SIMULATED ANNEALING FOR DAMAGE DETECTION

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Abstract. *The method of simulated annealing (SA) is a technique that has attracted significant attention as suitable for large scale optimization problems, especially where a desired global extreme is among many minima locals. The main objective of this paper consists in evaluating the performance of SA method for localization and quantification of damage in structural members using the concept of changes in structure's natural frequencies. The performance of method was analyzed crossing the results with the experimental tests in some metallic bars with simple support conditions. Different damage scenarios were tested in order to test the program in different conditions. In this case, the performance function is evaluated using the structure's experimental natural frequencies and the numerically calculated natural frequencies, obtained by a parametric numerical model. The performance of methods was available with comparisons between the actual damage state and the predictable ones in the analyzed tests. Another heuristic algorithm was used: a real coded genetic algorithm (GA). Comparisons between the genetic algorithm and SA were accomplished so that the processing time and accuracy of method could be evaluated regarding the site and level of the damage.*

Keywords*: simulated annealing, damage detection, natural frequencies, genetic algorithm.*

1. INTRODUCTION

Structure health monitoring has been receiving increasing interest in both academic research and industry applications for several decades. This works has been dedicated to deal with the problem of damage detection, evaluating the location and extent of damage in elastic structures by using the changes in their modal parameters, specifically, natural frequencies, modal damping and associated modal shapes, see Lin and Cheng (2007). The most mentioned used method use the changes in the natural frequencies. It is widely recognized that the natural frequencies are least contaminated by noise and can be captured with accuracy about 0.15%. The modal damping and modal shapes has a bigger accuracy than natural frequencies. It's about twenty times worst; see Friswell and Penny (1997).

In this last decade, a lot of works has been made with heuristic methods to optimize cost functions and to solve the problem of damage detection. In most cases, the objective function is based on the difference between numerically evaluated and experimentally measured shifts on modal frequencies, in order to match their results. The numerical differences could come from a FEM (Finite Element Method) model. When the differences in frequencies match, the FEM model stiffness will indicate the location and extension of damage; see Doebling *et al.* (1996).

2. PRINCIPLES OF SIMULATE ANNEALING

Recently, a number of global optimization methods have emerged as promising strategies to deal with complicated optimization problems like damage detection. The most popular of them are genetic algorithm (GA), evolution strategies (ES), evolutionary programming (EP), simulated annealing (SA) and some others. The common approach shared by all of these techniques is the similarity with physical processes and the avoidance of gradient-based search and thus reducing the possibility of getting stuck in the local optima.

The optimization task in the SA is carried out by a heuristic approach, where the background theory to the annealing process of physical systems in thermodynamic can be used. In this process, a physical system (a solid or a liquid) initially at a high-energy state is slowly cooled to reach the lowest energy state, see Hasançebi and Erbatur (2002). The main idea is to give time and energy to the molecules to organize themselves in the way of a lower energy state. The idea, which was used to solve other optimization problems, was suggested independently by Kirkpatrick *et al.* (1983) and Cerny (1985) by the analogy between the decrease the energy in the annealing process and later extended to lowering any other objective functions. The wide applicability and success of the SA has been empirically tested and verified on a variety of different disciplines in the literature, including structural optimization, damage detection, optimization of biomechanical systems, see Higginson *et al.* (2005). SA executes a search for the optimal solution by randomly altering the control variables within a neighborhood of the solution space and evaluating the performance of objective function.

The acceptance criteria for a new optimum variables are two: (a) based on the cost function, all set of variables that results cost function values in optimum direction are accepted; (b) for those set of variables that are against the optimization direction, the Metropolis criterion is used(1) to accept or not this new set as an optimum. This is a crucial part of the method. In case of minimization tasks, when the probability of a set of variables has a increase in the cost

function value M (decreasing the temperature) is greater than a random number (between 0 and 1), the point is accepted, and otherwise, the point is not accepted.

$$Prob(E_{i+1} > E_i) = e^{\left(\frac{\Delta E}{K.T}\right)}, \quad \left\{ \begin{array}{ll} \text{Accepted} & if \quad e^{\left(\frac{\Delta E}{K.T}\right)} > r \\ \text{Not\_accepted} & if \quad e^{\left(\frac{\Delta E}{K.T}\right)} \leq r \end{array} \right\} \tag{1}$$

where r is a random number generated between *0* and *1*, and ΔE means the energy difference between a state and another disturbed state, K is the Boltzman Constant related to the physical process (in this paper this constant was set to *1.0*, and this compensated in the initial temperature choice so that at early stages the rate of acceptance falls about *0.4* to *0.6*).

Another important criterion is the way the states are generated. The simplest way is to set upper and lower limits to the variables and raffle a number between these bounds. This length of the interval can be changed during the iterations so it can assume large values initially and than short ones later in order to keep a pre-set rate of acceptance.

With this criterion, the algorithm can escape from local optima and reach easily around the optimum point. In the analogy with the physical annealing process, the difference of energy is the difference of objective function. Keeping in mind the similarities and differences, the process can be easily applied to other cases of optimizations. Due to the stochastic heuristics involved in the algorithm, SA is able to overcome local optima and is relatively insensitive to the initial guess for the optimized variables; see Higginson *at al.* (2005). The convergence is given when an enough number of performance evaluations have been made without any improvement in the optimum. The algorithm can also end if a pre-set number of evaluations have been accomplished without convergence to any optimum.

3. BASICS OF GENETIC ALGORITHMS

Genetic Algorithms (GA) are optimization techniques based on the Darwin's Theory of evolution and survival of the fittest. The Darwin's Theory of Natural Selection says that: "… any being, if it varies slightly in any manner profitable to itself, will have better chance of surviving…" (Holland, 1975) GA simulates the evolutionary process numerically. They represent the parameters in a given problem by encoding them into a string. As in genetics, chromosomes are constituted by genes. Similarly, in simple GA, encoded strings are composed of bits. A string of bits can be decoded to the respective problem parameter value and the total evaluation of the string of bits for and individual may be weighted following some fitness function representing the phenotype to that string of bits. A simple genetic algorithm consists of three basic operations, these being reproduction, crossover and mutation. The algorithm begins with a population of individuals each of them representing a possible solution of the problem. The individuals, as in nature, perform the three basic operations and evolve in generations where it prevail the Darwin's Theory or in other words, a population of individuals more adapted emerges as result of natural selection. At the reproduction level, the evaluation of the objective (fitness) function indicates which individuals will have more chances to procreate and generate a larger offspring.

In the genetic operations, the genes of pair individuals are exchanged and as in nature this may be performed by several ways being called by crossover.

Differences between conventional techniques and the genetic algorithm (GA) can be summarized as follows (Goldberg, 1989):

-GA operates on coded form of the task parameters;

-GA works with a population which represents numerical values of a particular variable;

-Differently of most of optimization algorithms which requires objective functions evaluations, GA only requires the use of the objective function;

-Only probabilistic rules of natural selection are used with GA;

The binary representation has a historical importance due to first uses by Holland, (1975). When working with binary coded genetic algorithms, each of the real parameters bi to be optimized is translated to binary codes by the following Eq. (2):

$$s = bin_n[round(2^n - 1)\frac{(b_i(k) - P_{\min}(k))}{(P_{\max}(k) - P_{\min}(k))}] \tag{2}$$

where bin_n indicates a binary translation to a string s of n bits, n means the number of bits, $P(k)$ means maximum and minimum values allowed for each variable. To transform the binary codes to real values the following Eq. (3) is used in the sequence, see Silva *et al.* (2006):

$$b_i(k) = x(k)_{\min} + bin^{-1}(s)\frac{x(k)_{\max} - x(k)_{\min}}{2^n - 1} \tag{3}$$

where $bin^{-1}(s)$ means the translation of the binary coded values to respective real ones. It could be noted that with this formulation it is implicit that the mapping has a resolution of $((x(k)_{max} - x(k)_{min})/(2^n-1))$. This restricts the search space of the real parameters to discrete values which could induce to local maxima.

This could be outlined by using real coded genetic algorithms. This approach assumes real values to each variable. The main differences are found on the crossover operator. There are several methods to deal with the real coded genetic algorithms crossover such as flat crossover, simple crossover, arithmetical crossover, Wright´s crossover, linear BGA crossover, etc. In this paper the BLX-α is used because it uses an initial exploration of the parameters field followed by an exploitation phase to improve resolution. It may be described by:

$$\begin{aligned} &\Delta = \max[b_i(k), b_{i+1}(k)] - \min[b_i(k), b_{i+1}(k)] \\ &b(k) = \mathit{randon}\{\min[b_i(k), b_{i+1}(k)] - \alpha\Delta, \max[b_i(k), b_{i+1}(k)] + \alpha\Delta\} \end{aligned} \tag{4}$$

where, i and $i+1$ are referred to two parents' chromosomes, α means a decreasing exploration parameter and random() means a random number in the corresponding interval.

4. USING SA FOR DAMAGE DETECTION

The algorithm was programmed in FORTRAN90 (Chapman, 1995) based on the Goffe (2008) code. The objective of SA is to maximize an objective function based on natural frequencies. In this problem, a 24 dimension problem is available, because the damage on each of the 24 discretized finite element of the used parametric model are design variables.

4.1. Heuristic parameters

The first test performed using a heuristic algorithm is the determination of heuristics parameters. This parameters change for each available problem. In SA, these parameters are: initial temperature, convergence criterion, cooling schedule, stop criterion and the final temperature. These parameters interfere in the process changing the elapsed processing time and changing the possibility of premature convergence of the process (sometimes the problem may not converge as a result of the parameters choice). Some authors suggested values to start the execution the program (see Goffe, 2008), but some experience is necessary to determine the best values to specific problems.

•Initial Temperature: the initial temperature in the SA is a very important parameter. The temperature must to be enough for the program accept more than 50 % of optimization attempts. If the temperature is not enough, the algorithm can stay in the optima local. If it is very large, time to convergence might increase. In the paper the initial temperature used was the dimensionless value1.0.

•Convergence criterion: the convergence criterion can be targeted when there are not changes (a small threshold is adopted) in the objective function value during a determined number of iterations. In the analyzed cases, the determined number was 4.

•Cooling Schedule: there are many ways to decrease the temperature in the process. If the temperature decreases quickly, this can hinder the program to search for others optima variables. In the other hand, if the program spends a lot of time to decrease temperature, this may slow down the algorithm processing time and convergence. Although there are many cooling schedule schemes proposed in the literature, this work investigates the use of just three ways to decrease the temperature as described by the following Eq. (5), (6) and (7):

$$T_{j+1} = T_{initial} / ln(j) \tag{5}$$

$$T_{j+1} = T_{initial}^{1/\sqrt{j}} \tag{6}$$

$$T_{j+1} = \beta T_{initial} \tag{7}$$

where T_{j+1} means the temperature in the next iteration, j means number of iteration, β means a real constant in the range $(0,1)$. The value of β defines the curvature of the temperature decreasing curve. This parameter was tested for several cases.

- Stop criterion: this is the criterion to stop the program if some problem happens. It is based on the number of evaluations of the objective function. In this paper the parameter used was $1.0x10^5$.
- Final temperature: it must be as low as possible. In this work there is not a determined final temperature, this value is attained as a consequence of the program iterations. If the convergence criterion was not reached, the final temperature still decreases to get the optimum point.

4.2. Tested Cost Functions

The objective function should be chosen for all the optimization methods. Both methods used to solve the problem (GA and SA) were tested with the same cost functions.

There are many different objective functions for the damage detection approach suggested by the literature, see Doebling *et al.* (1996). The main idea is common for all of them. One of the differences is the use of natural frequencies or modal shapes to match the numerical and experimental values. In this paper, just functions that use the natural frequencies are shown, due the well-known poor accuracy in experimentally of modal shapes evaluation.

If the numerical and experimental values match each other, within the range of mode numbers considered, the cost function index yields *1* and if the opposite happens, the cost functions tends to *0*.

Two cost functions were analyzed in order to verify the algorithm performance. Both are shown above:

The first one was the MDLAC index (Multiple Damage Location Assurance Criterion), which was suggested by Messina *et al.* (1998):

$$MDLAC(\delta\mathbf{D}) = \frac{\left|\Delta\boldsymbol{\Omega}^T.\delta\boldsymbol{\Omega}(\delta\mathbf{D})\right|^2}{(\Delta\boldsymbol{\Omega}^T.\Delta\boldsymbol{\Omega}).[\delta\boldsymbol{\Omega}(\delta\mathbf{D})^T.\delta\boldsymbol{\Omega}(\delta\mathbf{D})]} \tag{8}$$

where $\Delta\Omega$ means a vector of experimental natural frequencies differences between the damaged and undamaged structure; $\delta\Omega$ means a vector of numerical natural frequencies differences between the damaged and undamaged structure; δD means a vector of multiple damage for the used parametric model. It has the size of the number of probable damaged sites allowed in the parametric model

The function below was suggested by Silva (2006). The advantage in this formulation lies in the normalization of the numerical and experimental vector of frequency differences by the maximum of the vector, respectively, as indicated by Eq. (9).

$$f(\delta\mathbf{D}) = \frac{1}{1+\sum_{i=1}^{N}\left(\frac{\delta\boldsymbol{\Omega}_i(\delta\mathbf{D})}{\max(\delta\boldsymbol{\Omega}_i)} - \frac{\Delta\boldsymbol{\Omega}_i}{\max(\Delta\boldsymbol{\Omega}_i)}\right)^2} \tag{9}$$

where *N* means the number of considered vibration modes and *i* is the mode order number. The same functions values ranges applies as far for the MDLAC function.

5. SOME COMPARISONS WITH GENETIC ALGORITHMS

Some comparisons between the genetic algorithm and simulated annealing were made in order to verify the differences and performance to solve damage detection scenarios. The details about the GA were not described because these are not the main goal of this work. Some information regarding the heuristic parameters used in GA, such as, number of individuals, number of generations, probability of cross-over etc, are reported in Tab. 1.

Table 1. Heuristic parameters of the genetic algorithm used in the comparisons.

Nº of individuals	200
Nº of generation	2000
Crossover Probability	100%
Mutation Probability	1%

5.1. Numerical Tests

The numerical test was accomplished in order to verify the performance of the methods. In this example, a simple supported beam with rectangular cross sectional area with height $h=0,24m$, width $b=0,14m$ and length of $L=2,4m$. The material has a Young Modulus of $E=2.5x10^{10}\ N/m^2$ and a material density of $\rho= 2.5x10^3\ kg/m^3$. The simplicity of this example resembles an experimental prototype in reduced scale that is being tested at the laboratory.

Figure 1 shows a sketch of the structural dimensions and the numbering of the discretized elements used in the finite element analysis. It was used 3D beam elements with six degrees of freedom per node, restrained in the plane of the structure, resulting in three degrees of freedom per node (two translational and one rotational).

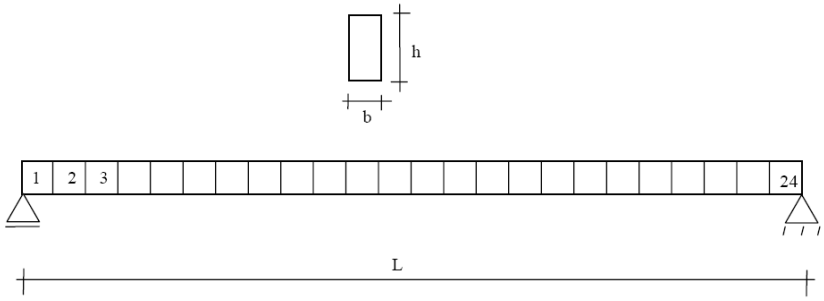

Figure 1. Single supported beam and cross section dimensions.

6. EXPERIMENTAL TEST

In the experimental test, a simple cantilever beam was used and tested, as indicated by Fig. 2.

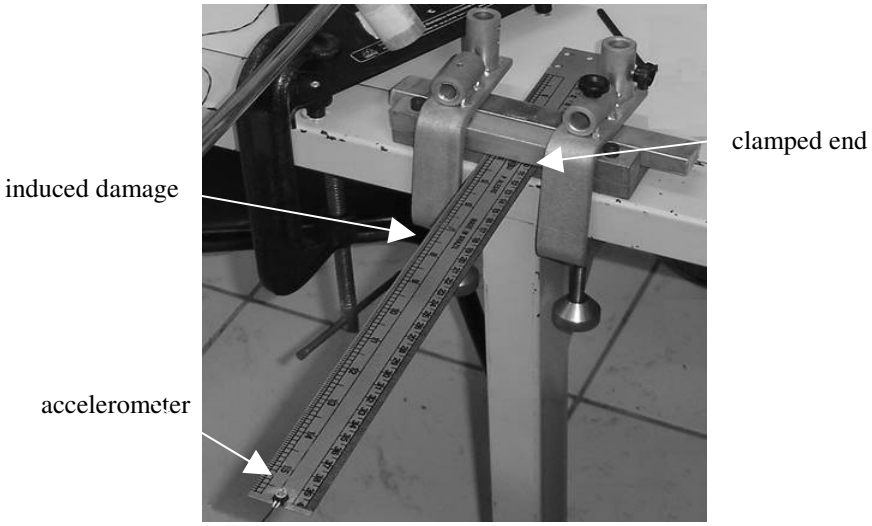

Figure 2. Experimental Setup and Cantilever beam used in the experimental test.

The structure has *35,2mm* x *0,9mm* cross section and is *303,5mm* length. The damage was introduced at *227.5mm* from the free end. The measured Young Modulus and mass density was *155GPa* and *7800 kg/m*^{3} respectively. The damage was increased by steps cutting the edges and the first seven natural frequencies were measured. The natural frequencies were measured with aid of a micro accelerometer (Endevco 2250A M1-10 model, *10 mV/g* sensitivity and mass of *0.4g*). The software used to measure and evaluate the FFT of the accelerometer's signal was Agilent Vee 7.0 (2005).

7. NUMERICAL TEST

The numerical test consists in assuming some damage scenarios (numerically induced damage on single or several elements in turn). Then the numerically evaluated frequencies for the healthy and damaged structure were used as input parameters for the SA or GA algorithms. The way the damage was induced on the elements consists in reducing the element stiffness by a defined amount. Generally speaking, the performance of numerical tests was good, the damaged scenarios were found by SA and GA. It was spent about 10 seconds for the worst cases with a Pentium IV, 3.0GHz, 1 GB RAM.

To investigate the algorithms, it was first assumed a vibration standard of 5 modes, it means, the first five frequencies of the damaged structure were used as input for the SA and GA maximize the cost function. When was necessary others modes to identify the damage, this number was indicated on the Figures.

The numerical frequencies are shown in Tab. 2 for each damage condition. As predicted, all the frequencies decreased with the increase of damage.

Table 2. Numerical frequencies for each damage condition.

Numerical Frequencies (Hz)							
	1	**2**	**3**	**4**	**5**	**6**	**7**
Undamaged	7,08	44.38	124.27	243.52	402.56	601.40	840.07
10% element 6	7.05	44.38	123.97	226.66	401.89	601.28	838.23
20% element 6	7.01	44.38	123.60	241.45	401.09	601.13	835.99
30% element 6	6.97	44.37	123.13	240.07	400.12	600.93	833.23
40% element 6	6.90	44.37	122.53	238.35	398.94	600.68	829.73
50% element 6	6.82	44.36	121.73	236.15	397.46	600.32	825.16
70% element 6	6.51	44.34	118.91	229.14	392.98	598.87	810.21

In the figures below, it is possible to see the performance of both methods to maximize the objective function suggested by Eq. (9):

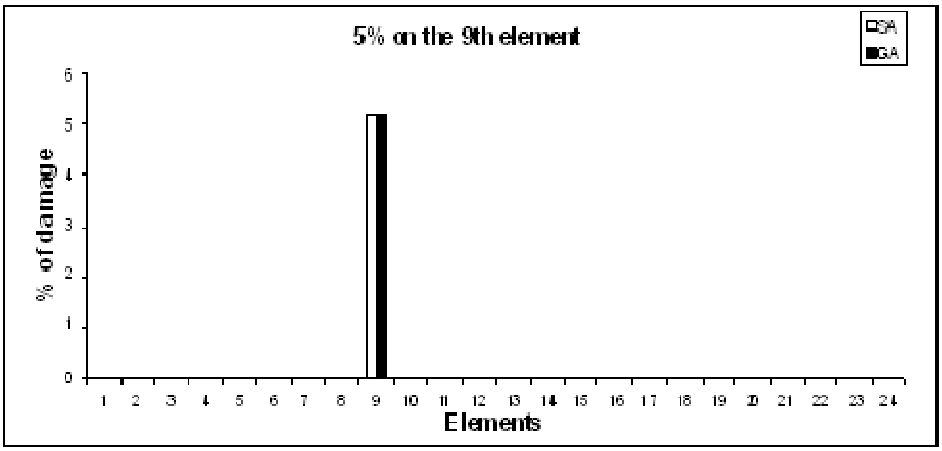

Figure 3. Bar chart of numerical test for 5% of damage on the 9th element.

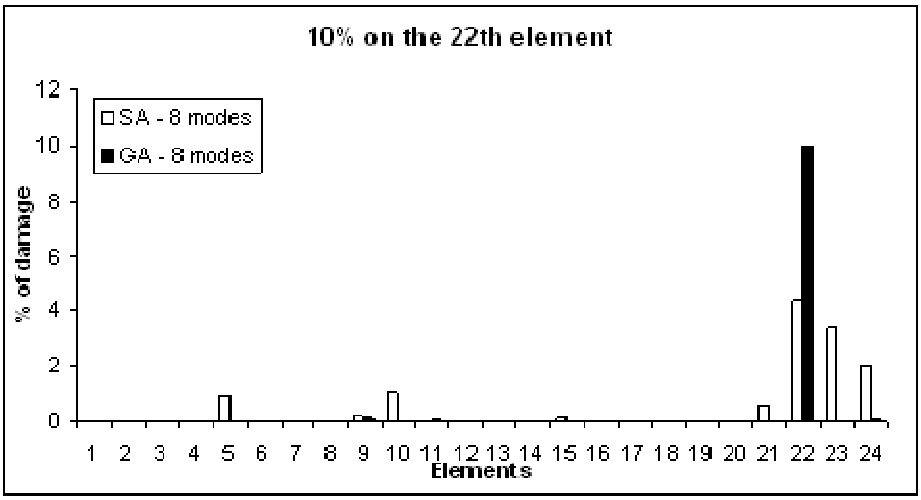

Figure 4. Bar chart of numerical test for 10% of damage on the 22th element.

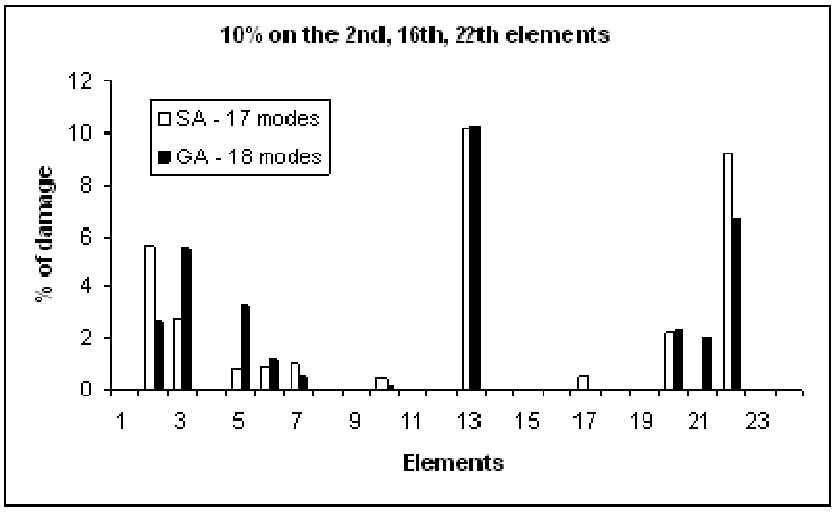

Figure 5. Bar chart of numerical test for 10% of damage on 2th, 16th and 22th elements.

For the localization of damage, just fell modes were necessary to find the site, but more modes were necessary to find the extent of damage. In the cases where the damage is near a boundary condition, it was more difficult to identify the damage. The same happens when the structure has more than one damaged element, such as in the case shown in Fig. 5.

8. EXPERIMENTAL TEST RESULTS

In this test, the experimentally measured frequencies in the cantilever beam were used as input parameters for the proposed algorithms and then the results are compared. The algorithms did not spend more than 10 seconds to find the damage as in the numerical tests using the same computer. It is important to say that in all cases, the algorithm converged, but for a number far of expected maximum point (cost function equal to one). To investigate the algorithm, it was first assumed a vibration standard of 7 modes, it that means, the first seven frequencies of the damaged structure were used as input for the SA and GA to maximize the objective function. On the studied cases, the cases of damage on the same site were made. Just the extent of then was increased to one case for the others.

A case with ten percent on the sixth element was investigated, but the algorithms couldn't identify the damage. Both algorithms show no damage on the structure for this case. It is easy to see that the location of damage is reasonable found, but the extent weren't identified in all cases. Just an idea of damage is given for both algorithms. This problem is cited for Messina *et al.* (1998) for the MDLAC index. The extent of damage had better values when using Silva's (2006) objective function. Table 3 shows the experimental frequencies are shown for the damage condition.

Table 3. Experimental frequencies for each damage condition.

Experimental Frequencies (Hz)							
	1	**2**	**3**	**4**	**5**	**6**	**7**
Undamaged	6.80	43.50	122.22	239.70	398.80	595.54	837.32
10% element 6	6.80	43.50	122.16	239.22	398.38	596.52	832.80
20% element 6	6.74	43.50	121.76	238.38	397.68	594.80	827.98
30% element 6	6.70	43.40	121.06	237.02	397.54	592.22	822.24
40% element 6	6.60	43.40	120.00	235.30	395.98	588.06	812.36
50% element 6	6.54	43.30	118.82	233.12	393.92	582.48	802.60
70% element 6	6.24	42.98	114.60	226.68	387.64	546.16	776.84

According to Tab. 3, all frequencies decreased or keep the same value with the increase of damage. Theoretically, all frequencies should decrease instead to keep the same value, but it can be justified for the presence of noise and uncertainties in the measured signal from the accelerometers. Figure 6 shows first frequency experimentally measured. Figures 7 to 9 show the results for the investigated damaged scenarious.

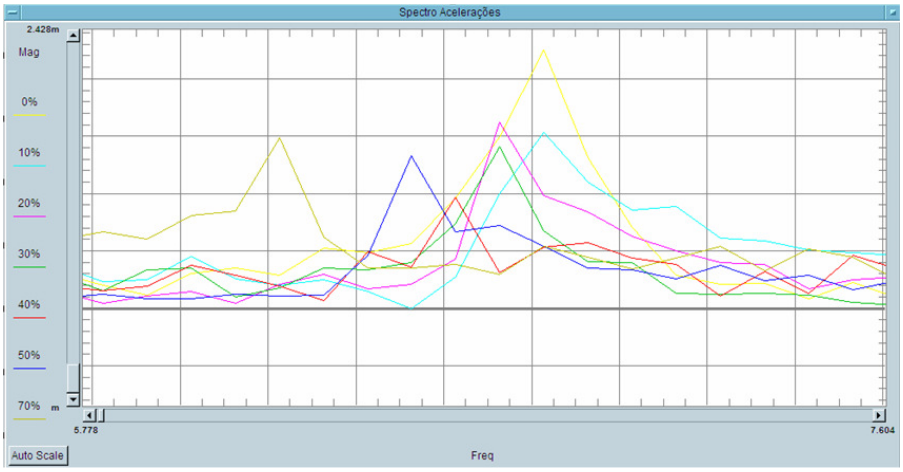

Figure 6. Acceleration Spectrum for the first frequency to all the damage studied cases.

On the figures below, it's possible to see the performance of both methods to maximize the objective function (9):

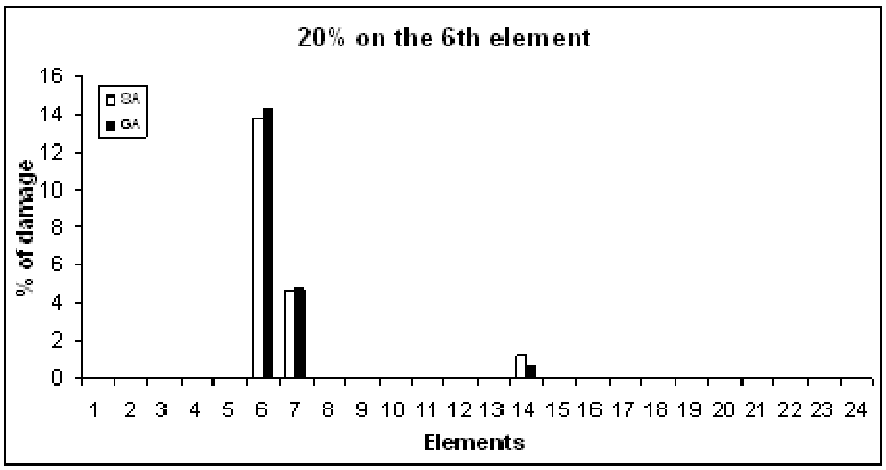

Figure 7. Bar chart of experimental test for 20% of damage on 6th element.

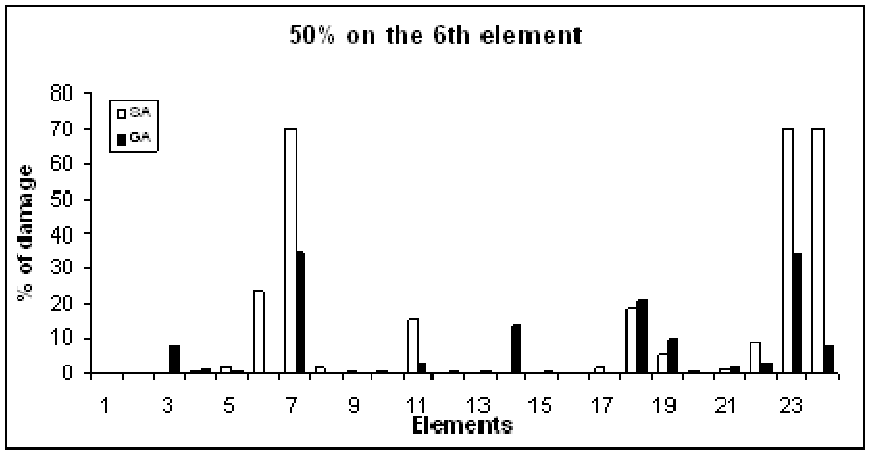

Figure 8. Bar chart of experimental test for 50% of damage on 6th element.

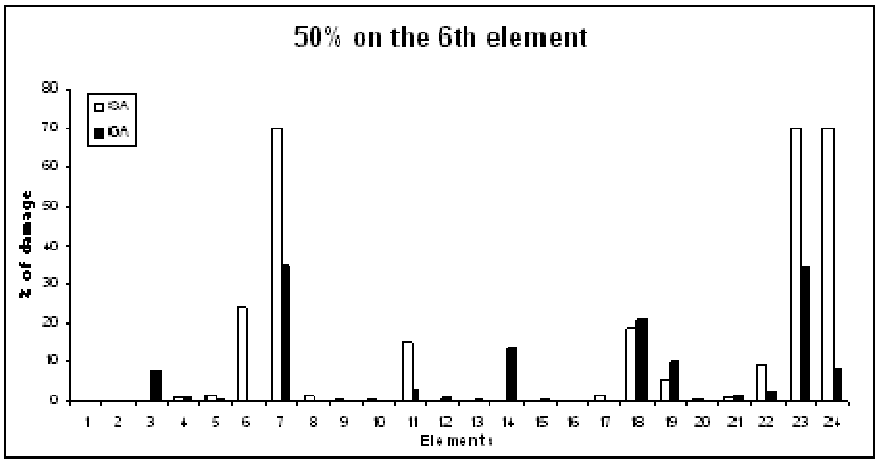

Figure 9. Bar chart of experimental test for 70% of damage on 6th element.

9. FINAL CONCLUSIONS

This paper has presented a benchmark test between the SA and GA for numerical and experimental tests. An investigation about processing time, robustness of methods regarding the noise in measurement, stability and accuracy was made in order to suggest the advantages and disadvantages using both methods. The performance of methods (GA and SA) was similar. Both methods were able to find the damage with a computation time about ten seconds for the numerical tests with nearly same accuracy. In some damaged cases the SA behaved better but in others the GA had similar performance as well. It was not possible to identify the reasons for this difference in performance.

In most of experimental tests, the site of damage was found but both algorithms did not evaluate the damage extent correctly. The experimental test suggests a special attention for the data quality. It seems that the accuracy of data was not enough to a good performance of both algorithms and this could have harmed the comparisons. This fact suggests that the algorithm used to optimize the objective does not have great importance when dealing with data with little accuracy. The methods did not show such a great robustness for the experimental studied cases, maybe because of already mentioned reasons.

10. ACKNOWLEDGMENT

The author acknowledges the CNPq financial support to this research.

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