APPLICATION OF AN EXTENDED TIME-DELAYED FEEDBACK CHAOS CONTROL METHOD TO A NONLINEAR PENDULUM

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Abstract. *Chaos control is employed for the stabilization of unstable periodic orbits (UPOs) embedded in chaotic attractors. The extended time-delayed feedback control uses a continuous feedback loop incorporating information from previous states of the system in order to stabilize unstable orbits. This article deals with the chaos control of a nonlinear pendulum employing the extended time-delayed feedback control method. The control law leads to delay-differential equations (DDEs) that contain derivatives that depend on the solution of previous time instants. A fourth-order Runge-Kutta method with linear interpolation on the delayed variables is employed for numerical simulations of the DDEs and its initial function is estimated by a Taylor series expansion. During the learning stage, the UPOs are indentified by the close-return method and control parameters are chosen for each desired UPO by defining situations where the largest Lyapunov exponent becomes negative. Analyses of a nonlinear pendulum are carried out by considering signals that are generated by numerical integration of the mathematical model using experimentally identified parameters. Results show the capability of the control procedure to stabilize UPOs of the dynamical system, highlighting some difficulties to achieve the stabilization of the desired orbit.*

Keywords: *Chaos, control, nonlinear dynamics, nonlinear pendulum, delayed-feedback control.*

1. INTRODUCTION

Inspired by nature, researchers are trying to design dynamical systems that can easily change from different kinds of responses. This is of special interest in order to confer flexibility to the system since it may quickly react to distinct kinds of response. In this regard, chaotic behavior has a great potential to be interesting in different situations due to its rich structure related to a wide range of behaviors. Chaos is a possible response of nonlinear systems that has a dense set of unstable periodic orbits (UPOs) and the idea that chaotic behavior may be controlled by small perturbations makes this kind of behavior to be desirable in different applications.

A chaos control method may be understood as a two stage technique. In the first step, the learning stage, the UPOs are identified and control parameters are evaluated. After that, there is the control stage where the desired UPOs are stabilized. The chaos control stabilization may be classified as discrete or continuous. The OGY (Ott-Grebogi-Yorke) method (Ott *et al.*, 1990) is a discrete technique that considers small perturbations promoted in the neighborhood of the desired orbit. On the other hand, continuous methods are exemplified by the so called time-delayed feedback control, proposed by Pyragas (1992), which states that chaotic systems can be stabilized by a feedback perturbation proportional to the difference between the present and a delayed state of the system. The semi-continuous approach lies between the continuous and the discrete time control (Hübinger *et al.*, 1994; Korte *et al.*, 1995).

Numerous research efforts are dedicated to overcome some limitations of these original techniques. Based on OGY method, De Paula & Savi (2009), Dressler & Nitsche (1992), Hübinger *et al.* (1994), Korte *et al.* (1995), Otani & Jones (1997) and So & Ott (1995) suggest some improvements concerning discrete approach. Savi *et al.* (2006) discusses some of these alternatives focused on mechanical systems. Pyragas (1995) presents a review about improvements and applications of time-delayed feedback control, while Ahlborn & Parlitz (2004), Fiedler *et al.* (2007), Kittel *et al.* (1995), Mausbach *et al.* (1997) and Socolar *et al.* (1994) present some extensions related to the original method of Pyragas (1992).

This article deals with an application of the extended time-delayed feedback control method to a mechanical system. The control law leads to delay-differential equations (DDEs). A nonlinear pendulum is considered as an application of the general formulation. This pendulum was previously addressed in De Paula *et al.* (2006) and its control was treated in Pereira-Pinto *et al.* (2004, 2005), Savi *et al.* (2006) and De Paula & Savi (2009). All signals are numerically generated by the integration of the equations of motion using experimentally identified parameters. Concerning the learning stage, the close-return method (Auerbach *et al.*, 1987) is employed to determine the UPOs embedded in the chaotic attractor. The control parameters are evaluated for each desired UPO by finding negative values of the largest Lyapunov exponent, which is calculated employing the algorithm due to Wolf *et al.* (1985). This is possible by assuming an approximation where the continuous evolution of the infinite-dimension delay-differential equation is replaced by a set of ordinary differential equations. Finally, the actuator is perturbed in order to achieve system stabilization considering different UPOs. Results confirm the possibility of the use of this approach to deal with chaos control in mechanical systems, showing some of the difficulties to achieve the stabilization of desired UPOs.

2. EXTENDED TIME-DELAYED FEEDBACK CONTROL

The time-delayed feedback control method (TDFC), also known as time-delay autosynchronization (TDAS), was proposed by Pyragas (1992) and is based on continuous-time perturbations to perform chaos control. This control technique deals with a dynamical system modeled by a set of ordinary nonlinear differential equations as follows:

$$\begin{aligned} \dot{x} &= Q(x, y) \\ \dot{y} &= P(x, y) + F(t) \end{aligned} \tag{1}$$

where x and y are the state variables, $Q(x, y)$ and $P(x, y)$ define the system dynamics, while $F(t)$ is associated with the control action. The TDFC is based on a feedback of the difference between the current and a delayed state, and the perturbation is given by:

$$F(t) = K[y_\tau - y] \tag{2}$$

where τ is the time delay, $y = y(t)$, $y_\tau = y(t-\tau)$ and K is a control parameter. This control method was successfully implemented, numerically and experimentally, to different systems including mechanical devices (Hikihara & Kawagoshi, 1996; Ramesh & Narayanan, 2001), electronic oscillators (Gauthier *et al.*, 1994; Pyragas & Tamasevicius, 1993) and laser (Bielawski *et al.*, 1993).

Despite this good performance, the TDFC fails when applied to orbits with high periodicity. This limitation may be overcome by a generalization of the feedback law presented in Eq. (2). Originally proposed by Socolar *et al.* (1994), the extended time-delayed feedback control (ETDFC) is also called extended time-delay autosynchronization (ETDAS). Essentially, this new control technique considers not only the information of one time-delayed state but other previous states of the system represented by the following equations:

$$\begin{aligned} F(t) &= K\left[(1-R)S_\tau - y\right] \\ S_\tau &= \sum_{m=1}^{\infty} R^{m-1} y_{m\tau} \end{aligned} \tag{3}$$

where $0 \le R < 1$, $S_\tau = S(t-\tau)$ and $y_{m\tau} = y(t - m\tau)$. The UPO stabilization can be achieved by a proper choice of R and K. Note that when R = 0, the ETDFC turns into the original TDFC feedback control law represented by Eq.(2). Moreover, note that for any R, perturbation of Eq.(3) vanishes when the system is on the UPO since $y(t - m\tau) = y(t)$ for all m if $\tau = T_i$, where T_i is the periodicity of the ith UPO.

The controlled dynamical system, composed by Eqs.(1)-(3), is represented by a delay differential equations (DDEs). The solution of this set of DDEs is done by establishing an initial function $y_0 = y_0(t)$ over the interval $(-m\tau, 0)$. This function can be estimated by a Taylor series expansion as proposed by Cunningham (1954):

$$y_{m\tau} = y - m\tau\,\dot{y} \tag{4}$$

Under this assumption, the following system is obtained:

$$\begin{aligned} \dot{x} &= Q(x, y) \\ \dot{y} &= P(x, y) + K\left[(1-R)S_\tau - y\right] \\ &\text{where } \begin{cases} S_\tau = \sum_{m=1}^{\infty} R^{m-1}\left[y - m\tau\,\dot{y}\right], & \text{for } (t-\tau) < 0 \\ S_\tau = \sum_{m=1}^{\infty} R^{m-1} y_{m\tau}, & \text{for } (t-\tau) \ge 0 \end{cases} \end{aligned} \tag{5}$$

Note that DDEs contain derivatives that depend on the solution at delayed time instants. Therefore, besides the special treatment that must be given for $(t - m\tau) < 0$, it is necessary to deal with time-delayed states while integrating the system. A fourth-order Runge-Kutta method with linear interpolation on the delayed variables is employed in this work for the numerical integration of the controlled dynamical system (Mensour & Longtin, 1997).

During the learning stage it is necessary to identify the UPOs embedded in the chaotic attractor, which is done by employing the close-return method (Auerbach *et al.*, 1987). Moreover, it is necessary to establish a proper choice of control parameters, K and R, for each desired orbits. This choice is done by analyzing Lyapunov exponents of the correspondent orbit, as presented in the next section. After this first stage, the control stage is performed, where the desired UPOs are stabilized.

3. UPO LYAPUNOV EXPONENTS

The idea behind the time-delayed feedback control is the construction of a continuous-time perturbation, as presented in Eqs. (2)-(3) (Kittel *et al.*, 1994; Pyragas, 1993), in such a way that it does not change the desired UPO of the system, but only its characteristics. This is achieved by changing the control parameters in order to force Lyapunov exponents related to an UPO to become all negatives, which means that the UPO becomes stable (Kittel *et al.*, 1995). In this regard, it is enough to determine only the largest Lyapunov exponent, evaluating values of K and R that change the sign of the exponents. In other words, it is necessary to look for a situation where the maximum exponent is negative, $\lambda(K,R)<0$, situation where the orbit becomes stable. Besides, Pyragas (1995) states that the minimum of $\lambda(K,R)$ provides a faster convergence rate of nearby orbits to the desired UPO and makes the method more robust with respect to noise.

The calculation of Lyapunov exponent from DDEs is more complicated than ODEs because the terms associated with the extended time-delayed feedback, Eq. (3), involves system states delayed in time. By considering three delayed states, for example, the second equation of Eq. (5) consists in a delay differential equation as follows:

$$\begin{aligned} \dot{x} &= Q(x,y) \\ \dot{y} &= P(x,y) + F(y, y_\tau, y_{2\tau}, y_{3\tau}) \end{aligned} \tag{6}$$

Therefore, the calculation of $y = y(t)$ for time instants greater than t implies that function $y(t)$ must be known over the interval $(t-3\tau,\ t)$. This is related to an infinite-dimensional system that presents an infinite number of Lyapunov exponents, from which only a finite portion of them can be determined by numerical analysis (Vicente *et al.*, 2005). Concerning the stability analysis of the UPO, however, it is enough to determine only the largest Lyapunov exponent (Pyragas, 1995).

In this work, the calculation of Lyapunov exponents is conducted by approximating the continuous evolution of the infinite-dimensional system by a finite number of elements where values change at discrete time steps (Farmer, 1982). In this regard, the function $y(t)$ over the interval $(t-3\tau,t)$ can be approximated by $N\text{-}1$ samples taken at intervals $\Delta t = 3\tau/(N-1)$. Therefore, instead of the two variables presented in Eq. (6), $N+1$ variables are now considered and represented by vector $\mathbf{z}$, where components $z_3,\ldots,z_{N+1}$ are related to delayed states of $y(t)$:

$$\mathbf{z} = (z_1, z_2, \ldots, z_{N-1}, z_N, z_{N+1}) = (x(t), y(t), y(t-\Delta t), \ldots, y(t-(N-1)\Delta t)) \tag{7}$$

There are different possibilities to perform this approximation. Here, the DDE is replaced by a set of ODEs following the procedure proposed by Sprott (2007). Under this assumption, the continuous infinite-dimensional system, Eq.(6), is represented in terms of $N+1$ finite-dimension ODEs:

$$\begin{aligned} \dot{z}_1 &= Q(z_1, z_2) \\ \dot{z}_2 &= P(z_1, z_2) + F(z_2, z_{(N-1)/3+2}, z_{2(N-1)/3+2}, z_{N+1}) \\ \dot{z}_i &= N(z_{i-1} - z_{i+1})/2T, \quad \text{for } \ 2 < i < N+1 \\ \dot{z}_{N+1} &= N(z_N - z_{N+1})/T \end{aligned} \tag{8}$$

where $N = 3\tau/\Delta t + 1$ and $T = 3\tau$. This system can be solved by any of the standard integration methods such as the fourth-order Runge-Kutta, and Lyapunov exponents can be calculated by using the algorithm proposed by Wolf *et al.* (1985). Moreover, in order to calculate the Lyapunov exponent of a specific UPO, the system follows the desired orbit as a fiducial trajectory, which can be done by using a time series.

4. NONLINEAR PENDULUM

As a mechanical application of the ETDFC method, a nonlinear pendulum, shown in Figure 1, is considered. The motivation of the proposed pendulum is an experimental set up discussed in De Paula *et al.* (2006). A mathematical model is developed to describe the pendulum dynamical behavior while the corresponding parameters are obtained from the experimental apparatus. Numerical simulations are employed in order to obtain time series related to the pendulum response assuming the uncontrolled situation, $K = R = 0$. Unstable periodic orbits are indentified from this time series using the close return method. Afterwards, control parameters are estimated for each UPO from the calculation of the Lyapunov exponents and their control is then simulated via extended time-delayed feedback control.

The nonlinear pendulum consists of an aluminum disc (1) with a lumped mass (2) that is connected to a rotary motion sensor (4). A magnetic device (3) provides adjustable energy dissipation. A string-spring device (6) provides torsional stiffness to the pendulum and an electric motor (7) excites the pendulum via the string-spring device. An actuator (5) is considered in order to provide the necessary perturbations to stabilize this system.

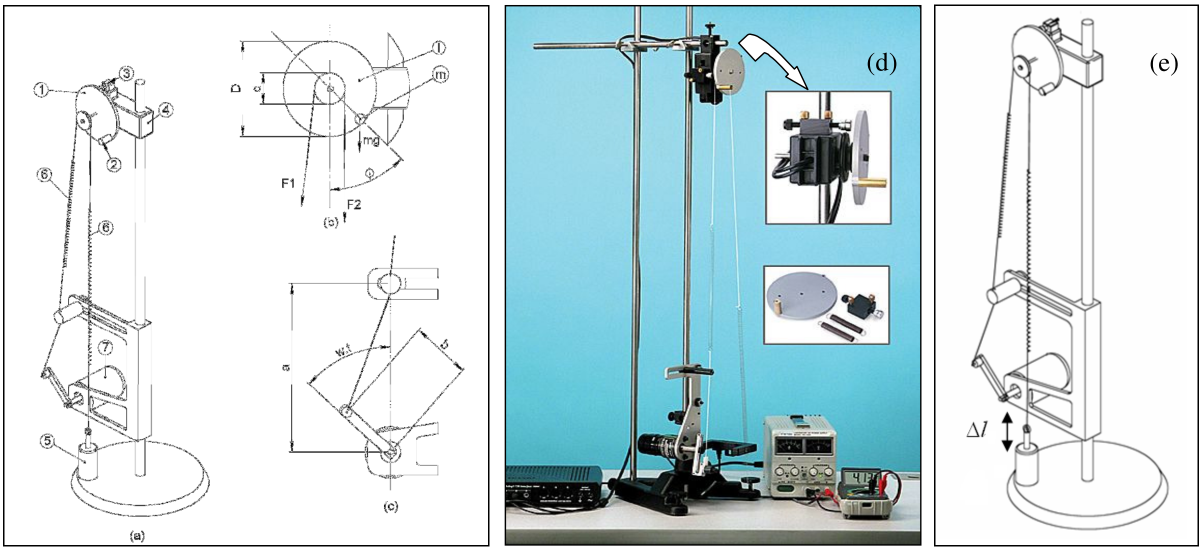

Figure 1. Nonlinear pendulum. (a) Physical Model. (1) Metallic disc; (2) Lumped mass; (3) Magnetic damping device; (4) Rotary Motion Sensor; (5) Actuator; (6) String-spring device; (7) Electric motor. (b) Parameters and forces on the metallic disc. (c) Parameters from driving device. (d) Experimental apparatus. (e) Actuator.

The pendulum dynamics is treated from a mathematical model by describing the time evolution of the angular position, ϕ. By assuming that ϖ is the forcing frequency, I is the total inertia of rotating parts, k is the spring stiffness, ζ represents the viscous damping coefficient and μ the dry friction coefficient, m is the lumped mass, a defines the position of the guide of the string with respect to the motor, b is the length of the excitation arm of the motor, D is the diameter of the metallic disc and d is the diameter of the driving pulley, the equation of motion is given by (De Paula *et al.*, 2006):

$$\ddot{\phi} + \frac{\zeta}{I}\dot{\phi} + \frac{\mu\,\mathrm{sgn}(\dot{\phi})}{I} + \frac{kd^2}{2I}\phi + \frac{mgD\sin(\phi)}{2I} = \Delta f(t) - \frac{kd}{2I}\Delta l(t) \tag{9}$$

where $\Delta f(t) = \frac{kd}{2I}\left[\sqrt{a^2 + b^2 - 2ab\cos(\varpi t)} - (a-b)\right]$ represents the forcing excitation; $\Delta l(t)$ is related to the perturbation provided by the linear actuator representing the length variation in the string (see Figure 1(e) for details). Moreover, the length variation may be evaluated by considering the extended time-delayed feedback control law, presented in Eq.(3), and the pendulum equations of motion, Eq.(9). Therefore:

$$\Delta l(t) = -\frac{2I}{kd} K[(1-R)S_\tau - \dot{\phi}] \tag{10}$$

where $S_\tau = \dot{\phi}_\tau + R\dot{\phi}_{2\tau} + R^2\dot{\phi}_{3\tau}$.

At this point, it is important to mention that the dry friction is modeled as a continuous function by assuming a relation as follows (De Paula *et al.*, 2006):

$$\mu\,\mathrm{sgn}(\dot{\phi}) = \mu\frac{2}{\pi}\arctan(10^6\dot{\phi}) \tag{11}$$

This mathematical model represents the pendulum dynamics and its numerical simulations are in close agreement with experimental data as can be observed in De Paula *et al.* (2006). Here, it is assumed the same parameters used in De

Paula *et al.* (2006) for all numerical simulations: $a = 1.6\times10^{-1}$ m; $b = 6.0\times10^{-2}$ m; $d = 4.8\times10^{-2}$ m; $D = 9.5\times10^{-2}$ m; $m = 1.47\times10^{-2}$ kg; $I = 1.738\times10^{-4}$ kg m^2; $k = 2.47$ N/m; $\zeta = 2.368\times10^{-5}\ \mathrm{kg\,m^2s^{-1}}$; $\mu = 1.272\times10^{-4}\ \mathrm{N\,m}$; $\omega = 5.61\,\mathrm{rad/s}$.

4.1. Calculation of the Lyapunov Exponent

In order to calculate Lyapunov exponents, it is proposed an alternative representation of the system. By assuming $x_1 = \phi$, $x_2 = \dot{\phi}$ and employing $x_3,\ldots,x_{N+1}$ for the approximation of $\dot{\phi}$ over the interval $(t-3\tau, t-h)$, the set of equations associated with Eq.(8) is given by:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -\dfrac{kd^2}{2I}x_1 - \dfrac{\zeta}{I}x_2 + \dfrac{kd}{2I}\Delta f(t) - \dfrac{mgD\,\mathrm{sen}(x_1)}{2I} - \dfrac{2\mu}{\pi I}\arctan(10^6 x_2) + \\ \qquad + K[(1-R)(x_{(N-1)/3+2} + R\,x_{2(N-1)/3+2} + R^2 x_{N+1}) - x_2] \\ \dot{x}_i = N(x_{i-1} - x_{i+1})/2T, \quad \text{if } 2 < i < N+1 \\ \dot{x}_{N+1} = N(x_N - x_{N+1})/T \end{cases} \tag{12}$$

where $N = 3\tau/h + 1$, $T = 3\tau$ and h is the integration time step. Note that $\Delta l \dfrac{kd}{2I} = K[(1-R)(x_{(N-1)/3+2} + R\,x_{2(N-1)/3+2} + R^2 x_{N+1}) - x_2]$ is related to the actuator perturbation. This N+1 first order ODE system is then numerically integrated by using the fourth-order Runge-Kutta method and the largest Lyapunov exponent is calculated using the algorithm proposed by Wolf *et al.* (1985).

The governing equations shown in Eq.(12) together with the system linearization form a set of $(N+1)^2+(N+1)$ ODEs. This system allows one to perform the calculation of the largest Lyapunov exponent by considering a set of (N+1) ODEs. Besides this, it is important to be pointed out that the fiducial trajectory associated with the original system (x_1, x_2) is replaced by a time series that represents the desired UPO, which reduces the system to a set of (N−1) ODEs.

In principle, the stabilization of the desired UPO can be achieved for control parameters that are related to negative values of the largest Lyapunov exponent and these parameters need to be chosen for values of the Lyapunov exponents near to its minimum. Therefore, in order to verify the capability of the ETDFC method to stabilize the desired UPO, the largest Lyapunov exponent of this orbit is calculated for different control parameter values, K and R.

5. NUMERICAL SIMULATIONS

Numerical simulations of the nonlinear pendulum are carried out in order to evaluate the capability of the ETDFC method to stabilized desired UPOs. All simulations use experimentally identified parameters obtained in De Paula *et al.* (2006). In the first stage of the control strategy, UPOs embedded in the chaotic attractor are identified using the close return method. After this identification, the largest Lyapunov exponent is calculated considering different control parameter values for each UPO of interest in order to find regions related to negative exponents. After the learning stage, the control stage starts where the actuator is perturbed in order to achieve system stabilization.

Initially, a period-1 UPO is of concern. Figure 2 shows this orbit and the largest Lyapunov exponent value evaluated for different values of the control parameters, R and K. This analysis indicates that stabilization can be achieved for all values of analyzed R, including R = 0 that represents the TDFC. System stabilization is now focused on by choosing control parameters that correspond to negative Lyapunov exponent. Figure 3 shows phase space, time history response and the actuator perturbation for R = 0 and K = 2.1, which is close to the minimum value of the largest Lyapunov exponent. The controller is capable to achieve the UPO stabilization and this is due to the low periodicity of the UPO.

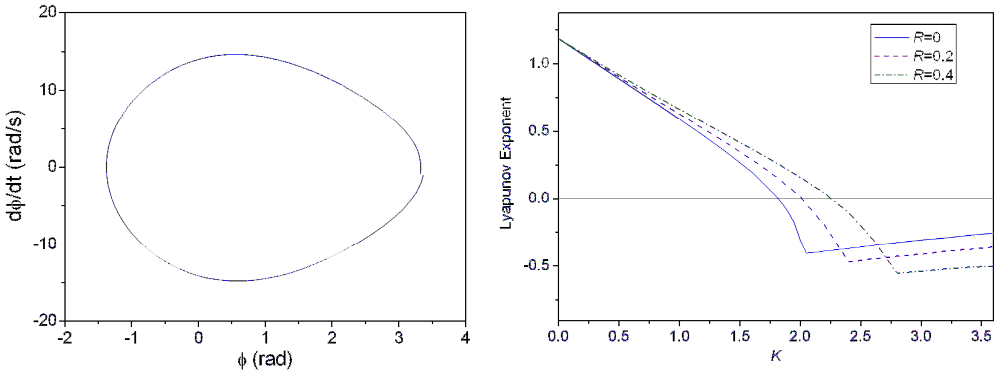

Figure 2. Period-1 UPO and the largest Lyapunov exponent for different control parameters.

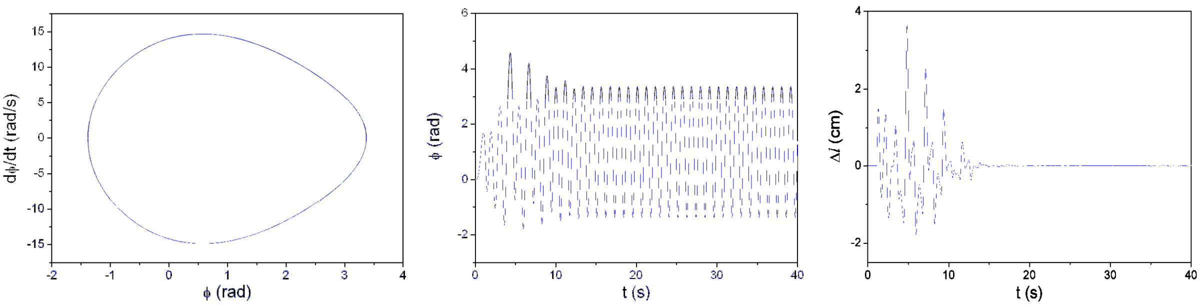

Figure 3. Period-1 UPO stabilized for R = 0 and K = 2.1:
(a) Phase space; (b) Time response; (c) Perturbation.

Let us now consider a period-2 UPO, presented in Figure 4 together with the largest Lyapunov exponent for different values of the control parameters, R and K. In this case, the stabilization of the orbit cannot be achieved for R = 0 since there is not a negative Lyapunov exponent associated with this parameter. This result shows the difference between the TDFC and ETDFC, highlighting the importance of the inclusion of parameter R in the definition of the control law. The stabilization of this period-2 UPO is presented in Figure 5 that shows phase space, time history response and the actuator perturbation for R = 0.2 and K = 1.1. Under this condition, the maximum Lyapunov exponent is close to the minimum value for this orbit, allowing the orbit stabilization.

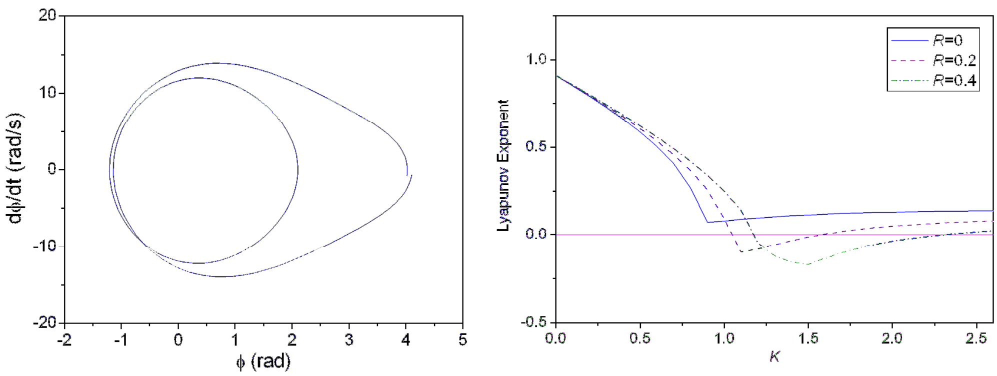

Figure 4. Period-2 UPO and the largest Lyapunov exponent for different control parameters.

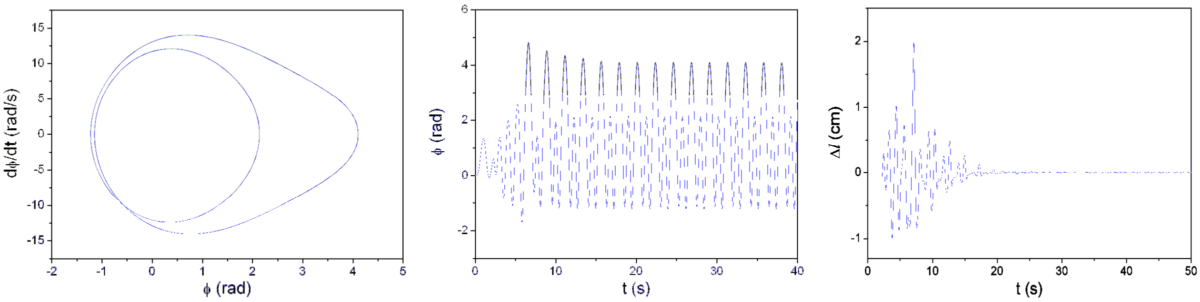

Figure 5. Period-2 UPO stabilized for R = 0.2 and K = 1.1:
(a) Phase space; (b) Time response; (c) Perturbation.

A period-3 UPO is now focused on. Figure 6 presents the UPO together with the largest Lyapunov exponent for different values of R and K. Once again, it is possible to identify regions associated with negative Lyapunov exponent which allows the UPO stabilization. Figure 7 shows phase space, time history response and the actuator perturbation for the stabilized period-3 UPO with R = 0.2 and K = 0.7.

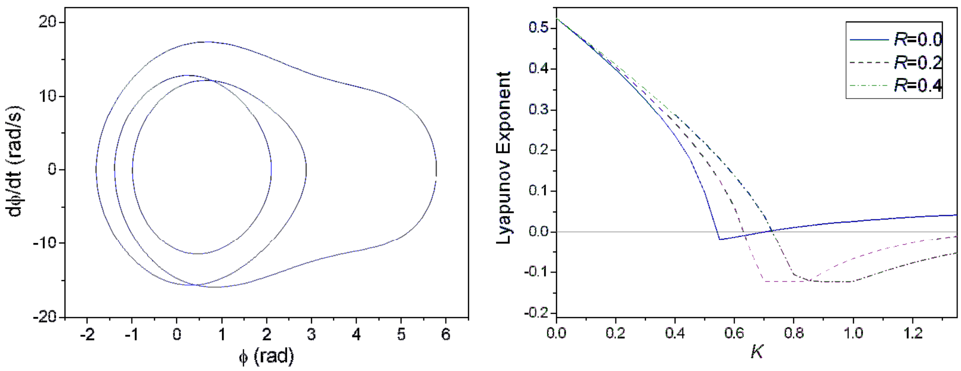

Figure 6. Period-3 UPO and the largest Lyapunov exponent for different controller parameters.

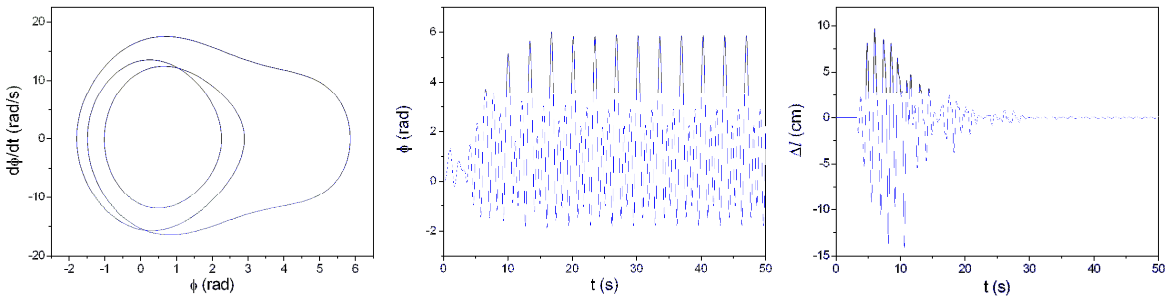

Figure 7. Period-3 UPO stabilized for R=0.2 and K=0.7:
(a) Phase space; (b) Time response; (c) Perturbation.

6. CONCLUSIONS

The extended time-delayed feedback control (ETDFC) method is applied to a nonlinear pendulum in order to stabilize the system trajectory into an UPO embedded in the chaotic attractor. The control law is associated with DDEs that contain derivatives that depend on the solution at previous time instants and consist of an infinite-dimensional system. A fourth-order Runge-Kutta method with linear interpolation on the delayed variables is used for the numerical integration of the DDEs. Delayed terms related to time instants $(t-\tau)<0$ are replaced by a Taylor series expansion. During the learning stage, the UPOs are indentified by the close-return method and control parameters are chosen for each desired UPO by defining situations where the largest Lyapunov exponent becomes negative. The Lyapunov exponent determination uses an alternative representation of the nonlinear pendulum where the continuous evolution of the infinite-dimensional system is approximated by a finite number of elements where values change at discrete time steps. This approximation allows one to change the DDE by a set of ODEs. Numerical simulations are carried out assuming experimentally identified parameters. Chaos control is applied to different UPOs and results show that the ETDFC method is effective in order to stabilize UPO embedded in the chaotic attractor. The TDFC method ($R = 0$), on the other hand, achieves the stabilization only for a period-1 UPO. For orbits with higher periodicity, only the ETDFC method ($0 < R < 1$) is capable to achieve system stabilization.

7. ACKNOWLEDGEMENTS

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