PREDICTION OF THE TEMPERATURE FIELD IN PIPELINES WITH BAYESIAN FILTERS AND NON-INTRUSIVE MEASUREMENTS

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Abstract. *One of the greatest challenges for the production of petroleum in deepwater is flow assurance. In fact, knowledge about the transient cool down behavior of the produced fluid is necessary to prevent the formation of hydrates and solid deposits during shutdown periods, which could result in a pipeline blockage and could result in large financial losses. In a typical subsea petroleum production system, the information provided by its monitoring system, regarding the temperature field is limited. One approach to predict the produced fluid temperature field in a pipeline system is to use Bayesian filters. In this paper, we compare the Kalman filter and the particle filter as applied to a problem of practical interest for the petroleum industry. Uncertainties in the state evolution and measurement models are taken into account by assuming that the errors involved are additive, normally distributed and with known means and covariance matrices.*

Keywords*: Flow assurance, State estimation problem, Bayesian filtering*

1. INTRODUCTION

Flow assurance in petroleum fields has become one of the greatest challenges on the hydrocarbon production in deepwater environments (Lorimer and Ellison, 2000, Cardoso *et al.*, 2003, Tebboth, 2003, Su, 2003, Carmargo *et al.*, 2004, Denniel *et al.*, 2004). This kind of environment presents high hydrostatic pressures and low sea bed temperatures, which can affect the flow of the produced multiphase fluids (oil, gas, condensate, and water) through pipelines up to the processing facilities.

The thermal management of offshore petroleum fields is, among other operational requirements, one of the main issues for petroleum exploitation operations. Such is the case because, when hydrocarbons are produced and transported over long distances, it is crucial for flow assurance to avoid and control solid deposits and hydrate formation (see figure 1) with thermal monitoring. There are different kinds of deposits that can be formed in pipelines and subsea equipment. The physical and chemical characteristics of the produced fluids may facilitate the accumulation of natural gas hydrates, wax, and other substances within the equipments (Lorimer and Ellison, 2000, Cardoso *et al.*, 2003, Su, 2003, Carmargo *et al.*, 2004, Denniel *et al.*, 2004). These accumulations may cause reduction of flow area and increase the wall roughness, thus increasing the head loss and reducing the flow capacity, which can eventually block the pipeline, resulting in large financial losses.

There are different physical and chemical techniques that can be applied to manage the potential deposits (Lorimer and Ellison, 2000, Cardoso *et al.*, 2003, Carmargo *et al.*, 2004, Denniel *et al.*, 2004). These techniques include "pigging", which represents a device to scrape the pipe walls, and the continuous injection of chemical inhibitors into the pipeline system to minimize the formation of these accumulations. One of the main strategies to mitigate these undesirable effects is to minimize heat losses from the system by using thermal insulation and/or active heating (Su, 2003). In fact, the accurate knowledge of the temperature field along the pipeline is one of the key requirements to maintain the produced fluid temperature above a minimum critical temperature (Alves *et al.*, 1992, Su and Cerqueira, 2001, Guo *et al.*, 2006, Escobedo *et al.*, 2006). Thermal analyses include both steady state and transient studies for the different stages of the field's lifetime and must serve as a design tool for the selection of thermal insulation and/or heating systems, in order to avoid the formation of deposits.

(a) Hydrate (b) Wax

Figure 1. Typical deposits that cause pipeline blockage

Recently, new technologies have emerged for detection, monitoring and control of critical parameters associated with the flow assurance, followed by the implementation of combined corrective actions, when anomalous conditions are identified (Brower and Prescott, 2004, Brower *et al.*, 2005, Zan *et al.*, 2005, Benson and Robins, 2007). For example, measurements of pressure, temperature, flow rate, fluid composition and strain, among other parameters, may be used to predict the onset of operational problems, thus allowing for timely corrective actions.

As one of the most important aspects for the management of deposits is based on the accurate knowledge of the temperature field inside the pipeline and/or subsea equipment, the main objective of this work is to apply Bayesian filters (Maybeck, 1979, Andrieu *et al.*, 2004, Kaipio and Somersalo, 2004, Scott and McCann, 2005, Orlande *et al.*, 2008) to predict the unsteady temperature field in a pipeline cross section during shutdown periods. The temperature field is predicted from limited temperature data available at the surface of the pipeline. Uncertainties in the state evolution and measurement models are taken into account, by assuming that the errors involved are additive, normally distributed and with known means and covariance matrices. The accurate estimation of the temperature field allows for the prediction of cold regions in the oil-gas-water mixture inside the pipeline. As a result, preventive actions can be taken beforehand in order to avoid the formation of deposits.

2. STATE ESTIMATION

In state estimation problems (Maybeck, 1979, Kaipio and Somersalo, 2004, Scott and McCann, 2005) observations obtained during the evolution of the system, are used together with prior knowledge about the physical phenomena and the measuring devices, in order to sequentially produce estimates of the desired dynamic variables. State estimation problems can be solved with the so-called Bayesian filters (Maybeck, 1979, Kaipio and Somersalo, 2004, Scott and McCann, 2005).

In order to define the state estimation problem, consider a model for the evolution of the state variables **x** in the form:

$$\mathbf{x}_k = \mathbf{f}_k\left(\mathbf{x}_{k-1}, \mathbf{v}_k\right) \tag{1}$$

where **f** is, in the general case, a non-linear function of $\mathbf{x}$ and of the state noise or uncertainty vector given by $\mathbf{v}_k \in \mathbf{R}^{\mathbf{n}_v}$. The vector $\mathbf{x}_k \in \mathbf{R}^{\mathbf{n}_x}$ is called the state vector and contains the variables to be dynamically estimated. This vector advances in time in accordance with the *state evolution model* (1). The subscript k =1, 2, 3, …, denotes a time instant t_k in a dynamic problem.

The observation model describes the dependence between the state variable **x** to be estimated and the measurements **z** through the general, possibly non-linear, function **h**. This can be represented by

$$\mathbf{z}_k = \mathbf{h}_k\left(\mathbf{x}_k, \mathbf{n}_k\right) \tag{2}$$

where $\mathbf{z}_k \in \mathbf{R}^{\mathbf{n}_Z}$ are available at times t_k, k=1, 2, 3,…. Eq. (2) is referred to as the *observation/measurement model*. The vector $\mathbf{n}_k \in \mathbf{R}^{\mathbf{n}_n}$ represents the measurement noise or uncertainty.

The *evolution and observation models*, given by Eqs. (1) and (2), respectively, are based on the following assumptions (Kaipio and Somersalo, 2004, Scott and McCann, 2005):

(a) The sequence $\mathbf{x}_k$ for k=1, 2, 3, …, is a Markovian process, that is,

$$\pi\left(\mathbf{x}_k \middle| \mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_{k-1}\right) = \pi\left(\mathbf{x}_k \middle| \mathbf{x}_{k-1}\right) \tag{3.a}$$

(b) The sequence $\mathbf{z}_k$ for k=1, 2, 3, …, is a Markovian process with respect to the history of $\mathbf{x}_k$, that is,

$$\pi\left(\mathbf{z}_k \middle| \mathbf{x}_0, \mathbf{x}_1, \ldots, \mathbf{x}_k\right) = \pi\left(\mathbf{z}_k \middle| \mathbf{x}_k\right) \tag{3.b}$$

(c) The sequence $\mathbf{x}_k$ depends on the past observations only through its own history, that is,

$$\pi\left(\mathbf{x}_k \left|\mathbf{x}_{k-1}, \mathbf{z}_1, \mathbf{z}_2, \ldots, \mathbf{z}_{k-1}\right.\right) = \pi\left(\mathbf{x}_k \left|\mathbf{x}_{k-1}\right.\right) \tag{3.c}$$

where $\pi\left(\mathbf{a}\left|\mathbf{b}\right.\right)$ denotes the conditional probability of **a** when **b** is given.

For the state and observation noises, the following assumptions are made (Kaipio and Somersalo, 2004, Scott and McCann, 2005):

(a) For $i \neq j$, the noise vectors $\mathbf{v}_i$ and $\mathbf{v}_j$, as well as $\mathbf{n}_i$ and $\mathbf{n}_j$, are mutually independent and also mutually independent of the initial state $\mathbf{x}_0$.
(b) The noise vectors $\mathbf{v}_i$ and $\mathbf{n}_j$ are mutually independent for all *i* and *j*.

Different problems can be considered for the evolution-observation model described above, such as (Kaipio and Somersalo, 2004, Scott and McCann, 2005):

(i) The prediction problem, when the objective is to obtain $\pi\left(\mathbf{x}_k \left|\mathbf{z}_{1:k-1}\right.\right)$;

(ii) The filtering problem, when the objective is to obtain $\pi\left(\mathbf{x}_k \left|\mathbf{z}_{1:k}\right.\right)$;

(iii) The fixed-lag smoothing problem, when the objective is to obtain $\pi\left(\mathbf{x}_k \left|\mathbf{z}_{1:k+p}\right.\right)$, where $p \geq 1$ is the fixed lag.

(iv) The whole-domain smoothing problem, when the objective is to obtain $\pi\left(\mathbf{x}_k \left|\mathbf{z}_{1:K}\right.\right)$, where $\mathbf{z}_{1:K} = \left\{\mathbf{z}_i, i = 1,\cdots,K\right\}$ is the complete set of measurements.

3. BAYESIAN FILTERS

The most widely known Bayesian filter is the Kalman filter, with its application limited to linear models with additive Gaussian noises. In such cases where the models are non-linear or the errors non-Gaussian, Monte Carlo methods can be applied to solve state estimation problems (Maybeck, 1979, Carpenter *et al.*, 1999, Doucet *et al.*, 2000, Arulampalam *et al.*, 2001, Kaipio and Somersalo, 2004, Andrieu *et al.*, 2004, Scott and McCann, 2005, Del Moral *et al.*, 2006, Del Moral *et al.*, 2007, Johansen and Doucet, 2008, Orlande *et al.* 2008). In this work we apply the Kalman filter and the particle filter to predict the temperature field in a pipeline system, as described below.

3.1. Kalman filter

This method, published in 1960, is a set of mathematical equations that recursively estimates the state variables of a system (Kalman, 1960, Sorenson, 1970, Maybeck, 1979, Ristic, 2004, Kaipio and Somersalo, 2004, Scott and McCann, 2005, Welch and Bishop, 2005, Orlande *et al.*, 2008). The Kalman filter is one of the most well-known and used Bayesian filters, but its application is limited to linear models with additive Gaussian noises.

Considering the classical discrete-time state estimation problem in the case of linear models, the evolution equation that describes the time dependence of the state variable **x** can be written in the form:

$$\mathbf{x}_k = \mathbf{F}_k \mathbf{x}_{k-1} + \mathbf{s}_k + \mathbf{v}_k \tag{4}$$

where $\mathbf{F}_k$ is the linear evolution matrix of the state variable **x**, and the vector $\mathbf{s}_k$ is assumed to be known sources for the problem. The state uncertainty or noise $\mathbf{v}_k$ is assumed to be a Gaussian random variable with zero mean and covariance **Q**.

The linear observation model can be represented in the form:

$$\mathbf{z}_k = \mathbf{H}_k \mathbf{x}_k + \mathbf{n}_k \tag{5}$$

where $\mathbf{z}_k$ is the measurement vector and $\mathbf{H}_k$ is the linear observation matrix. The observation noise $\mathbf{n}_k$ is assumed to be a Gaussian random variable with zero-mean and known covariance **R**. The state and observation noises are assumed to be mutually independent.

The algorithm of the Kalman filter is presented below in tables 1 and 2, as applied to the state estimation problem given by equations (4,5) (Maybeck, 1979, Kaipio and Somersalo, 2004, Scott and McCann, 2005, Welch and Bishop, 2005, Orlande *et al.*, 2008).

Table 1 – Discrete time evolution update equations

$$\mathbf{x}_k^- = \mathbf{F}_k\,\mathbf{x}_{k-1} \tag{6.a}$$

$$\mathbf{P}_k^- = \mathbf{F}_k\,\mathbf{P}_{k-1}\mathbf{F}_K^T + \mathbf{Q}_k \tag{6.b}$$

Table 2 – Measurement update equations

$$\mathbf{K}_k = \mathbf{P}_k^-\,\mathbf{H}_k^T\left(\mathbf{H}_k\,\mathbf{P}_k^-\,\mathbf{H}_k^T + \mathbf{R}_k\right)^{-1} \tag{7.a}$$

$$\mathbf{x}_k = \mathbf{x}_k^- + \mathbf{K}_k\left(\mathbf{z}_k - \mathbf{H}_k\mathbf{x}_k^-\right) \tag{7.b}$$

$$\mathbf{P}_k = \left(\mathbf{I} - \mathbf{K}_k\,\mathbf{H}_k\right)\mathbf{P}_k^- \tag{7.c}$$

Here, **K** is known as Kalman's gain matrix and **P** is the covariance matrix of the estimated state variables.

3.2. The Particle Filter

The Particle Filter Method is a Monte Carlo technique for solution of the state estimation problem, where the main idea is to represent the required posterior density function by a set of random samples with associated weights and to compute the estimates based on these samples and weights (Maybeck, 1979, Carpenter *et al.*, 1999, Doucet *et al.*, 2000, Arulampalam *et al.*, 2001, Kaipio and Somersalo, 2004, Andrieu *et al.*, 2004, Scott and McCann, 2005, Del Moral *et al.*, 2006, Del Moral *et al.*, 2007, Johansen and Doucet, 2008, Orlande *et al.* 2008). In this study, we use the so-called *Sequential Importance Resampling* (SIR) algorithm for the particle filter, which includes a resampling step at each time instant, as described in (Arulampalam *et al.*, 2001, Orlande *et al.* 2008). The SIR algorithm makes use of an importance density, which is a density proposed to represent another one that cannot be exactly computed. Then, samples are drawn from the importance density instead of the actual density.

Let $\left\{\mathbf{x}_{0:k}^i,\ i = 0,\cdots,N\right\}$ be the particles with associated weights $\left\{\mathbf{w}_k^i,\ i = 0,\cdots,N\right\}$ and $\mathbf{x}_{0:k} = \left\{\mathbf{x}_j,\ j = 0,\cdots,k\right\}$ be the set of all states up to t_k, where N is the number of particles. The weights are normalized, so that $\sum_{i=1}^{N} w_k^i = 1$. Then, the posterior density at t_k can be discretely approximated by:

$$\pi\left(\mathbf{x}_{0:k}\left|\mathbf{z}_{1:k-1}\right.\right) \approx \sum_{i=1}^{N} w_k^i\,\delta\left(\mathbf{x}_{0:k} - \mathbf{x}_{0:k}^i\right) \tag{8}$$

where $\delta(.)$ is the Dirac delta function. By taking hypotheses (3.a-c) into account, the posterior density in Eq. (8) can be written as (Arulampalam *et al.*, 2001):

$$\pi\left(\mathbf{x}_k\left|\mathbf{z}_{1:k-1}\right.\right) \approx \sum_{i=1}^{N} w_k^i\,\delta\left(\mathbf{x}_k - \mathbf{x}_k^i\right) \tag{9}$$

A common problem with the SIS particle filter is the degeneracy phenomenon, where after a few states all but one particle may have negligible weight. The degeneracy implies that a large computational effort is devoted to updating particles whose contribution to the approximation of the posterior density function is almost zero. This problem can be overcome by increasing the number of particles, or more efficiently by appropriately selecting the importance density as the prior density $\pi\left(\mathbf{x}_k\left|\mathbf{x}_{k-1}^i\right.\right)$. In addition, the use of the resampling technique is recommended to avoid the degeneracy of the particles.

Resampling involves a mapping of the random measure $\left\{\mathbf{x}_k^i, \mathrm{w}_k^i\right\}$ into a random measure $\left\{\mathbf{x}_k^{i^*}, N^{-1}\right\}$ with uniform weights. It can be performed if the number of effective particles with large weights falls below a certain threshold number. Alternatively, resampling can also be applied indistinctively at every instant t_k, as in the Sampling Importance Resampling (SIR) algorithm described in (Arulampalam *et al.*, 2001). This algorithm can be summarized in the steps presented in Table 3, as applied to the system evolution from t_{k-1} to t_k (Arulampalam *et al.*, 2001).

Although the resampling step reduces the effects of the degeneracy problem, it may lead to a loss of diversity and the resultant sample can contain many repeated particles. This problem, known as sample impoverishment, can be severe in the case of small process noise. In this case, all particles collapse to a single particle within few instants t_k (Arulampalam *et al.*, 2001, Kaipio and Somersalo, 2004). Another drawback of the particle filter is related to the large computational cost due to the Monte Carlo method, which may limit its application only to fast computing problems.

Table 3 – Sampling Importance Resampling Algorithm

Step 1
For $i = 1, \cdots, N$ draw new particles $\mathbf{x}_k^i$ from the prior density $\pi\left(\mathbf{x}_k \middle| \mathbf{x}_{k-1}^i\right)$ and then use the likelihood density to calculate the correspondent weights $w_k^i = \pi\left(\mathbf{z}_k \middle| \mathbf{x}_k^i\right)$.
Step 2
Calculate the total weight $T_w = \sum_{i=1}^{N} w_k^i$ and then normalize the particle weights, that is, for $i = 1, \cdots, N$ let $w_k^i = T_w^{-1} w_k^i$.
Step 3
Resample the particles as follows : Construct the cumulative sum of weights (CSW) by computing $c_i = c_{i-1} + w_k^i$ for $i = 1, \cdots, N$, with $c_0 = 0$. Let $i = 1$ and draw a starting point u_1 from the uniform distribution $U\left[0, N^{-1}\right]$ For $j = 1, \cdots, N$ Move along the CSW by making $u_j = u_1 + N^{-1}\left(j-1\right)$ While $u_j > c_i$ make $i = i+1$. Assign sample $x_k^j = x_k^i$ Assign sample $w_k^j = N^{-1}$

4. PHYSICAL PROBLEM AND MATHEMATICAL FORMULATION

The physical problem in this work considers a critical operational condition involving a pipeline shutdown situation. The problem consists of a pipeline cross-section represented by a circular domain filled with a stagnant fluid and bounded by a constant thickness pipe wall (Jamaluddin *et al.*, 1991, Su and Cerqueira, 2001, Escobedo *et al.*, 2006). The fluid is considered as homogeneous, isotropic and with constant thermal properties. The idealized pipeline will be treated here with an unsteady heat conduction problem in a single medium, thus not taking into account the pipe wall. By considering axial symmetry, the dimensionless formulation of this heat conduction problem in cylindrical coordinates is given by

$$\frac{\partial \theta(R,\tau)}{\partial \tau} = \frac{\partial^2 \theta(R,\tau)}{\partial R^2} + \frac{1}{R}\frac{\partial \theta(R,\tau)}{\partial R} \qquad 0 \le R < 1 \, , \, \tau > 0 \tag{10.a}$$

$$\frac{\partial \theta(R,\tau)}{\partial R} + Bi\,\theta(R,\tau) = 0 \qquad R = 1 \, , \, \tau > 0 \tag{10.b}$$

$$\theta(R,0) = 1 \qquad 0 \le R < 1 \, , \, \tau = 0 \tag{10.c}$$

where the dimensionless groups were defined as

$$\theta(R,\tau) = \frac{T(r,t) - T_\infty}{T(r,0) - T_\infty} \tag{11.a}$$

$$\tau = \frac{\alpha t}{R^{*2}} \tag{11.b}$$

$$R = \frac{r}{R^*} \tag{11.c}$$

$$Bi = \frac{h\,R^*}{k} \tag{11.d}$$

Here, T_∞ is the surrounding environment temperature, h is the convective heat transfer coefficient, k is the thermal conductivity coefficient, R^* is the external radius and Bi is the Biot number.

The solution for such a mathematical model can be obtained with finite-differences, thus resulting in the following linear system of algebraic equations (Ozisik, 1993):

$$\mathbf{\Theta}^{k+1} = \mathbf{F}\,\mathbf{\Theta}^{k} + \mathbf{S} \tag{12}$$

where,

$$\mathbf{\Theta} = \begin{bmatrix} \Theta_1 \\ \vdots \\ \Theta_N \end{bmatrix} \qquad \mathbf{F} = \begin{bmatrix} (1-4B) & 4B & & & \\ \left(B-\frac{B}{2}\right) & (1-2B) & \left(B+\frac{B}{2}\right) & & \\ & \ddots & \ddots & \ddots & \\ & & \ddots & \ddots & \ddots \\ & & \left(B-\frac{B}{2(N-1)}\right) & (1-2B) & \left(B+\frac{B}{2(N-1)}\right) \\ & & & 2B & C \end{bmatrix}$$

$$\mathbf{S} = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix} \qquad \mathbf{B} = \frac{\Delta\tau}{\Delta R^2} \qquad \mathbf{C} = \left(1 - 2B - 2\Delta R Bi B - \frac{\Delta R\, B\, Bi}{N}\right)$$

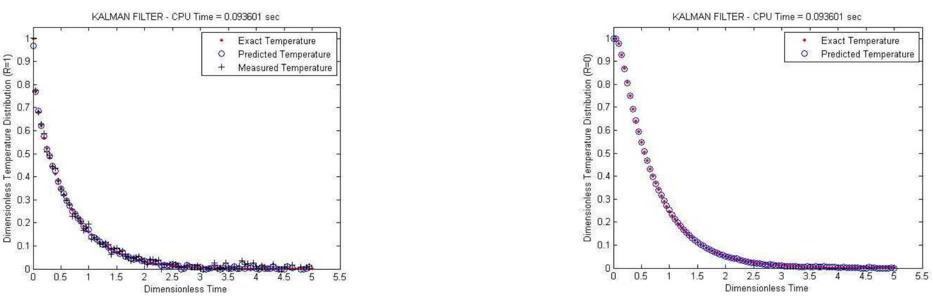
Here, N is the number of internal nodes in the finite-difference solution, **F** is an N x N coefficient matrix, $\mathbf{\Theta}$ is a temperature vector of order N x 1 and **S** is a known vector of order N x 1.

5. RESULTS AND DISCUSSIONS

We now present the results obtained for the state estimation problem under analysis, by using simulated experiments. The simulated measurements contain additive, uncorrelated, Gaussian errors, with constant standard deviation. Two test cases are examined below, involving different standard deviations for the measurement errors, namely: (i) Test case 1 with standard deviation of 1^{o}C; and (ii) Test case 2 with standard deviation of 5^{o}C. It is assumed that the simulated measurements are taken with a sensor located at the outer boundary surface, that is, at R = 1.0. Initial temperature of the oil was uniform at 80^{o}C and the surrounding temperature was T_∞ = 4^{o}C. The state variables to be estimated are the transient temperatures inside the pipeline cross section at the equidistant finite difference nodes.

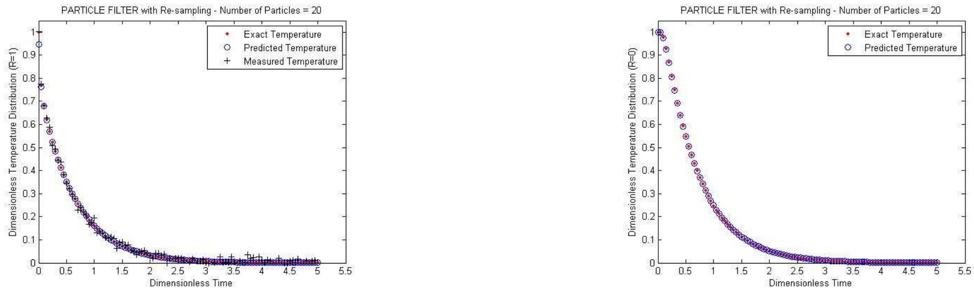
For test case 1, we simulate transient measured temperatures containing Gaussian errors with standard deviation of 1^{o}C and compare the exact temperature (obtained with an analytic solution with separation of variables, which is omitted here for the sake of brevity), measured and predicted temperatures at positions R = 0 and R = 1.0. The predicted temperatures were obtained with the Kalman filter and with the particle filter (implemented in accordance with the SIR algorithm described above). Twenty particles were used for each state variable in the particle filter.

Figures 2.a,b, and 3.a,b present the results obtained with the Kalman filter and the particle filter, respectively, for a situation involving a standard deviation of the evolution model errors of 0.5^{o}C. These figures show an excellent agreement between predicted and exact temperatures, as a result of the small errors in the evolution and observation models.



(a) Transient temperatures at R=1 (b) Transient temperatures at R=0

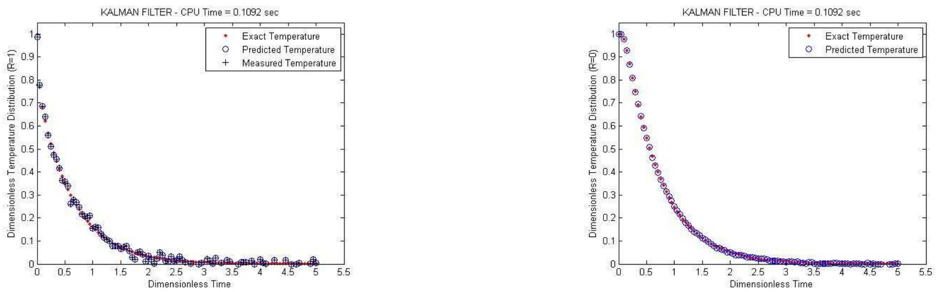
Figure 2. - Standard deviation for the evolution model errors of 0.5^{o}C – Kalman filter – Test Case 1



(a) Transient temperatures at R=1 (b) Transient temperatures at R=0

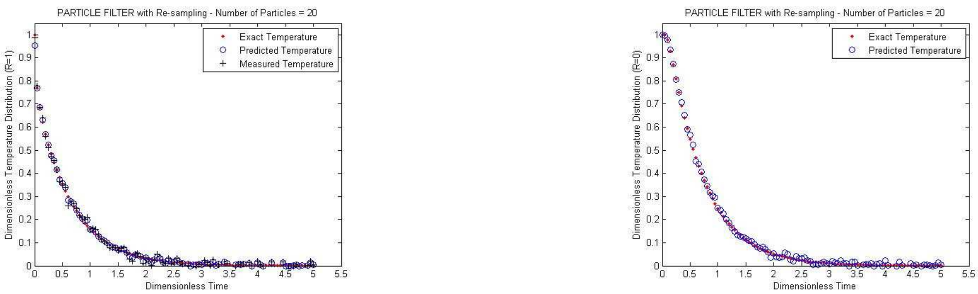
Figure 3. - Standard deviation for the evolution model errors of 0.5^{o}C – Particle filter– Test Case 1

We now address the solution for a case involving a standard deviation of evolution model errors of 5^{o}C. The results obtained with this case are presented in figures 4.a,b and 5.a,b for the Kalman and particle filters, respectively. Note in these figures that the predictions tend to follow the measurements instead of the evolution model at R = 1.0, because of the large evolution model errors. On the other hand, the predictions are in excellent agreement with the exact temperatures at positions where no measurements are taken, such as at R = 0.



(a) Transient temperatures at R=1 (b) Transient temperatures at R=0

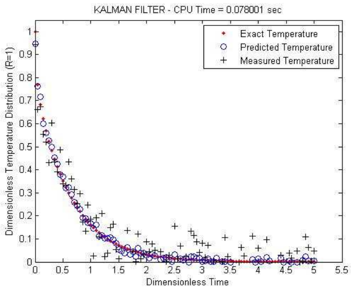
Figure 4. - Standard deviation for the evolution model errors of 5^{o}C – Kalman filter - Test Case 1



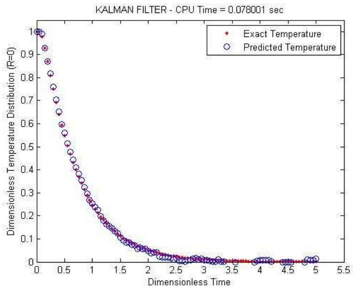
(a) Transient temperatures at R=1 (b) Transient temperatures at R=0

Figure 5. - Standard deviation for the evolution model errors of 5^{o}C – Particle filter with re-sampling - Test Case 1

A similar analysis is now made for test case 2, where the simulated measured temperatures contain errors with standard deviation of 5^{o}C. Figures 6.a,b and 7.a,b present the results obtained with a standard deviation of the evolution model errors of 1^{o}C, for the Kalman and particle filters, respectively. These figures show that, despite the larger measurement errors, the two Bayesian solution techniques examined in this work are capable of accurately predicting the temperatures inside the domain of interest, even at points quite distant from the measurement locations. A comparison of figures 6.a,b and 7.a,b reveals that the particle filter provided more accurate estimates than the Kalman filter for this case, at R = 0 and R = 1.0. Similar results were obtained with larger errors in the evolution model, as illustrated by figures 8.a,b and 9.a,b. These figures show the predicted and exact temperatures for a standard deviation of the evolution model of 5^{o}C.

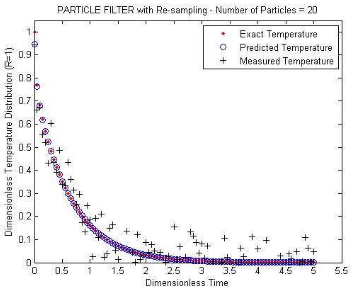


(a) Transient temperatures at R=1

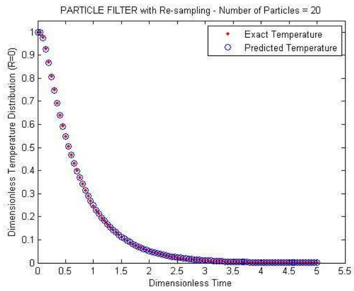


(b) Transient temperatures at R=0

Figure 6. - Standard deviation for the evolution model errors of $1^{o}C$ – Kalman filter – Test Case 2

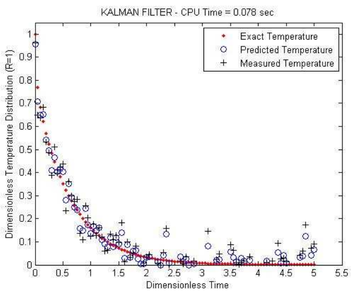


(a) Transient temperatures at R=1

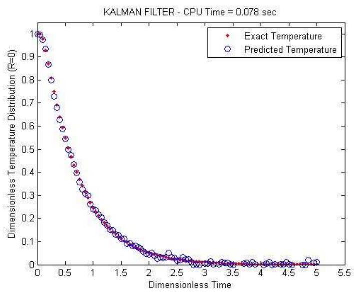


(b) Transient temperatures at R=0

Figure 7. - Standard deviation for the evolution model errors of $1^{o}C$ – Particle filter – Test Case 2

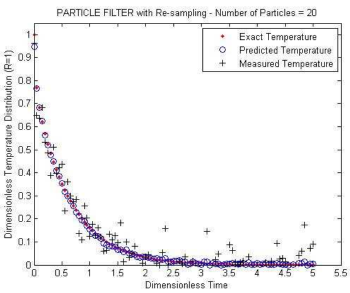


(a) Transient temperatures at R=1

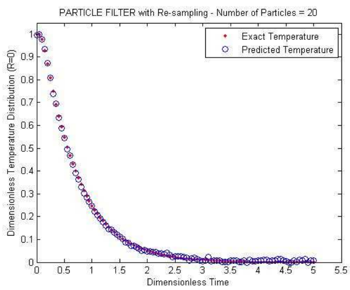


(b) Transient temperatures at R=0

Figure 8. - Standard deviation for the evolution model errors of $5^{o}C$ – Kalman filter - Test Case 2



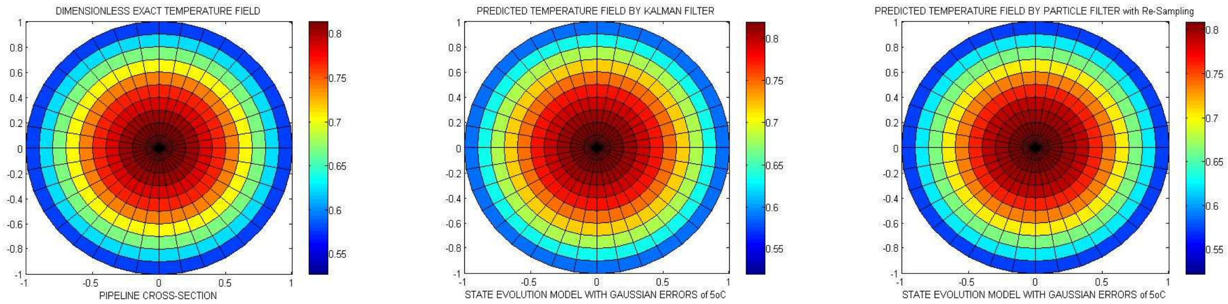
(a) Transient temperatures at R=1



(b) Transient temperatures at R=0

Figure 9. - Standard deviation for the evolution model errors of $5^{o}C$ – Particle filter with re-sampling - Test Case 2

Figures 10.a-c were prepared in order to illustrate the capabilities of the Bayesian filters as a prediction tool for the temperature field in a pipeline cross-section from limited temperature data available at this surface. These figures show the exact temperature field, as well as those predicted with the Kalman and particle filters, respectively, at a specific dimensionless time. In the case presented in figures 10.a-c, the standard deviation for the measurement errors and the standard deviation for the evolution model errors are both of $5^{o}C$.



(a) Exact temperature field | (b) Predicted temperature field by Kalman filter | (c) Predicted temperature field by particle filter

Figure 10. Temperature field inside the pipeline predicted with Bayesian filters

6. CONCLUSIONS

The objective of this work was to apply Bayesian filters to the estimation of the transient temperature field inside a pipeline, by using limited temperature data available at its outer surface. The state estimation problem under analysis was solved with the Kalman filter and with the particle filter, for linear evolution and observation models, with additive and uncorrelated Gaussian noises. The Kalman filter and the particle filter provided results of similar accuracy for a test case involving measurement errors of small magnitude. On the other hand, the predictions obtained with the particle filter are in much better agreement with the exact temperatures than those obtained with the Kalman filter, when the measurement errors are large.

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