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THEORETICAL AND EXPERIMENTAL NONLINEAR DYNAMICS OF ELASTIC SAGGED CABLES

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Abstract. *The paper presents a comprehensive overview of recent advancements in theoretical and experimental research on modeling, analysis, and nonlinear/nonregular phenomena in the resonant dynamics of sagged, horizontal or inclined, elastic cables. Hints for proper reduced order modeling in cable dynamics are obtained from asymptotic solutions or experimental investigations, and challenging issues arising in the characterization of involved bifurcation scenarios to complex dynamics are specifically addressed.*

Keywords*: Suspended cable, Nonlinear vibrations, Asymptotic solution, Experimental analysis, Bifurcation and chaos, Reduced order models.*

1. INTRODUCTION

Suspended cables are lightweight, flexible structural elements used in several applications in mechanical, civil, electrical, ocean and space engineering, due to their capability of transmitting forces, carrying payloads and conducting signals across large distances. At the same time, the suspended cable is a basic element of theoretical interest in applied mechanics, and an archetypal model of various phenomena in nonlinear dynamics for being prone to large amplitude vibrations.

Upon classical analyses of linear vibrations (Irvine, 1981; Triantafyllou and Grinfogel, 1986), further complemented by recent achievements on experimental validation of theoretical phenomena (Russell and Lardner, 1998) and unified treatment of shallow and non-shallow cables (Lacarbonara *et al*., 2007), finite amplitude cable vibrations have been a subject of intensive research in the last two decades, as documented by a few review articles concerned with deterministic (Rega, 2004) and stochastic (Ibrahim, 2004) regimes. Focusing on the former, nonlinear vibrations under various conditions of planar and nonplanar, internal, external and/or parametric, resonances are addressed by means of variably refined theoretical models, through purely analytical, numerical or mixed treatments. Owing to the inherent combination of system quadratic and cubic nonlinearities, a rich variety of nonlinear dynamic features of cable response has been unveiled by previous research in the field.

However, description and understanding of the overall system nonlinear dynamics still suffers from a number of limitations, which are herein schematically summarized. From the modeling and analysis viewpoint, they are concerned with the solely consideration (i) of approximate continuous cable models, (ii) of quite low-dimensional finite representations and analysis of such models, and (iii) of only shallow horizontal or nearly taut inclined cables, with the ensuing incomplete and/or unsatisfactory description of actual cable dynamics. From the viewpoint of system response, (iv) just partial description of the many involved (solely in-plane or in-plane/out-of-plane) interaction phenomena possibly occurring in various internal/external/parametric resonance conditions has been obtained, along with (v) a limited knowledge of transition scenarios to nonregular dynamics. Moreover, still (vi) an incomplete cross-validation of analytical and numerical solutions, and (vii) a lack of experimental results are to be noticed. Based on previous achievements in the field, the theoretical and experimental research in the last few years is being aimed at overcoming some of the above mentioned issues.

Along these lines, the present paper provides an overview of the most important topics, concerned with modeling, analysis and system phenomenology, that are being tackled at both theoretical and experimental level in cable nonlinear dynamics. The paper is organized as follows. Continuous modeling is discussed in Sect. 2, pointing out the main relevant approximations. Multimode discretization aimed at obtaining reduced-order models to be tackled via mainly analytical techniques and to be reliably referred to for highlighting some main features of resonant nonlinear dynamic response of the underlying infinite-dimensional system are addressed in Sect. 3 and 4, respectively. Then, challenging issues concerned with experimental characterization of cable nonlinear dynamics are discussed (Sect. 5), paying special attention to the description and understanding of involved bifurcation scenarios and of the ensuing complex dynamics (Sect. 6). Further expected research developments are mentioned at the end of the paper.

2. CONTINUOUS MODELING

The continuous model mostly widely used for analyzing the large amplitude, forced, damped, three-dimensional vibrations of a suspended cable refers to a perfectly flexible, homogeneous, linearly elastic, system, with negligible torsional, bending and shear rigidities, is based on the quasi-static assumption (also called static condensation) of the axial stretching, and considers a horizontally hanging shallow cable with two supports at the same level (Rega, 2004). Any of the previous assumptions is somehow relaxed in more recent continuous formulations.

Still keeping the elastic material assumption and accounting for the solely axial rigidity, a more general model of suspended cable is based on a refined kinematical description of the cable element deformation and considers an arbitrarily sagged and inclined system (Srinil *et al.*, 2003, 2004). The corresponding *exact* partial differential equations (PDEs) of 3D cable motion read:

$$
\begin{aligned}
&\left(\frac{EA+EA(1+e_o)u'}{\sqrt{1+y_o'^2}}-\frac{EA(1+u')}{\sqrt{(1+u')^2+(y_o'+v')^2+w'^2}}\right)'=\left(\frac{w_C}{g}\ddot{u}+c_u\dot{u}\right)\frac{\sqrt{1+y_o'^2}}{1+e_o}-F_u\sqrt{1+y_o'^2},\\
&\left(\frac{EAy_o'+EA(1+e_o)v'}{\sqrt{1+y_o'^2}}-\frac{EA(y_o'+v')}{\sqrt{(1+u')^2+(y_o'+v')^2+w'^2}}\right)'=\left(\frac{w_C}{g}\ddot{v}+c_v\dot{v}\right)\frac{\sqrt{1+y_o'^2}}{1+e_o}-F_v\sqrt{1+y_o'^2}\\
&\left(\frac{EA(1+e_o)w'}{\sqrt{1+y_o'^2}}-\frac{EAw'}{\sqrt{(1+u')^2+(y_o'+v')^2+w'^2}}\right)'=\left(\frac{w_C}{g}\ddot{w}+c_w\dot{w}\right)\frac{\sqrt{1+y_o'^2}}{1+e_o}-F_w\sqrt{1+y_o'^2}.
\end{aligned}
\tag{1}
$$

where $u(x, t)$, $v(x, t)$, $w(x, t)$ are the dynamic horizontal, vertical and out-of-plane displacement components in a Cartesian coordinate frame measured from the static equilibrium configuration $y_o(x)$ attained by the cable under its own gravity g, e_o is the initial static strain, E, A and w_C are the cable Young's Modulus, cross-sectional area and self-weight per unit unstretched length, respectively, c_u, c_v, c_w are viscous damping coefficients, and F_u, F_v, F_w are uniformly distributed harmonic external forcing. A prime (dot) represents partial differentiation with respect to the horizontal space coordinate x (time t).

Based on the assumptions of small static strain and moderately large vibration amplitudes associated with small dynamic strain, *approximate*, third-order, nonlinear PDEs of motion are derived (Srinil *et al.*, 2007). They exhibit both quadratic and cubic nonlinear terms even in the absence of initial sag (i.e, in the taut string case), and account for spatially non-uniform extensional dynamic strain, with the ensuing spatio-temporal variation of the nonlinear dynamic tension. Along with the exact model, the approximate model nonlinearly couples the longitudinal and transverse (in-plane or out-of-plane) cable dynamics, thus being referred to as a kinematically *non-condensed* model to distinguish it from the above mentioned *condensed* model, governed by integro-PDEs accounting for the solely transverse dynamics, typically considered in the cable literature. Besides some further, minor, kinematical assumptions, the latter model basically results from neglecting the longitudinal inertia and viscous damping effects in the PDEs of motion, which corresponds to assuming that the cable nonlinearly stretches in a quasi-static manner in the absence of longitudinal external loading, and entails a space-independent dynamic tension.

As regards cable sag and inclination, a closed-form solution of the inclined cable static equilibrium approximate up to the cubic order has to be used in the relevant equations of motion, based on a relatively small sag assumption, in order to account for the system *asymmetry* in also the nonlinear range (Rega and Srinil, 2007). It allows to describe the frequency *avoidance* (or veering) phenomenon and the associated hybrid modes (Triantafyllou and Grinfogel, 1986), which actually distinguish the linear dynamics of sagged inclined cables with respect to the frequency *crossover* and the associated symmetric/antisymmetric modes of sagged horizontal cables. Again, this is in contrast with the parabolic, *symmetric*, equilibrium profile usually considered in the nonlinear dynamics literature of nearly taut inclined cables, which exhibit the same linear dynamic behavior as that of the corresponding small-sag horizontal cables.

3. MULTIMODE DISCRETIZATION AND ANALYSIS

The *exact* cable model described by Eqs. (1) is solely referred to in *purely numerical* treatments of a given nonlinear dynamic problem based on, e.g., the space-time finite difference method coupled with a predictor-corrector iteration algorithm (Srinil *et al*., 2003, 2004). The major advantage of such a treatment stands in allowing to capture the spatial richness of cable response and its time-varying content, and in obtaining information about the possibly significant involvement of higher-order modes which ensues from the considered multidegree-of-freedom model. In contrast, if the interest is in highlighting the characterizing features of system nonlinear dynamics in different resonance conditions, any of the solely handable *approximate* models, whose analysis can be pursued via analytical or *mixed analytical-numerical* approaches, are referred to. The ensuing static and nonplanar/planar linear and nonlinear dynamic results can be thoroughly validated against those of the exact model via numerical techniques (Runge-Kutta/shooting, finite elements, finite differences, see Srinil *et al*. (2007)), thus allowing for a proper approximate continuous model selection in different technical situations.

In a reduced-order model and analytical solution perspective, a *multimode discretization* of the PDEs of the approximate non-condensed model is made based on a Galerkin projection accounting for the full-basis eigenspectrum of cable linear modes, and an asymptotic analysis of the ensuing infinite set of nonlinearly coupled ODEs is developed through any perturbation technique (typically, the multiple time scales method) (Srinil *et al.*, 2007). Provided enough modes are retained in the discretization, the relevant outcomes are substantially equivalent to those furnished by the direct application of the asymptotic method to the original, approximate, PDEs, with no a priori assumptions of the displacement solution form (Lacarbonara *et al.*, 2002).

Depending on the generalized Irvine elasto-geometric parameter governing the dynamic behavior of horizontal/inclined cables (Irvine, 1981; Triantafyllou and Grinfogel, 1986), a very rich pattern of *internal resonance* conditions (1:1, 2:1, and/or 3:1) involving different modes occurs at both crossover (avoidance) frequencies of horizontal (inclined) cables and far away from them.

Multiple scale analyses (Nayfeh, 1981, 2000) of the various resonant dynamics are accomplished up to the second-order, with the aim of capturing the combined effects due to higher-order quadratic and cubic nonlinearities. Indeed, in the 1:1 and 3:1 resonances – which are associated with cubic nonlinearities – the secular effects directly appear at the ε^3 order (with ε being a small parameter), whereas in the 2:1 resonance – associated with quadratic nonlinearities – they are split between the ε^2 order (due to quadratic nonlinearities only) and the ε^3 order (due to higher-order quadratic as well as cubic nonlinearities). In particular, the second-order effects of quadratic nonlinearities play a meaningful role in the involvement of non-negligible (both low- and higher-order) *non-resonant* modes in cable response (Srinil and Rega, 2007a, 2007b; Rega and Srinil, 2007; Lacarbonara *et al.*, 2007b). Detailed analyses and comparisons of the contributions of various – resonant and non-resonant – modes to system dynamics, as determined by the governing nonlinearities in the asymptotic solutions, allow to make convergence analyses of the response of various cables in terms of resonantly coupled amplitudes and frequencies, and to get hints about the proper selection of variable *reduced order models*.

3.1. Asymptotic solutions at various resonance conditions

For each considered internal resonance condition, the multiple scale analysis provides the amplitude and phase modulation equations (APMEs) of the involved resonant modes, whose fixed points correspond to the periodic motions of the original ODEs. Analysis of the interaction coefficients in the multiple scale solution of *free* nonlinear vibrations highlights possible restrictions as to the nature of the involved cable modes (e.g., symmetric and/or antisymmetric) in order for the internal resonance to be actually activated (Lacarbonara and Rega, 2002). In this respect, remarkable qualitative differences occur between horizontal and inclined cables, based on the associated existence or non-existence, respectively, of non-linear orthogonality properties of the relevant normal modes (Rega and Srinil, 2007). The multiple scale analysis in various internal resonance conditions also furnishes the coupled cable dynamic configurations associated with the *nonlinear normal modes* of the system, which meaningfully account for the spatial corrections, with respect to the reference linearly resonant modes, due to the quadratic nonlinearity effects of all infinite modes or of the considered non-resonant modes in a finite discretization (Lacarbonara *et al.*, 2002; Srinil and Rega, 2007a). This is also of major importance as regards the evaluation of cable nonlinear dynamic tension and of its actual space-time modifications. In turn, the multiple scale analytical predictions are validated against the numerical outcomes of finite elements (Gattulli *et al.*, 2004) or finite difference (Srinil and Rega, 2007a, 2008) solutions of the original, exact or approximate PDEs of motion, under specified initial conditions.

In the case of *forced* planar vibrations under uniformly-distributed vertical harmonic excitation at primary resonance with some internally resonant mode, analysis of the APMEs with the associated nonlinear interaction coefficients, as provided by the multiple scale analyses, allows to draw a general description of various possible resonant solutions occurring for horizontal (symmetric) and inclined (asymmetric) cables (Rega and Srinil, 2007). Depending on (i) the generalized elasto-geometric parameter λ/π, (ii) the kind of activated internal resonance, and (iii) the primary resonance of either a high (s) or low frequency (r) mode, a summary of the existence of *uncoupled* (UC) and/or *coupled* (C) mode planar solutions for horizontal (inclined) cables at crossover (avoidance), or away from it, is reported in Table 1.

Table 1. A summary of regular planar solutions in horizontal/inclined cables at various internal resonances.

$s{:}r$	λ/π	n	horizontal (sym.) cables		inclined (asym.) cables	
			$\Omega = \omega_s+\varepsilon^n\sigma_f$	$\Omega = \omega_r+\varepsilon^n\sigma_f$	$\Omega = \omega_s+\varepsilon^n\sigma_f$	$\Omega = \omega_r+\varepsilon^n\sigma_f$
1:1	CR vs. AV	2	UC/C	UC/C	C	C
2:1	CR vs. AV Non-CR vs. Non-AV	1	UC/C	C*	UC/C	C
3:1	Non-CR vs. Non-AV	2	UC/C	C	UC/C	C

*At CR, only with non-vanishing excitation on the low-frequency mode

In whatever internal resonance condition, UC solutions only involve the directly excited resonant mode whereas C solutions also involve the non excited mode which is indirectly driven into the response via an internal resonance enhanced mechanism of energy transfer. Both UC and C solutions coexist for 1:1 resonant crossover cables, whereas only C solutions exist for avoidance cables, regardless of the mode being directly excited. In turn, as regards 2:1 and 3:1 resonances, UC and C (only C) solutions exist when directly exciting a high-frequency (low-frequency) mode, irrespective of the considered horizontal (inclined) cable being at crossover (avoidance) or away from it. This implies that 2:1 and 3:1 (1:1) resonant solutions do (do not) depend on the mode being directly excited, while they do not (do) depend on the cable geometry being symmetric or asymmetric. It is thus evident how moving from horizontal to inclined cables – with the associated modification from purely symmetric/antisymmetric to hybrid linear modes – entails a substantially different scenario of nonlinear response in the 1:1 internal resonance condition.

4. MAJOR ISSUES IN THE RESONANT VIBRATIONS OF HORIZONTAL/INCLINED CABLES

Based on non-condensed/condensed models, multimode discretization and second-order multiple scale analyses, the non-linear interactions occurring in free or resonantly forced planar vibrations of various horizontal/inclined cables under different internal resonance conditions are investigated. *Steady-state* and *periodic* responses of the APMEs, as well as their *stability* and *bifurcation* features with a varying control parameter, are evaluated through continuation of frequency- and force-response diagrams. Direct numerical simulations of the APMEs provided by the multiple scale analyses are also utilized to validate continuation results and to characterize the post-bifurcation dynamics in terms of possibly *nonregular* responses.

Within the large amount of diverse dynamical aspects and the many criteria to be possibly adopted for discussing and comparing them, some major issues are herein schematically addressed by distinguishing among effects of continuous modeling approximations, features of internally resonant dynamics of horizontal or inclined cables with the ensuing differences in terms of proper reduced-order modeling, and nonlinear/complex phenomena characterizing the dynamics of various response classes.

4.1. Non-condensed versus condensed modeling

Depending on internal resonance condition and system elasto-geometric and control parameters, even for shallow cables the condensed model may lead to significant quantitative and qualitative discrepancies in the non-linear dynamic responses, bifurcation properties, as well as non-linear tensile or compressive stresses, with respect to the non-condensed model (Srinil and Rega, 2007b). Actual errors are seen to be significant in coupled – particularly higher-amplitude – responses, with respect to the uncoupled ones. Moreover, the kinematic condensation assumption shows to be definitely questionable when considering a larger-sagged or higher-extensible (Srinil and Rega, 2008a) resonant cable.

In particular, *longitudinal dynamics* and *space-dependent dynamic tension* are accounted for in only the non-condensed model, and play an increasingly meaningful role as the cable sag, inclination and/or extensibility increase. Overall, all of these parameters have significant effects on cable nonlinear dynamics. Yet, appreciable time-varying differences between maximum and minimum total tensions may occur in even shallow horizontal cables.

4.2. Internal resonances and contributing modes

Conditions for activation of internal resonances and nonlinear modal contributions to the ensuing dynamics may vary considerably for diverse resonant horizontal or inclined cables.

For horizontal cables, 2:1 internal resonance is activated only when the involved high-frequency mode is symmetric, whereas for inclined cables, owing to the asymmetry effects of inclined configurations which entail modal hybridity, it is activated nearly always – depending on frequency-tuning and hybridity capacity – and occurs over a wide range of system parameters. In turn, the modification from purely symmetric/antisymmetric normal modes at crossover to hybrid normal modes at avoidance significantly makes the planar 1:1 resonant forced nonlinear dynamics of inclined cables different from that of horizontal cables.

A major issue is concerned with the variable contribution of *resonant* and *non-resonant modes* to the overall response. Analysis of second-order quadratic modal contributions in 2:1 resonance shows that, besides the two resonant modes, only symmetric non-resonant modes affect the solution of (non-crossover) horizontal cables, whereas all non-resonant modes – irrespective of their order or spatial character – do contribute for inclined cables. Some non-resonant modes may play a role even greater than the resonant ones. At avoidances, due to the system high modal density and strong coupling, the non-modeled hybrid mode – out of the two modes therein coexisting – may contribute to the response even greater than the directly-modeled hybrid mode. This highlights the necessity to account for both of them and the possible involvement of a larger number of coupled modes in avoidance cables than in crossover cables. Along the same line, in the 1:1 resonance at avoidance (crossover), the asymmetry (symmetry) features of inclined (horizontal) cables make the contribution from non-resonant modes greater (lower): for cables at first ($\lambda/\pi \cong 2$) or second ($\lambda/\pi \cong 4$)

avoidance with different inclinations (θ) see, e.g., (Fig. 1) the percent contributions of each resonant (encircled) and non-resonant (lower- and/or higher-order) modes to the second-order quadratic terms embedded in various nonlinear interaction coefficients K of the asymptotic solution (Rega and Srinil, 2007).

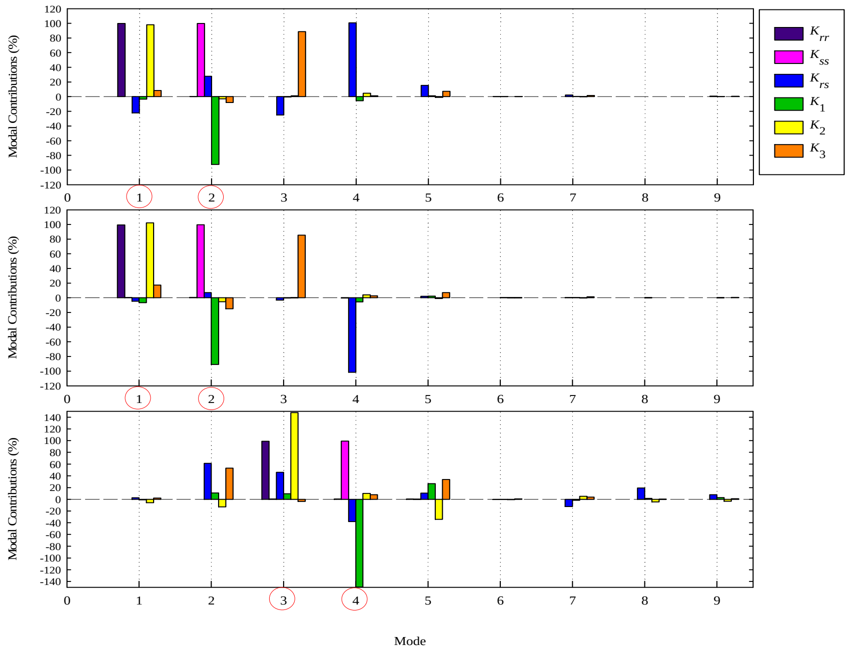


Figure 1. Mode contributions to various nonlinear interaction coefficients: θ=30 (upper, lower), 45 (middle) deg; $\lambda/\pi \cong 2$ (upper, middle), 4 (lower).

Considerable effects occur as regards the proper *reduced-order models* to be referred to in the discretization for capturing the main response features of the actual underlying infinite-dimensional system. Generally speaking, there is a clear evidence about the significance of accounting for both resonant and non-resonant (usually higher-order) modes in all resonance cases, although the actual mode selection depends on system parameters and coupled amplitudes. In the 2:1 resonance, the minimal (two-degree-of-freedom) model involving only the resonant modes shows capable of providing reliable results only for very low-sagged cables. In turn, for relatively low-sagged cables, it may be sufficient to account for non-resonant modes in evaluating the non-linear amplitudes and frequencies only, thereby developing an improved first-order multiple scale solution (Srinil and Rega, 2007a). In contrast, they have to be accounted for in also the nonlinear dynamic displacements (the full second-order multiple scale solution) – where they provide possibly meaningful spatial corrections – as the cable sag and/or inclination become significant.

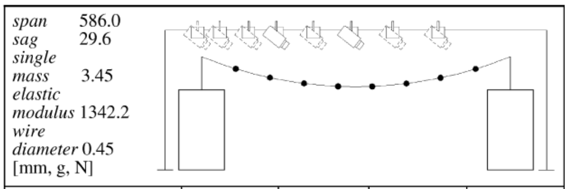
4.3. Modulated, non-regular, and multi-harmonic responses

Overall, depending on the considered resonance cases, system control parameters, and initial conditions, a multiplicity of (stable/unstable) equilibrium and periodic solutions may occur, along with meaningful transitions from periodic to quasi-periodic and chaotic responses. For 1:1 internally resonant inclined cables at avoidance, isolated coupled-mode solution branches coexist with frequency islands, both experiencing saddle-node and Hopf bifurcations. Limit cycles may undergo cyclic-fold as well as direct/reverse period-doubling bifurcations. A whole cornucopia of non-linear phenomena are observed: sequences of period-doubling bifurcations to chaos, funnel-shaped chaos, on-off intermittency mechanisms, sudden switching of solutions via boundary crises, with also competing effects of dynamic solutions. Multi-harmonic response features occur mainly due to contributions from higher-order non-resonant modes. The dynamic deflections occurring in chaotically resonant vibrations exhibit non-periodic multi-mode features, with time-varying amplitudes which may become significantly large. As a result, the chaotic dynamics are endowed with remarkable asymmetric features of spatially non-uniform, strongly time-varying, tensile/compressive dynamic tensions (Rega and Srinil, 2007).

5. EXPERIMENTAL CHARACTERIZATION OF CABLE DYNAMICS

Of course, besides theoretical and numerical investigations, there is a need to describe the actual nonlinear behavior of sagged cables through also *physical models*. As a matter of fact, confirmation of regular system response and meaningful information about possible occurrence of new/complex phenomena associated with the governing nonlinearities but often un-modeled in the theoretical analyses, are solely provided by experimental investigations of suspended cables aimed, more generally, at understanding their actual nonlinear dynamic behavior and complementing or validating theoretical and numerical predictions. Sophisticated experimental techniques allow us, among others, (i) to characterize response dimensionality in terms of both time and spatial complexity, (ii) to identify the minimum number and features of the spatial configuration variables actually needed to describe complex motions, (iii) to unveil possibly actual low-dimensionality of both regular and nonregular response, with the ensuing identification of suitable reduced, and minimal, theoretical models able to describe the system dynamics in a certain control parameter range. Experimental analyses may also provide important information about robust bifurcation features of regular responses towards complex dynamics.

Quite a systematic investigation of cable nonlinear experimental dynamics has been accomplished for a horizontal elastic cable/mass hanging at (in-phase or out-of-phase) vertically and harmonically moving supports, and realizing, for relatively low excitation frequencies, a fairly reliable model of bare suspended cable (Fig. 2). System parameters (crossover or slacker cable) are adjusted to produce two different conditions of multiple (1:1 plus 2:1) internal resonances involving up to four modes (symmetric and antisymmetric, vertical (V) in-plane and horizontal (H) out-of-plane). Nonlinear dynamics are investigated in various frequency ranges, which include meaningful external (primary and subharmonic) resonance conditions of either modes (Benedettini and Rega, 1997; Rega and Alaggio, 2001).



mode	H1	V2	H2	V1
shape h-plane / v-plane				
frequency ω [Hz]	3.49	6.68	6.77	8.82
damping	0.0021	0.0003	0.0008	0.027

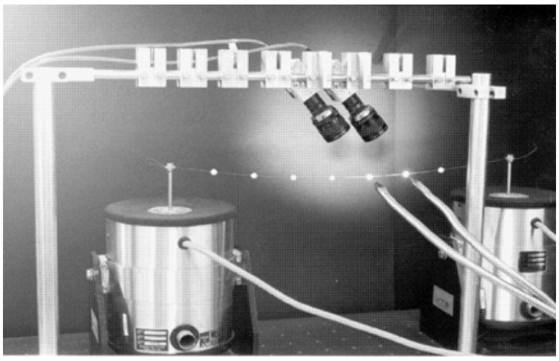


Figure 2. Mechanical model with slacker system parameters and dynamic characteristics; experimental setup.

5.1. System dimensionality and reduced order models

Upon analyzing and classifying local and overall system responses in regular or non-regular regimes, based on various dynamical systems tools, attention is focused on characterizing some main features of system complex response and of the relevant bifurcation mechanisms by means of proper reconstruction techniques of the dynamics from experimental measurements. The *dimensionality* of the response is evaluated in terms of *time* and *spatial complexity* (Alaggio and Rega, 2000). Steady nonregular responses are characterized through the delay-embedding technique for the attractor reconstruction (Takens, 1981) and the proper orthogonal decomposition of the spatio-temporal flow (Holmes *et al.*, 1996), the latter technique allowing to identify the experimental eigenfunctions, i.e. the *proper orthogonal modes* (poms), mostly contributing to system nonregular dynamics. System dimensionality is evaluated both by relating the dimension of the reconstructed attractors to the dimension of the linear phase space, and from the analysis of the spatial structure of nonregular flows and of the relevant dominating poms.

Though being the system potentially infinite-dimensional, not only its *regular* but also its *complex* response is shown to be relatively *low-dimensional* in several cases. Indeed, more than 90% of the nonregular signal power is representable by using up to three or four poms among those already responsible for the higher-dimensional coupled regular dynamics. In addition, overall heuristic correspondence between the main poms and the main linear physical modes of the system is seen to occur.

These results are of notable interest to the aim of associating to each class of complex response of the system, a class of *reduced* (and minimal) *theoretical models* able to describe the complex dynamics of the experimental system. Within the framework of a reduction procedure, these can be built specifically in each region of control parameter space either by using just the identified poms, or - getting hints from them - by projecting the infinite-dimensional dynamics on the known sub-optimal basis of corresponding linear modes (Alaggio and Rega, 2001).

6. BIFURCATION SCENARIOS AND COMPLEX DYNAMICS

A challenging task in the experimental nonlinear dynamics of flexible systems as the sagged cable consists in characterizing *bifurcation scenarios* to complex response and the response itself. Generally speaking, this can be done by realizing a profitable *feedback between experiments and theory* allowing us (i) to possibly trace preliminary experimental results back to a canonical scenario from dynamical systems theory, (ii) to exploit hints from the latter to improve and steer the experimental analyses, (iii) to systematically pursue ahead the physical investigation by detailing the most robust features of system response and clarifying to what extent they can be referred to theoretical scenarios, (iv) to improve cable modeling, and (v) to identify and analyze a proper reduced order cable model aimed at (partially) reproducing the highlighted experimental scenarios.

A general overview of the richness and robustness of different global bifurcation scenarios to chaos, occurring with different support motion in various regions of the excitation control parameter space, is reported in Table 2. Two main routes to chaos, possibly coexisting and competing with each other, do occur, namely a quasiperiodic (3-Tori breakdown) scenario and a scenario involving the global bifurcation of an homoclinic (or heteroclinic) invariant set of the flow (Rega and Alaggio, 2001).

Table 2. A summary of non-regular response regimes, transition scenarios and involved poms.

Experimental transition to nonregular dynamics								
cable	support motion							
	in-phase				anti-phase			
	external resonance condition				external resonance condition			
	Primary		subharmonic-½		primary		subharmonic-½	
	scenario	poms	scenario	poms	Scenario	poms	scenario	Poms
slacker	**QP**	**V1 H1 H2**	**HOM (HET)**	**V5 H5 V3 H3**	**HOM**	**V2 H2**	**HOM (QP)**	**V4 H4 (+H1^{T} H2)**
crossover	**no chaos**		**HOM**	**V5 H5 V3 H3**	**HOM**	**V2 H2**	**HOM**	**V4 H4**

6.1. Quasiperiodic transition to chaos

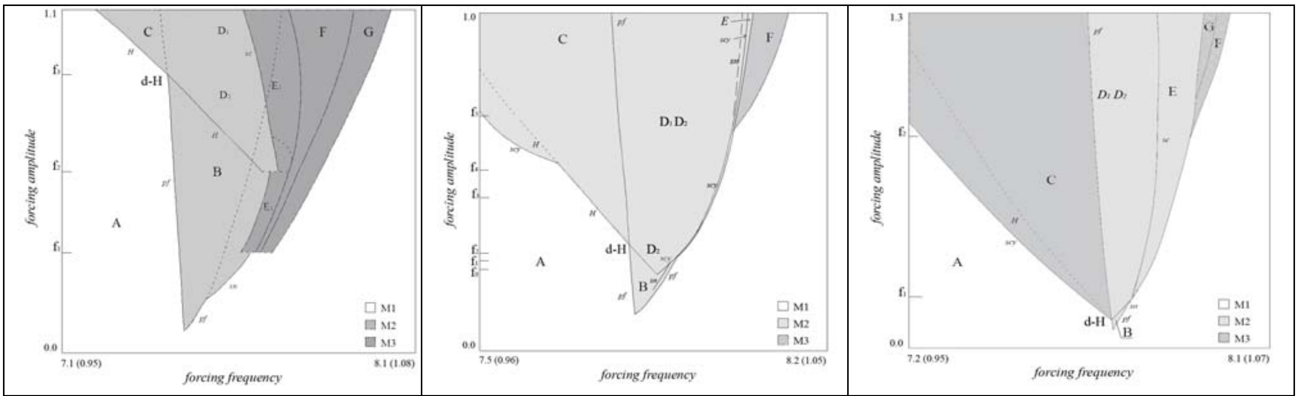
Quasiperiodic transition to chaos is robust for the slacker cable at primary resonance under in-phase support motion, where it involves well identified experimental poms constituting the optimal basis for decomposing the spatio-temporal flow. Specifically, the first antisymmetric out-of-plane (H2) mode adds to the underlying first symmetric in-plane (V1) and out-of-plane (H1) modes, and plays the decisive role as regards transition to chaos. The experimental transition mechanism (Alaggio and Rega, 2000) looks quite rich and involved due to complicated interactions between various internally resonant and non-resonant modes, motion on a 2-Torus, and phase locked three-mode motions on 3-Tori. Yet, it is substantially traceable to a canonical scenario of bifurcation of flows via breakdown of regular dynamics on 3-Tori, known from dynamical systems theory and numerics. However, not all of the experimental response features can be explained according to the torus breakdown paradigm, due to the occurrence of intermittent synchronization on high-periodicity solutions and competing complex phase-modulated regimes which are conjectured to represent toroidal chaos.

6.2. A divergence-Hopf bifurcation organizing regular nonlinear dynamics and homoclinic chaos

Homoclinic bifurcation involves just two main – though variable – poms, and shows to be quite a robust scenario with respect to variations of both cable geometrical-mechanical parameters and excitation conditions, thus being of *general* interest (Rega and Alaggio, 2001).

To characterize in-depth the features of this seemingly paradigmatic scenario, the attention is focused on the homoclinic bifurcation of the slacker cable under anti-phase support motion at primary resonance, which is associated with the passage from a substantially unimodal (in-plane, V2) antisymmetric motion to a ballooning-type (in-plane/out-of-plane, V2/H2, modes) antisymmetric motion evolving from periodic to quasi-periodic up to chaotic. Yet, the systematic experimental investigation needed for in-depth characterization of both classes of motion and transition scenario can be feasibly and reliably accomplished only working with a proper, thermally conditioned, experimental setup, which has the advantage of stabilizing the response of the very flexible system and of making it mechanically accessible without cable loosening effects, while at the same time allowing to consider the temperature as a further controllable parameter in addition to the excitation amplitude and frequency (Rega and Alaggio, 2009).

The analysis shows how the overall cable dynamics possibly ending up with homoclinic chaos is organized by a codimension-two divergence-Hopf (d-H) bifurcation point where two (a Hopf and a pitchfork) bifurcation loci of the Poincaré map of the experimental attractor cross with each other in the excitation frequency-amplitude parameter plane. However, investigations made at different values of setup temperature highlights (i) a substantial decrease of the excitation amplitude value at which the codimension-two bifurcation does occur, along with (ii) meaningful changes in the picture of the ensuing classes of regular and/or non-regular motion extending up to regions relatively far away from the organizing centre. This is clearly observable by comparing the response charts relevant to three different temperature values (Fig. 3). The relevant subfigures and tables show the most robust classes of motion occurring in each zone, along with the possibly competing classes (denoted by lower case labels and arrows in the second column of the corresponding table) alternatively attained for *T*=12°C when considering different initial conditions and/or sweeping directions of the control parameter value, herein represented by the excitation frequency. Based on systematic construction of bifurcation diagrams and spectra of singular values of the covariance matrix of measurements results (Rega and Alaggio, 2009), the various classes of motion are characterized both qualitatively and quantitatively in terms of periodicity (P), quasiperiodicity (QP) – and their degree – or chaoticity (CH), of dimension of the manifold (M) where the motion develops, of correlation dimension (D_C) of the attractor, of dimension of the possibly resonant (^{R}) invariant torus (D-T), of number (typically corresponding to the manifold dimension) of contributing poms that provide more than 90% of the experimental signal power and, finally, of mechanical meaning of the corresponding, vertical (V) or horizontal (H), linear modes.



Zone		Attractor	Dimension			Modes
			Dc	D-T	poms	
A		P1M1	1	1	1	V_2
C		QP1M1	2	2	1	V_2
B		$P1M2^{(SC)}$	1	2^R	2	$V_2 H_2$
D1		$QP2M2^{(S)}$	2	2	2	V_2H_2
		$QP2M2^{(SC)}$	2	2	2	V_2H_2
D2		$QP1M2^{(SC)}$	2	2	2	V_2H_2
E1	(a)	QP1M3	2	3^R	3	$V_2H_2H_1$
	(b)	QP2M3	2	3^R	3	$V_2H_2H_1$
	←	P1M1	1	1	1	V_2
E2	(a)	QP1M3	2	3^R	3	$V_2H_2H_1$
	(c)	P2M3	1	3^R	3	$V_2H_2H_1$
	←→(d)	P1M1	1	1	1	V_2
F	(a)	CHM3	2.8		3	$V_2H_2H_1$
	(b)	P2M3	1	3^R	3	$V_2H_2\,H_1$
	←	P1M1	1	1	1	V_2
G	(b)	P2M3	1	3^R	3	$V_2H_2H_1$
		P1M1	1	1	1	V_2

Zone	Attractor	Dimension			Modes
		Dc	D-T	poms	
A	P1M1	1	1	1	V_2
C	$QP1M1^{(+1)}$	2	2	1(+1)	$V_2(H_1)$
B	$P1M2^{(SC)}$	1	2^R	2	V_2H_2
D_1	$QP1M2^{(S)}$	2	2	2	V_2H_2
D_2	$QP1M2^{(SC)}$	2	2	2	V_2H_2
E	$CHM2^{(*)}$			2	V_2H_2
F	CHM3	2.7		3	$V_2H_2H_1$
	P1M1	1	1	1	V_2

(*) unstable

Zone	Attractor	Dimension			Modes
		Dc	D-T	poms	
A	P1M1	1	1	1	V_2
B	P1M1	1	2^R	2	V_2
C	$QP1M1^{(+2)(S)}$	2	2	1(+2)	$V_2(H_1V_1)$
D_1	$QP1M2^{(S)}$	2	2	2	V_2H_2
D_2	$QP1M2^{(SC)}$	2	2	2	V_2H_2
E	CHM2	2.3		2	V_2H_2
G	QP5M2	2	2	2	V_2H_2
F	CHM3	3.1		3	$V_2H_2H_1$
	P1M1	1	1	1	V_2

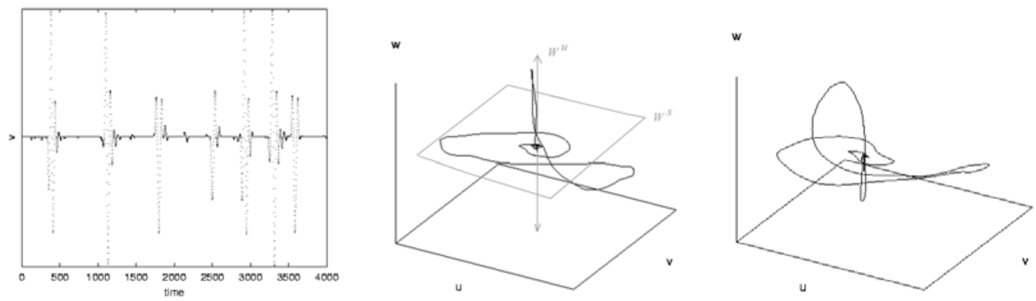
Figure 3. Response charts at various temperatures (*T*=12°C, 6°C and 4°C in first, second and third column, respectively) with zones of periodic (P), quasiperiodic (QP) and chaotic (CH) response, and with qualitative and quantitative characterization of the relevant classes.

Besides lowering the critical forcing amplitude corresponding to d-H codimension-two bifurcation, lowering the temperature – which indirectly induces a material damping decrease – progressively entails a clearer scenario of regular response, along with the possibility to distinguish between chaotic motions characterized by either two (CHM2) or three (CHM3) basic poms, the latter involving also the out-of-plane symmetric mode H1 in addition to the reference in-plane (V2) and out-of-plane (H2) antisymmetric modes. Overall, with decreasing temperatures, the CHM2 class of motion – into which quasiperiodic two-mode motions (QP1M2) end up at also low excitation amplitudes – becomes structurally stable and robust, and exhibits a clear evidence of low-dimensional homoclinic chaos, as shown by the results of a delay embedding reconstruction of phase space from a relevant time series. As a matter of fact, the dynamics in the second order Poincaré section of the reconstructed attractor is organized by an unstable fixed point characterized by a two-dimensional focus-stable manifold W^s and a one-dimensional saddle-unstable manifold W^u, and an invariant of the flow responsible for re-injection toward the fixed point. The fixed point on the second order Poincarè section corresponds to an unstable two-dimensional invariant of the flow resembling the formerly stable quasiperiodic solution QP1M2. In Fig.

4, besides the time series (a), two of the recorded homoclinic orbits are reported (b,c), showing the ejection along the two opposite directions of the unstable manifold (*w* direction), and the re-injection onto the stable manifold (the *uv* plane), respectively.

Overall, the availability of the temperature as a third control parameter shows to be fundamental: (i) to qualitatively refer the *experimental unfolding* of *regular* and *nonregular* cable dynamics in the considered frequency range to the *theoretical* unfolding of the divergence-Hopf bifurcation normal form; (ii) to unfold the system dynamics not only in the strict neighbourhood of the organizing d-H bifurcation but also in the ensuing postcritical regions where the dependence of material damping on temperature affects secondary bifurcations to homoclinic chaos; (iii) to show the variable involvement, in either quasiperiodic or chaotic responses, of a further proper orthogonal mode with respect to the reference two-mode normal form scenario ending up with an *homoclinic chaos* (Rega and Alaggio, 2009).

Parallel ongoing theoretical studies (Alaggio and Rega, 2008) are concerned with developing a consistent phenomenologically-based two-degree-of-freedom model of the suspended cable. Availability of such a reduced order model and its analytical-numerical solution will allow us to check (i) the pursued theoretical interpretation of the dominant experimental scenario along with the possibility to (partially) reproduce it, and (ii) the likely need to account for also the resonant contribution of a third degree-of-freedom for actually reproducing the richness of experimental results in the post-critical range.



(a) (b) (c)

Fig.4. Time delay reconstruction: (a) time series, (b) and (c) homoclinic orbits due to homoclinic tangency.

7. FURTHER RESEARCH DEVELOPMENTS

Research developments on cable nonlinear dynamics are expected on several of the above mentioned topics. To name just a few: (i) Concerned with modeling, attention is already being paid to introducing material nonlinear constitutive laws and/or hysteretic behaviour, as well as cable bending and torsion capacities, in view of specific analyses and applications. (ii) A more complete description of nonlinear multimodal interaction phenomena in the fully 3D dynamics is being accomplished, along with consideration of further internal resonance conditions. (iii) Improving the understanding of bifurcation scenarios and systematically describing large amplitude responses through properly tailored reduced order theoretical models are topics of major interest.

Further challenging issues are concerned with companion topics of both theoretical and practical importance. They include, among others, considering cables with moving loads, analyzing the distinguishing features of fluid-cable interactions, to be accomplished based on reliable formulation and treatment of various (structural-induced, fluid-induced and fluid-structure interface-induced) problem nonlinearities (Ibrahim, 2004; Rega and Sorokin, 2007), as well as the many possibilities to control unwanted cable nonlinear vibrations, and the associated phenomena, by means of a considerable variety of passive, active or hybrid control techniques (Gattulli, 2007).

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9. RESPONSIBILITY NOTICE

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