p_i is close to p_o, in the drill collar region (at the bottom) A_0 is around ten times A_i what turns the system stiffer at the bottom.

The fluid damping matrix depends on the flow velocity as well as on the viscous parameters of the fluid which have not well established values. A detailed analysis of the damping is not addressed in the present paper.

Finally, the fluid force vector $\mathbf{f}_\mathrm{f}$ is a constant axial force induced by the fluid and it is the only fluid force in the axial direction.

2.3 Bit-rock interaction model

The model used in this work is the one developed in [Tucker and Wang, 2003], which can be written as

$$f_{\text{bit}} = -\frac{\dot{u}_{\text{bit}}}{a_2 Z(\omega_{\text{bit}})^2} + \frac{a_3 \omega_{\text{bit}}}{a_2 Z(\omega_{\text{bit}})} - \frac{a_1}{a_2}\,,$$

$$t_{\text{bit}} = -\frac{\dot{u}_{\text{bit}} a_4 Z(\omega_{\text{bit}})^2}{\omega_{\text{bit}}} - a_5 Z(\omega_{\text{bit}})\,,$$

(7)

where f_{bit} is the axial force (also called weight-on-bit), t_{bit} is the torque about the x-axis, $Z(\omega_{\text{bit}})$ is the regularization function and $a_1, \ldots, a_5$ are positive constants that depend on the bit and rock characteristics as well as on the average weight-on-bit. Note that these forces couple the axial and torsional vibrations.

2.4 Final discretized system of equations

The dynamics is considered about the following prestressed configuration:

$$\mathbf{u}_{\text{S}} = [K]^{-1}(\mathbf{f}_{\text{g}} + \mathbf{f}_c + \mathbf{f}_{\text{f}})\,.$$

(8)

where $\mathbf{f}_{\text{g}}$ is the gravity, $\mathbf{f}_c$ is the reaction force at the bit and $\mathbf{f}_{\text{f}}$ is the fluid axial force. After assemblage, the final discretized system is written as

$$([M] + [M_{\text{f}}])\ddot{\overline{\mathbf{u}}} + ([C] + [C_{\text{f}}])\dot{\overline{\mathbf{u}}} + ([K] + [K_{\text{f}}] + [K_{\text{g}}(\mathbf{u}_{\text{S}})])\overline{\mathbf{u}} = \mathbf{f}_{\text{NL}}(t, \overline{\mathbf{u}}, \dot{\overline{\mathbf{u}}}, \ddot{\overline{\mathbf{u}}})\,,$$

(9)

in which $\overline{\mathbf{u}} = \mathbf{u} - \mathbf{u}_{\text{S}}$. The response $\overline{\mathbf{u}}$ is represented in a subspace $V_m \subset \mathbb{R}^m$, where m equals the number of degrees of freedom of the system. $[M]$, $[C]$ and $[K]$ are the usual mass, damping and stiffness matrices, $[M_{\text{f}}]$, $[C_{\text{f}}]$ and $[K_{\text{f}}]$ are the fluid mass, damping and stiffness matrices, $\mathbf{f}_{\text{f}}$ is the fluid force vector, $[K_{\text{g}}(\mathbf{u}_{\text{S}})]$ is the geometric stiffness matrix and $\mathbf{f}_{\text{NL}}(t, \overline{\mathbf{u}}, \dot{\overline{\mathbf{u}}}, \ddot{\overline{\mathbf{u}}})$ is the nonlinear force vector that is written as

$$\mathbf{f}_{\text{NL}}(t, \overline{\mathbf{u}}, \dot{\overline{\mathbf{u}}}, \ddot{\overline{\mathbf{u}}}) = \mathbf{f}_{\text{ke}}(\overline{\mathbf{u}}, \dot{\overline{\mathbf{u}}}, \ddot{\overline{\mathbf{u}}}) + \mathbf{f}_{\text{se}}(\overline{\mathbf{u}}) + \mathbf{f}_{\text{ip}}(\overline{\mathbf{u}}) + \mathbf{f}_{\text{br}}(\dot{\overline{\mathbf{u}}}) + \mathbf{g}(t)\,.$$

(10)

where $\mathbf{f}_{\text{ke}}$ is composed by the quadratic terms of the kinetic energy, $\mathbf{f}_{\text{se}}$ is composed by the quadratic and higher order terms of the strain energy, $\mathbf{f}_{\text{ip}}$ is the force vector due to the impact and rubbing between the column and the borehole, $\mathbf{f}_{\text{br}}$ is the force vector due to the bit-rock interactions (see Section 2.3) and $\mathbf{g}(t)$ is the force that corresponds to the Dirichlet boundary condition (rotational speed imposed at the top).

2.5 Reduced model

Usually the final discretized FE system have big matrices (dimension $m \times m$) and the dynamical analysis may be time consuming, which is the case of the present analysis. One way to reduce the system is to project the nonlinear dynamical equation on a subspace V_n, with $n << m$, in which V_n is spanned by an algebraic basis of $\mathbb{R}^n$. In the present paper, the basis used for the reduction corresponds to the normal modes projection. The normal modes are obtained from the following generalized eigenvalue problem

$$([K] + [K_{\text{f}}] + [K_{\text{g}}(\mathbf{u}_{\text{S}})])\boldsymbol{\phi} = \omega^2([M] + [M_{\text{f}}])\boldsymbol{\phi}\,,$$

(11)

where $\boldsymbol{\phi}_i$ is the i-th normal mode and ω_i is the i-th natural frequency. Using the representation $\overline{\mathbf{u}} = [\Phi]\,\mathbf{q}$, where $[\Phi]$ is a $(m \times n)$ real matrix composed by n normal modes, and projecting Eq. (9) on the subspace spanned by these normal modes yields

$$[M_r]\,\ddot{\mathbf{q}}(t) + [C_r]\,\dot{\mathbf{q}}(t) + [K_r]\,\mathbf{q}(t) = [\Phi]^T \mathbf{f}_{\text{NL}}(t, \overline{\mathbf{u}}, \dot{\overline{\mathbf{u}}}, \ddot{\overline{\mathbf{u}}})\,,$$

(12)

where

$$[M_r] = [\Phi]^T([M] + [M_{\text{f}}])[\Phi], \quad [C_r] = [\Phi]^T([C] + [C_{\text{f}}])[\Phi]\,,$$

$$[K_r] = [\Phi]^T([K] + [K_{\text{f}}] + [K_{\text{g}}(\mathbf{u}_{\text{S}})])[\Phi]$$

(13)

are the reduced matrices.

3. PROBABILISTIC MODEL FOR THE BIT-ROCK INTERACTION

The parametric probabilistic approach allows physical-parameter uncertainties to be modeled. It should be noted that the underlying deterministic model defined by Eq. (7) exhibits parameters a_1, a_2, a_3, a_4 and a_5 which do not correspond to physical parameters. Consequently, it is difficult to construct an *a priori* probabilistic model using the parametric probabilistic approach. For instance, there is no available information concerning the statistical dependence of these parameters. Then, we propose to apply the nonparametric probabilistic approach to model uncertainties [Soize, 2000] which consists in modeling the operator of the constitutive equation (Eq. (7)) by a random operator. Such an approach allows both system-parameters uncertainties and modeling errors to be globally taken into account.

The nonparametric probabilistic approach has been applied for linear operators [Soize, 2005]. Recently it was extended [Mignolet and Soize, 2008] but the type of problem studied in the present paper is completely different from the geometrically nonlinear dynamical system studied in [Mignolet and Soize, 2008]. We are dealing with a nonlinear operator (related to a local nonlinearity). Therefore, it requires a different methodology. Let $\mathfrak{f}_{\mathrm{bit}}(\dot{\mathbf{x}})$ and $\dot{\mathbf{x}}$ be such that

$$\mathfrak{f}_{\mathrm{bit}}(\dot{\mathbf{x}}) = \begin{pmatrix} f_{\mathrm{bit}}(\dot{\mathbf{x}}) \\ t_{\mathrm{bit}}(\dot{\mathbf{x}}) \end{pmatrix} \quad \text{and} \quad \dot{\mathbf{x}} = \begin{pmatrix} \dot{u}_{\mathrm{bit}} \\ \omega_{\mathrm{bit}} \end{pmatrix} . \tag{14}$$

In the fist step of the methodology proposed, we look for a symmetric positive-definite matrix $[A_b(\dot{\mathbf{x}})]$ depending on $\dot{\mathbf{x}}$ such that the virtual power of the bit-rock interactions be written as

$$\delta \mathcal{P}_{\mathrm{bit}}(\dot{\mathbf{x}}) = \; <\mathfrak{f}_{\mathrm{bit}}(\dot{\mathbf{x}}), \delta\dot{\mathbf{x}}> \; = \; -<[A_b(\dot{\mathbf{x}})]\dot{\mathbf{x}}, \delta\dot{\mathbf{x}}>, \tag{15}$$

and such that force $\mathfrak{f}_{\mathrm{bit}}(\dot{\mathbf{x}})$ be given by:

$$\mathfrak{f}_{\mathrm{bit}}(\dot{\mathbf{x}}) = \nabla_{\delta\dot{\mathbf{x}}} \, \delta\mathcal{P}_{\mathrm{bit}}(\dot{\mathbf{x}}) \,, \tag{16}$$

Equation (7) can be rewritten as

$$\mathfrak{f}_{\mathrm{bit}}(\dot{\mathbf{x}}) = -[A_b(\dot{\mathbf{x}})]\dot{\mathbf{x}} = - \begin{pmatrix} \dfrac{a_1}{a_2} + \dfrac{\dot{u}_{\mathrm{bit}}}{a_2 Z(\omega_{\mathrm{bit}})^2} - \dfrac{a_3 \omega_{\mathrm{bit}}}{a_2 Z(\omega_{\mathrm{bit}})} \\[3mm] \dfrac{a_4 Z(\omega_{\mathrm{bit}})^2 \dot{u}_{\mathrm{bit}}}{\omega_{\mathrm{bit}}} + a_5 Z(\omega_{\mathrm{bit}}) \end{pmatrix} . \tag{17}$$

From Eqs. (15) to (17) it can be deduced that

$$\begin{aligned} [A_b(\dot{\mathbf{x}})]_{11} &= \frac{a_1}{a_2 \dot{u}_{\mathrm{bit}}} + \frac{1}{a_2 Z(\omega_{\mathrm{bit}})^2} - \frac{a_3 \omega_{\mathrm{bit}}}{a_2 Z(\omega_{\mathrm{bit}}) \dot{u}_{\mathrm{bit}}} \,, \\[2mm] [A_b(\dot{\mathbf{x}})]_{22} &= \frac{a_4 Z(\omega_{\mathrm{bit}})^2 \dot{u}_{\mathrm{bit}}}{\omega_{\mathrm{bit}}^2} + \frac{a_5 Z(\omega_{\mathrm{bit}})}{\omega_{\mathrm{bit}}} \,, \\[2mm] [A_b(\dot{\mathbf{x}})]_{12} &= [A_b(\dot{\mathbf{x}})]_{21} = 0 \,. \end{aligned} \tag{18}$$

It can be seen that for all $\dot{\mathbf{x}}$ belonging to its admissible space $\mathcal{C}$, $[A_b(\dot{\mathbf{x}})]$ is positive-definite.

The second step consists, for all deterministic vector $\dot{\mathbf{x}}$ belonging to $\mathcal{C}$, in modeling matrix $[A_b(\dot{\mathbf{x}})]$ by a random matrix $[\mathbf{A}_b(\dot{\mathbf{x}})]$ with values in the set $\mathbb{M}_2^+(\mathbb{R})$ of all positive-definite symmetric (2×2) real matrices. Note that matrix $[\mathbf{A}_b(\dot{\mathbf{x}})]$ should be written as $[\mathbf{A}_b(\dot{\mathbf{x}}(t))]$ which shows that $\{[\mathbf{A}_b(\dot{\mathbf{x}}(t))], t > 0\}$ is a stochastic process with values in $\mathbb{M}_2^+(\mathbb{R})$. Thus, for all $\dot{\mathbf{x}}$ in $\mathcal{C}$, the constitutive equation defined by Eq. (17) becomes a random constitutive equation which can be written as

$$\mathfrak{F}_{\mathrm{bit}}(\dot{\mathbf{x}}) = -[\mathbf{A}_b(\dot{\mathbf{x}})]\dot{\mathbf{x}} \,. \tag{19}$$

The third step consists in constructing the probability distribution of random variable $[\mathbf{A}_b(\dot{\mathbf{x}})]$ for all fixed vector $\dot{\mathbf{x}}$ in $\mathcal{C}$. Following the methodology of the nonparametric probabilistic approach and using the Cholesky decomposition, the mean value of $[A_b(\dot{\mathbf{x}})]$ is written as

$$[A_b(\dot{\mathbf{x}})] = [L_b(\dot{\mathbf{x}})]^T [L_b(\dot{\mathbf{x}})] \,, \tag{20}$$

and random matrix $[\mathbf{A}_b(\dot{\mathbf{x}})]$ is defined by

$$[\mathbf{A}_b(\dot{\mathbf{x}})] = [L_b(\dot{\mathbf{x}})]^T [\mathbf{G}_b][L_b(\dot{\mathbf{x}})] . \tag{21}$$

In the above equation, $[\mathbf{G}_b]$ is a random matrix satisfying the following available information [Soize, 2005]: (1) random matrix $[\mathbf{G}_b]$ is positive-definite almost surely, (2) $E\{[\mathbf{G}_b]\} = [\,I\,]$ and (3) $E\{||[\mathbf{G}_b]^{-1}||_F^2\} = \mathbf{c}_2$, $|\mathbf{c}_2| < +\infty$. In which $E\{\cdot\}$ is the mathematical expectation, $||\cdot||_F$ denotes the Frobenius norm such that $||[A]||_F = (tr\{[A][A]^T\})^{1/2}$ and $[\,I\,]$ is the identity matrix. It should be noted that, in the construction proposed, random matrix $[\mathbf{G}_b]$ neither depends on $\dot{\mathbf{x}}$ nor on t. Let the dispersion parameter δ be such that

$$\delta = \left\{ \frac{1}{2} E\{||[\mathbf{G_b}] - [I]||_F^2\} \right\}^{\frac{1}{2}} . \tag{22}$$

Taking into account the above available information and using the Maximum Entropy Principle [Shannon, 1948, Jaynes, 1957a, Jaynes, 1957a] yield the following probability density function of $[\mathbf{G}_b]$ [Soize, 2005],

$$p_{[\mathbf{G}_b]}([G_b]) = \mathbb{1}_{\mathbb{M}_2^+(\mathbb{R})}([G_b]) C_{\mathbf{G}_b} \det([G_b])^{3\frac{(1-\delta^2)}{2\delta^2}} \exp\left\{ -\frac{3}{2\delta^2} tr([G_b]) \right\}, \tag{23}$$

where $\det(\cdot)$ is the determinant, $tr(\cdot)$ is the trace. The constant of normalization is written as

$$C_{\mathbf{G}_b} = \frac{\left(\frac{3}{2\delta^2}\right)^{3/(2\delta^2)}}{(2\pi)^{1/2}\Gamma\left(\frac{3}{2\delta^2}\right)\Gamma\left(\frac{3}{2\delta^2} - \frac{1}{2}\right)} , \tag{24}$$

where $\Gamma(z)$ is the gamma function defined for $z > 0$ by $\Gamma(z) = \int_0^{+\infty} t^{z-1}e^{-t}dt$. The random generator of independent realizations of random matrix $[\mathbf{G}_b]$ for which the probability density function defined is by Eq. (23) is given in [Soize, 2005].

4. NUMERICAL RESULTS

The drill-string is discretized with 56 finite elements. For the construction of the reduced dynamical model, 158 lateral modes, 4 torsional modes, 3 axial modes and also the two rigid body modes of the structure (axial and torsional) are used. As boundary conditions, the lateral displacements and the rotations about the y and z-axis are zero at the top. The lateral displacements at the bit are also zero. A constant rotational speed about the x-axis Ω_x is imposed at the top. Stabilizers are considered as elastic elements: $F_y|_{x=x_{\text{stab}}} = k_{\text{stab}}\, v|_{x=x_{\text{stab}}}$ and $F_z|_{x=x_{\text{stab}}} = k_{\text{stab}}\, w|_{x=x_{\text{stab}}}$, where x_{stab} is the stabilizer location and k_{stab} is the stabilizer stiffness. As initial conditions, all the points move with constant axial speed and constant rotational speed about the x-axis, and the column is deflected laterally.

For the time integration procedure, the implicit Newmark integration scheme has been implemented with a predictor and a fix point procedure to equilibrate the system response at each time step. The data used for computations are representative values that are found in the literature [Christoforou and Yigit, 2003, Tucker and Wang, 2003, Khulief, 2007, Sampaio et al., 2007] (see Appendix A). All the numerical results presented below correspond to the forced response (deterministic case) and to the stationary response (stochastic case) for which the transient part of the response induced by the initial conditions has vanished. The results presented are the time response and the frequency spectrum defined as the modulus of its Fourier transform. The dynamical system is excited by a constant rotational speed about the x-axis at the top and with value 0.83 Hz.

To better visualize the bit-rock interaction, Fig. 2 shows the torque and force at the bit versus the rotational speed of the bit. Fig. 2(a) shows that for a rotational speed of the bit about 4 rd/s we have the maximum absolute value of the torque, then, if the rotational speed of the bit is higher or lower than 4 rd/s the absolute value of the torque gets lower. Fig. 2(b) shows that the relation between the force at the bit and the rotational speed of the bit is not that simple, however, the variation of the force at the bit is very low. The curves of the torque and force at the bit versus the axial speed of the bit are similar to what is shown in Fig. 2.
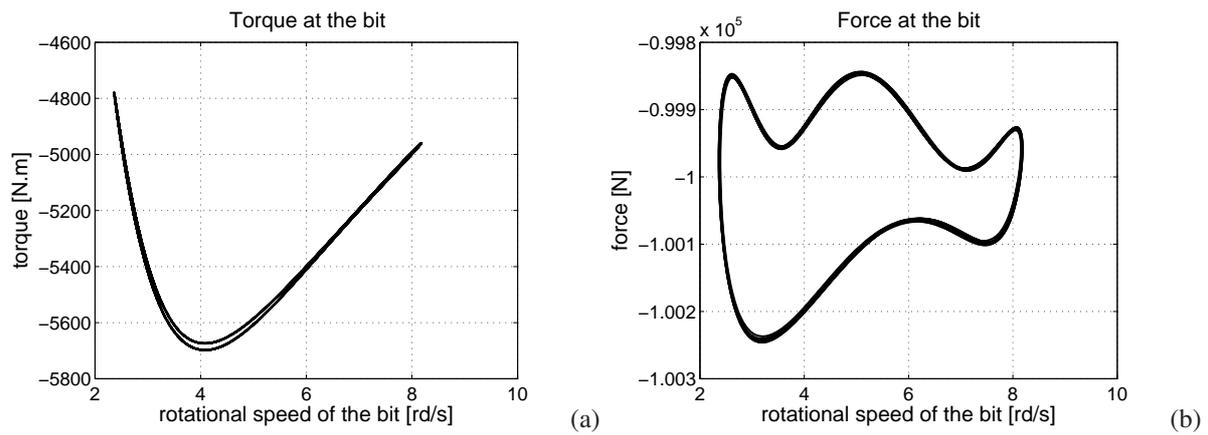
Figure 2. (a) Torque at the bit versus rotational speed of the bit and (b) Force at the bit versus rotational speed of the bit.

4.1 Stochastic response

The stochastic response is computed for two values of δ which are 0.001, 0.1. Figure 3 displays the random ROP for $\delta = 0.001$. This figure shows the response of the mean model together with the mean response of the stochastic model and the 95% envelope (which means that the confidence region is constructed with a probability level of 0.95). The upper and lower envelopes of the confidence region are calculated using the method of quantiles [Serfling, 1980].
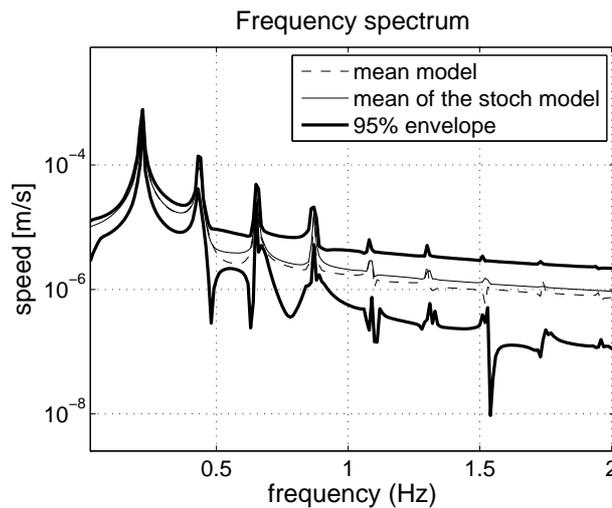
Figure 3. Stochastic response for $\delta = 0.001$. Frequency spectrum of the ROP.

Fig. 3 shows that the dispersion of the random ROP is already significant in the high part of the frequency band. However, the stochastic response in the low part of the frequency band is robust for the level of uncertainties considered. Figure 4 shows the random radial displacement at $x = 700$ m (middle point of the drill pipe). It can be seen that the lateral vibrations are also affected by the probabilistic model of the bit-rock interaction.
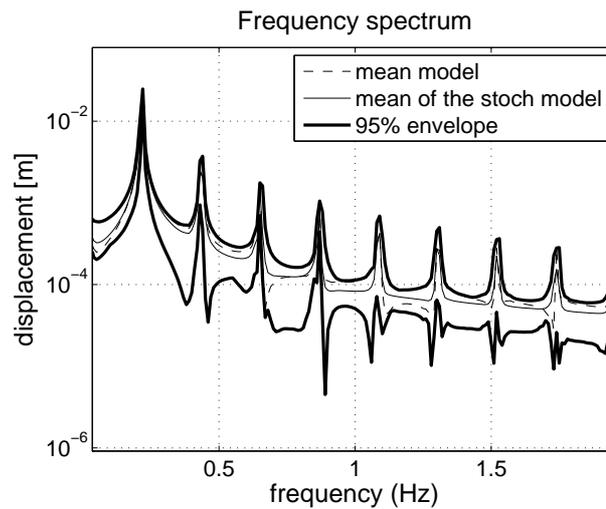
Figure 4. Stochastic response for $\delta = 0.001$. Frequency spectrum of the radial displacement at $x = 700$ m.

Figure 5 shows the random rotational speed of the bit for $\delta = 0.1$. It can be noted that, for this level of uncertainty, the dispersion of the stochastic response is significant for all the frequency band analyzed. Figure 6 shows some Monte Carlo realizations of the stochastic ROP. The arrows in Fig. 6 indicate that, for some realizations, the bit loses contact with the soil.
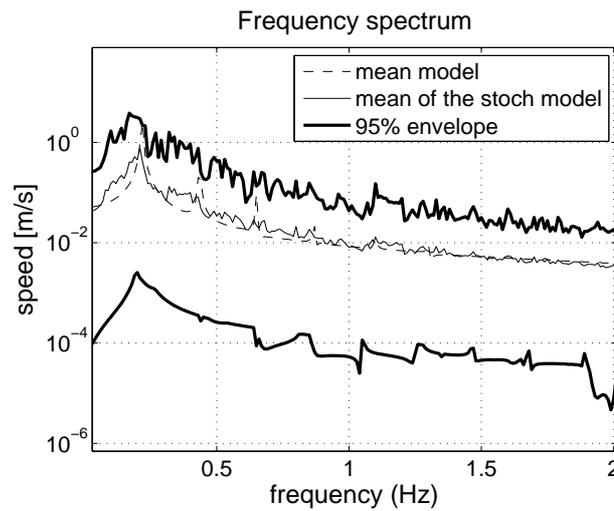
Figure 5. Stochastic response for $\delta = 0.1$. Frequency spectrum of the rotational speed of the bit ω_{bit}.
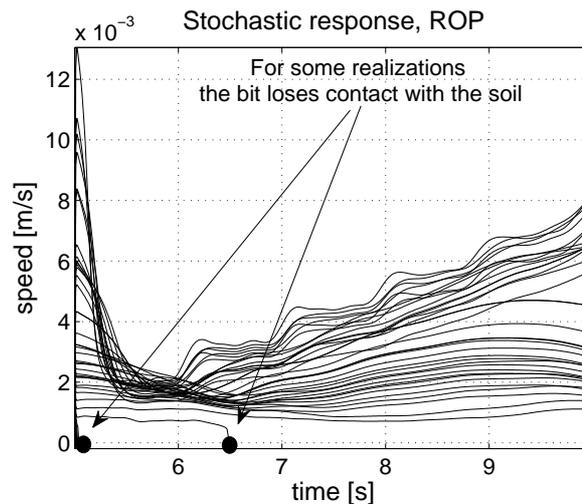
Figure 6. Random ROP for $\delta = 0.1$.

The probabilistic model proposed for the bit-rock interaction model allows us to simulate cases such as the bit losing contact with the soil and the column impacting the borehole. The nonparametric probabilistic approach permits both parameters and modeling errors to be taken into account for the bit-rock interaction model.

5. CONCLUDING REMARKS

A computational nonlinear dynamical model taking into account uncertainties has been developed to simulate the drill-string dynamics and it has been shown to be well suited to describe the problem. A probabilistic model has been proposed for the bit-rock interaction model. Since the parameters of the bit-rock interaction do not correspond to physical parameters, these parameters are not adequate to the use of the parametric probabilistic approach. Then, the nonparametric probabilistic approach has been applied. This corresponds to a completely novel approach to take into account model uncertainties in a nonlinear constitutive equation. Since the dynamical system is globally nonlinear, an adapted strategy has been developed to implement the stochastic simulation.

The Timoshenko beam model has been used and the main forces that affect the dynamics of the drill-string have been considered such as the bit forces, the fluid forces and impacts. Finite strains have been taken into account without neglecting the higher order terms and the vibration has been computed about a prestressed configuration.

6. ACKNOWLEDGEMENTS

A DATA USED IN THE SIMULATIONS

$\Omega_x = 0.83$ Hz (rotational speed about the x-axis at the top), $L_{\mathrm{dp}} = 1400$ m (length of the drill pipe), $L_{\mathrm{dc}} = 200$ m (length of the drill collar), $D_{o\mathrm{dp}} = 0.127$ m (outside diameter of the drill pipe), $D_{o\mathrm{dc}} = 0.2286$ m (outside diameter of the drill collar), $D_{i\mathrm{dp}} = 0.095$ m (inside diameter of the drill pipe), $D_{i\mathrm{dc}} = 0.0762$ m (inside diameter of the drill collar), $D_{\mathrm{ch}} = 0.3$ m (diameter of the borehole (channel)), $x_{\mathrm{stab}} = 1400$ m (location of the stabilizer), $k_{\mathrm{stab}} = 17.5$ MN/m (stiffness of the stabilizer per meter), $E = 210$ GPa (elasticity modulus of the drill string material), $\rho = 7850$ kg/m^3 (density of the drill string material), $\nu = 0.29$ (poisson coefficient of the drill string material), $k_s = 6/7$ (shearing correcting factor), $k_{\mathrm{ip}} = 1 \times 10^8$ N/m (stiffness per meter used for the impacts), $\mu_{\mathrm{ip}} = 0.0005$ (frictional coefficient between the string and the borehole), $u_{in} = 1.5$ m/s (flow speed in the inlet), $\rho_{\mathrm{f}} = 1200$ kg/m^3 (density of the fluid), $C_{\mathrm{f}} = 0.0125$ (fluid viscous damping coefficient), $k = 0$ (fluid viscous damping coefficient), $g = 9.81$ m/s^2 (gravity acceleration), $a_1 = 3.429 \times 10^{-3}$ m/s (constant of the bit-rock interaction model), $a_2 = 5.672 \times 10^{-8}$ m/(N.s) (constant of the bit-rock interaction model), $a_3 = 1.374 \times 10^{-4}$ m/rd (constant of the bit-rock interaction model), $a_4 = 9.537 \times 10^6$ N.rd (constant of the bit-rock interaction model), $a_5 = 1.475 \times 10^3$ N.m (constant of the bit-rock interaction model), $e = 2$ rd/s (regularization parameter).

The damping matrix is constructed using the relationship $[C] = \alpha([M] + [M_{\mathrm{f}}]) + \beta([K] + [K_{\mathrm{f}}] + [K_{\mathrm{g}}(\mathbf{u_S})])$ with $\alpha = 0.01$ and $\beta = 0.0003$.

B   ACKNOWLEDGEMENTS

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