DAMAGE IDENTIFICATION USING FREQUENCY RESPONSE FUNCTION

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Abstract. *This paper presents a methodology based on an iterative method that uses acceleration and bending moment frequency response functions (FRF), instead of using modal parameters, in order to show damage.*
A Goal Programming optimization software which minimizes the difference between experimental and numerical FRFs was developed in order to identify, locate and quantify the damage. Some numerical simulations were performed on a clamped beam to evaluate the efficiency of the method. The obtained results show good performance of the implemented methodology.

Keywords*: damage detection, model updating, goal programming*

1. Introduction

Structural identification has been used in the last decades with the aim of damage assessment of operational structures. The possibility of preventing the appearance of anomalies, which can jeopardize the structural integrity, has given confidence to the development of damage identification methodologies. There are many methods that can be used to locate damages in structures and an extensive review can be observed in Doebling *et ali* (1996), Hoons *et ali* (2003) and Salawu (1997). Most of them try to adjust numerical models to experimental data, based on dynamic parameters such as natural frequency, modal damping and modal shape by comparisons of different structural time conditions. Therefore, damages may be expressed through the observed discrepancies.

Despite the effort dispended, there is no spread out methodology that can be applied to locate and quantify damages. This paper presents a methodology to locate damage based on an iterative method, which allows the simultaneous adjustment of several physical parameters, by minimizing the difference between experimental and numerical responses for the analyzed structure and its corresponding numerical model. In order to do so, a computer program, which uses the Goal Programming optimization technique to choose the best set of variables that minimizes the difference between experimental and numerical Frequency Response Functions (FRFs) was implemented.

It is quite common to use just Acceleration FRFs in experimental analyses. However, in this paper, Bending Moment FRFs are also used in order to provide redundant information about the structural condition. Some numerical simulations were performed on a clamped beam to test the efficiency of this method. The results show that such method is efficient to locate and identify damages in the performed simulations.

2. Methodology of the damage identification

A finite element program was coupled with an optimization routine based on Goal Programming technique (Ignizio, 1976 and Neves, 1997) in order to identify damages (Fig. 1). Basically, the program performs comparisons between experimental and numerical FRF data. It means that based on experimental result, the program will search a set of variables that minimizes errors between these FRFs, obeying some physical restrictions for the control variables. Thus, the final set will represent the actual condition of the analyzed structure.

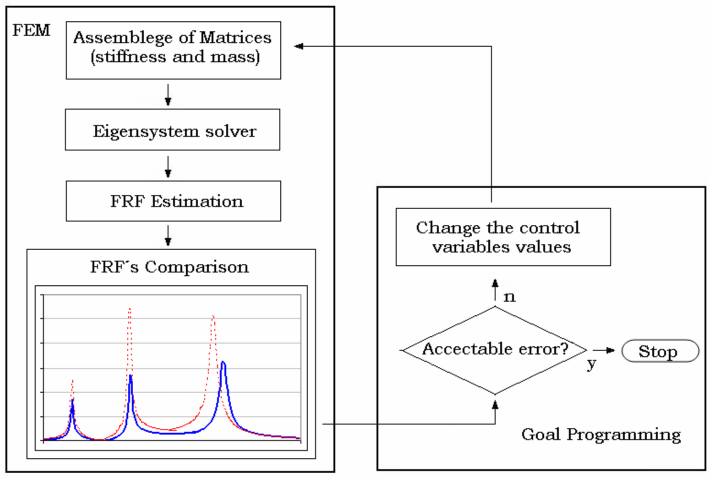

Fig. 1. Illustrative scheme of the software developed for damage identification.

The numerical FRFs are estimated from a random set of initial parameters, and, then, they are compared with the experimental FRFs. The difference between these FRFs is the error that has to be minimized. For each iteration, new values for the control variables have been chosen according to the error of the previous step. Therefore, this procedure is repeated until the maximum number of iteration is reached or until the error becomes acceptable.

To perform this analysis it is necessary to define some operational parameters, such as control variables, the objective function and the restrictions. Hence, the problem is to find the optimum value for a given function $f_i(\boldsymbol{x}):R^n \rightarrow R$ subjected to a set of restrictions $g_i(\boldsymbol{x})$ *and* $\boldsymbol{x}_i^l \leq \boldsymbol{x}_i \leq \boldsymbol{x}_i^u$, which for each *i-th* restricted objective there is an expected value *(b_i)*. It can be mathematically written as:

$$\begin{cases} f_i(\boldsymbol{x}) \geq b_i, \text{ or} \\ f_i(\boldsymbol{x}) \leq b_i, \text{ or} \\ f_i(\boldsymbol{x}) = \mathrm{b_i} \\ \\ \text{subjected to: } g_i(\boldsymbol{x}) \geq 0 \text{ e} \\ \qquad\qquad \boldsymbol{x}_\mathrm{i}^\mathrm{l} \leq \boldsymbol{x} \leq \boldsymbol{x}_\mathrm{i}^\mathrm{u} \end{cases} \tag{1}$$

The Goal Programming also allows to give priority to each objective and restriction. In this case, it is important to make sure that the objectives will be attained only when all restrictions are fulfilled.

2.1. Determination of the Control Variables

The control variables are the geometrical and physical characteristics of the modeled structure. There are two types of variables: Global and Element. The first one represents the global structure, such as modal damping and concentrated masses, while the second one represents the properties of the implemented element (spatial frame) as shown in Table 1.

Table 1. Element Variables

Variable	Description
E	Young Modulus
ν	Poisson rate
A	Section Area
I (I_x, I_y, I_z)	Moment of Inertia (x, y, z)
ρ	Density
K (x,y,z,θx,θy,θz)	Spring stiffness (Tx, Ty, Tz ,θx ,θy, θz)

The upper limits values of the elements variables could be chosen from “as built” or in case of its absence, design or experimental data may be used. For the lower limits, however, there is an indetermination because it is exactly what is wanted. In such situation, it is suggested to use very low values to guarantee that the optimal solution is inside the domain, so that the physical coherence is maintained. To the global variables, some experimental information is necessary, mainly for modal damping.

It is important to observe that the ideal limit must be wide enough to contain the solution; however, when it is greater than the necessary, it may demand more time to its convergence and sometimes it may converge to a local minimum.

2.2. The Objective Function

The objective function is the numerical FRF obtained through a finite element program. Every time the system receives a new set of variables, it estimates a new FRF and calculates differences (errors) between this FRF and the reference one (experimental). After that, the optimization routine sends to the main program a new set of variables based on the errors calculated for the previous set. These comparisons are made until the encountered error is acceptable or until the maximum number of iterations is reached.

The error was estimated by a function named "combined error" which had been presented before in reference Roitman, Gadea and Magluta (2004). In this paper, two weight factors were added to normalize discrepancies between amplitude and frequency components, as seen in Eq. (2).

$$E_{p,q} = \beta_1 \cdot \sum_{r=1}^{N} \left(\left| \frac{A^e_{p,q,r} - A^n_{p,q,r}}{A^e_{p,q,r}} \right| \right) + \beta_2 \cdot \sum_{r=1}^{N} \left(\left| \frac{F^e_{p,q,r} - F^n_{p,q,r}}{F^e_{p,q,r}} \right| \right) \tag{2}$$

where:

$E_{p,q}$ - error for a pair of correlated FRFs;
A^e, A^n - amplitude of the experimental and numerical FRF peaks, respectively;
F^e, F^n - resonance frequency of numerical and experimental FRF, respectively;
β_1, β_2 - weights associated to the discrepancies between amplitudes and frequencies, respectively;
p - index that indicates the point of the applied force;
q - index that indicates the response point ;
r - index of the modes;
m - number of modes;

The total error to be minimized is formulated by the addition of all compared FRFs, as observed in Eq. (3).

$$E^{tot} = \sum_{p=1}^{ni} \sum_{q=1}^{no} E_{p,q} \tag{3}$$

where:

ni - number of input forces;
no - number of output points;

This error formulation demands continuous comparisons between correlate modes; however, it is quite common to evaluate more modes numerically, than those observed experimentally, thus it is necessary to choose the correct set of appropriate modes to estimate the numerical FRFs; therefore, the Modal Assurance Criterion - MAC (Allemang, 1982) was implemented to associate the numerical and experimental modes in each iteration.

The developed approach estimates the numerical FRFs from its analytical equation. In order to do so, the structure was considered elastic linear and time-invariant, where the modal superposition principle is perfectly applicable. Thus, the Acceleration FRF for a structure subjected to an excitation in a point (p) and monitored with a sensor in position (q), for N modes can be written as shown in (4).

$$H_{p,q} = \sum_{r=1}^{N} \left[\frac{R_{p,q,r}}{(i\omega - \lambda_r)} + \frac{R^*_{p,q,r}}{(i\omega - \lambda^*_r)} \right] \tag{4}$$

where

$H_{p,q}$ - estimated FRF for excitation in (p) response in (q);
$R_{p,q,r}$ - residue of mode r for excitation in (p) and response in (q);
p - index that indicates the point of the applied force;
q - index that indicates the response point ;
r - index that indicates the modes;

λ_r - system pole for mode r;
ω - variable in the frequency domain;
N - number of analyzed modes;
$()^*$ - the complex conjugate.

The Bending Moment FRF for a structure subjected to an excitation in a point (p) and monitored with a sensor in the position (q), for N modes can be written as shown in (5). This equation can be observed in details in reference Gadea (2005).

$$\mathcal{H}^{f}_{ip} = \sum_{r=1}^{N} \left[\frac{\mathcal{R}^{f}_{ipr}}{i\omega - \lambda_r} + \frac{\mathcal{R}^{f*}_{ipr}}{i\omega - \lambda^*_r} \right] \tag{5}$$

where:

$\mathcal{H}^{f}_{ipr}$ - Bending Moment FRF for excitation in (p) and response in (q), for a f-th degree of freedom of the i-th element;
$\mathcal{R}^{f}_{ipr}$ - residue of mode r for excitation in (p) and response in (q), for a f-th degree of freedom of the i-th element;
p - index that indicates the point of applied force;
q - index that indicates the point of the response;
r - index of the modes;
f - index of the degree of freedom of the element;
i - index of the analyzed element;
λ_r - system pole for mode r;
ω - variable in the frequency domain;
N - number of analyzed modes;
$()^*$ - the complex conjugate.

3. The Analyzed Structure

The structure to be adjusted is a clamped steel beam, 1467 mm long and cross section of 76,2x7,93 mm^2. The reference "experimental" FRFs were estimated for acceleration in point A at the right-hand side (free side), and for bending moment in B, at the clamped side, as indicated in Fig. 2. The excitation point was the point A.

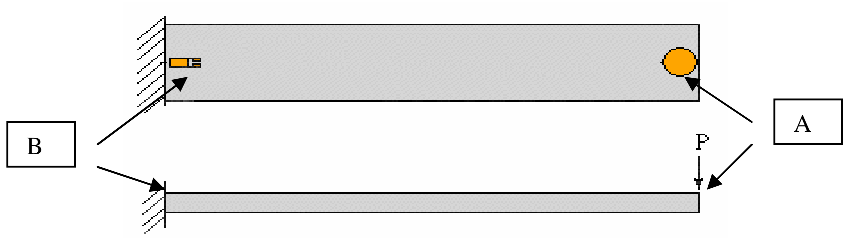

Fig. 2. Illustrative scheme of the analyzed structure: A steel clamped beam.

The numerical modeling of the beam was performed with 11 nodes and 10 frame elements (see Fig. 3). As mentioned before, the boundary conditions for this structure were clamped-free, and 205 GPa for the steel the elastic modulus (E).

3.1. The Simulated Damage

In order to test the developed program, two simultaneous damages were imposed to elements #4 and #7, as shown in Fig. 3. The imposed damages were simulated by decreasing the steel elastic modulus (E) in 15%. Although it is a non-realistic scheme, this approach allows reduction of the number of variables, besides inducing the same damage obtained by simultaneous reduction in cross-sectional area and moment of inertia. Table 2 presents a comparison between the natural frequencies obtained from the undamaged and damaged structure.

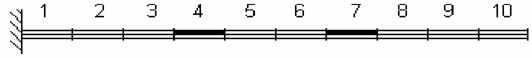

Fig. 3. Numerical model with the simulated damages.

Table 2. Natural Frequencies (Hz).

Case	1st mode	2nd mode	3rd mode	4th mode
Undamaged	3.017	18.694	51.819	100.487
Damaged	2.984	18.315	50.439	100.100
Reduction (%)	-1.094	-2.027	-2.663	-0.385

According to Table 2, the 3rd mode is the most sensitive to the imposed damage. This can also be seen in Fig. 4, which shows the obtained FRFs to damaged and undamaged cases (acceleration and bending moments). Although the other analyzed modes are less sensitive than the 3rd mode, all modes were considered in these analyses in order to preserve useful information about the structure.

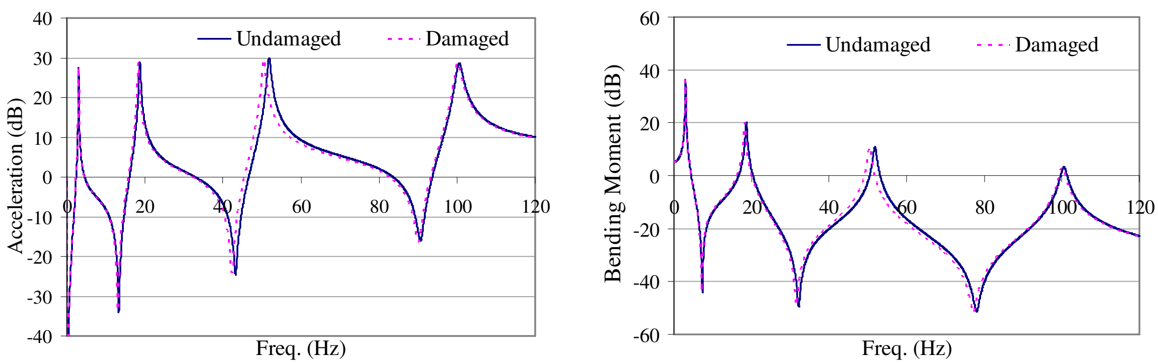

Fig. 4. Comparisons between damage *vs.* undamaged FRFs.

3.2. The Damage Detection

The chosen control variables were the elastic moduli of bars #1 to #10, and their limits were 0.02GPa to 2.05 GPa, for lower and upper limits, respectively. The modal damping was considered constant for all modes, in spite of damages. For these analyses both types of FRF were used in order to increase the method performance.

The Goal Programming is very influenced by the initial parameters. thus, it was necessary to perform several analyses using different initial values of the control variables, in order to obtain confident responses.

Table 3 shows the results of ten different analyses using arbitraries initial values for the control variables. In each case, an optimization problem was evaluated, which the minimized errors (calculated through Eq. 3) were obtained from acceleration and bending moment FRFs.

Table 3. Analyses cases: Damage estimated for the elements (%).

Element	Analysis Case										Average (%)
	1	2	3	4	5	6	7	8	9	10	
1	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.1	0.0	0.0	0.0
2	0.0	0.0	0.0	0.0	0.0	0.0	1.4	0.6	0.0	0.0	0.2
3	0.0	0.0	0.0	0.0	0.0	0.0	1.8	0.0	0.0	0.0	0.2
4	15.0	15.0	15.0	15.0	15.0	14.6	10.1	13.9	15.0	15.0	14.4
5	0.0	0.0	0.0	0.0	0.0	0.9	0.2	0.0	0.0	0.0	0.1
6	0.0	0.0	0.0	0.0	0.0	0.8	0.0	0.0	0.0	0.0	0.1
7	15.0	15.0	15.0	15.0	15.0	12.8	16.1	15.1	15.0	15.0	14.9
8	0.0	0.0	0.0	0.0	0.0	2.5	2.1	0.9	0.0	0.0	0.6
9	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0
10	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0	0.0

According to the simulation, the exact solution is 15% of damage for elements #4 and #7, and 0% for the other elements. Based on results (Table 3), it is possible to affirm that the elements #4 and #7 are damaged, since their estimated level of damage is close to the simulated in all evaluated cases. It is quite clear that there is no fault on additional elements, once none of them have presented level of damage larger than 3%.

This result shows the efficiency of the methodology once it was capable of detecting damage in a very realistic scenario, that is, just 15% of damage and less than 3% of variation in the natural frequency (Table 2).

The damage location with numerical data is easier than those with experimental data, because, in general, numerical simulations have a well defined optimal point. In experimental analyses; however, it is necessary to have experimental FRFs estimated with uncertainties lower than the variations induced by the damage, to assure reliable solutions. This way, it is more difficult to predict lower rates of damages in real structures. For this reason, in actual structures, the smaller the damage is, the more difficult it is to be predicted.

4. Conclusions

The problem of damage detection is very complex, even in numerical simulations. There are many parameters to be chosen and in general, if they are badly selected, they can lead to unrealistic solutions, then many simulations must be performed until one has enough confidence to decide if there is some damage in the structure. There is no guarantee that the damage will be found, but when it is found, it is necessary to know if it is a real fault or just an unfortunate parameter combination. That is why it is so important to have reliable analysis.

Although the developed software is able to identify and quantify the simulated damages, it is not a trivial task. The Goal Programming needs some knowledge about the structures to facilitate the damage detection. However, in complex structures, the damage identification is always a difficult task.

The developed methodology was capable of detecting the simulated damage even when there was almost no difference between the FRFs for the damaged and the healthy structure.

5. Acknowledgements

The Authors would like to thank the financial support from ANP – National Petroleum Agency and FINEP – Brazilian state Agency of Studies and Projects Financing, through the program of human resources of ANP for the department of Petroleum and Natural Gas - PRH-ANP/MME/MCT

6. References

Allemang, R. J., Brown, D. L., 1982, “A Correlation Coefficient for Modal Vector Analysis”, 1st International Model Analysis Conference, 110-116. Canada.

Doebling, S. W., Farrar, C. R., Prime, M. E., Shevitz, D. W. , 1996, “Damage Identification and Health Monitoring of Structural and Mechanical Systems from Changes in Their Vibration Characteristics: A Literature Review”, Los Alamos Laboratory Report LA-13070-MS, Los Alamos, New Mexico, USA.

Gadea, A. S. M., 2005, “Structural Damage Identification based on the Frequency Response Function Error Minimization through Optimization Methods, D.Sc. Thesis, COPPE/UFRJ, Rio de Janeiro, RJ, Brazil (in Portuguese).

Hoons, S., Farrar, C. R., Hemez, F. M., Shunk, D. D., Stinemates, D. W., Nadler, 2003., “A Review of Structural Health Monitoring Literature: 1996-2001”, Los Alamos Laboratory Report LA-13976-MS, Los Alamos, New Mexico, USA

Ignizio, 1976, J. P., “Goal Programming and Extensions, Lexington Books”.

Neves, 1997, F. A., “Programação com Multiobjetivos Aplicada à Otimização de Projeto de Pontes Estaiadas”, Tese de Doutorado, COPPE/UFRJ, Rio de Janeiro, RJ, Brasil. (in Portuguese)

Roitman, N., Gadea, A. S. M., Magluta, C., 2004, “Damage Identification in Offshore Platforms Using Frequency Response Funcions”, Proceedings of the 23rd International Conference on Offshore Mechanics and Arctic Engineering June 20-25, Vancouver, British Columbia, Canada.

Salawu, O. S., 1997, “Detection of Structural Damage through Changes in Frequency: a Review”, Engineering Structures, Vol. 19, No. 9, p.718-723.

5. Responsibility notice