OPTIMIZATION OF A MULTI-RESPONSE GMAW-P WELDING PROCESS BY DESIGN OF EXPERIMENTS AND PRINCIPAL COMPONENT ANALYSIS

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Abstract. Most previous applications of Design of Experiments – DOE- statistical method in manufacturing systems have only focused in a single-response problem. However, multiple correlated responses normally occurs in industrial applications instead of a few ones and only limited attention has been dedicated to study this problem. In this work an alternative procedure on the basis of principal component analysis (PCA) and Derringer's Desirability algorithm to optimize a multi-response is discussed and an application in welding process is described. With the PCA, a group of original responses can be transformed into a set of uncorrelated components (named eigenvalues). The case study evaluated five responses under the influence of pulsed parameters in a Gas Metal Arc Welding – GMAW- process. The results showed that the differents methods conduct to the same results. Additionally an optimum process condition for the responses analysed was obtained and its levels showed a good correlation with the center points of the Response Surface Methodology – RSM- design.

Keywords: DOE, RSM, PCA, Desirability, Welding, Pulsed GMAW.

1. Introduction

The selection of arc welding procedure which fulfils several specifications, generally follows a nearly structured approach widely based upon subjective appreciations. This strategy, currently used in industry, has a lot of drawbacks. To overcome these drawbacks, the Design of Experiments - DOE - statistical strategies have been employed in order to achieve a multiple attribute requirements. However, the most previous approaches used have focused in customer's preferences and needs, only addressing the problem with a single-response approach. Even though some researches have been treating the problem through simultaneous optimization algorithms, a fact that is often neglected: the presence of more than one correlated response, which is a predominant characteristic in the manufacturing processes as a whole. According to Tong et al. (2004), difficulties arise in multi-response optimization direction since moderate or high conflicting tendencies exists among the optimization factor/level combinations for these responses. Such considerations are extremely adequate to the present work. Here, a set of correlated responses from the geometric weld bead shape represented by weld penetration, reinforcement, width, convexity and total area of weld bead is modeled considering the experimental variation in a pulsed GMAW through the use of four independent variables (Peak current - Ip, Background current - Ib, Duty Cycle - Ca and Wire-feed rate - Va) through DOE techniques. Moreover, the Derringer's Desirability function was employed to treat the multiple optimization case, as well as, to show the possibility that most part of the response variables above have any degree of correlation. Hence, an effective procedure based in a principal component analysis (PCA) to optimize the multi-response problem is proposed with the use of a PCA, a set of original responses that can be transformed into a set of uncorrelated components. Therefore, the conflict for determining the optimal settings of the design parameters can be reduced and a better global desirability coefficient using PCA approach is expected.

2. Response Surface Methodology

The Response Surface Methodology (RSM) is a collection of mathematical and statistical techniques that are useful for the modeling and analysis of problems in which a response of interest is influenced by several variables with the objective to optimize it (Montgomery, 1997). RSM also quantifies relationships among one or more measure responses and the main input factors. The relationship between the responses and the variables investigated is commonly approximated by polynomial functions, whilst the model parameters can be evaluated by DOE statistical methods.

Suppose that the expected response E(Y) is a function of K predictors variables $x_1, x_2, ..., x_k$. In case that a curvature in the process is present, the relationship between the response and the factors can be expressed as a second-order polynomial expression, as noted in Eq. (1):

$$y = \beta_0 + \sum_{i=1}^k \beta_i x_i + \sum_{i=1}^k \beta_{ii} x_i^2 + \sum_{i< j} \sum_{i< j} \beta_{ij} x_i x_{j+} \varepsilon$$
 (1)

To determine the β parameters in the model, it is commonly used the least squares method, as expressed in Eq. (2):

$$\hat{\boldsymbol{\beta}} = (\boldsymbol{X}^T \boldsymbol{X})^{-1} \boldsymbol{X}^T \boldsymbol{y} \tag{2}$$

where *X* is the RSM experimental matrix and *y*, the response set.

In this work, it was used the Minitab statistical software commercial package to implement the DOE procedures.

2.1. Principal Components Analysis

The Principal Components Analysis (PCA) is a very interesting statistical tool that can be used in a large variety of applications. This technique has basically two main purposes, i.e., reduction of dimensionality and interpretation of the original characteristics from the point of view of new variables with linear combinations of the originals characteristics. The concept of PCA was first introduced by Karl Pearson (1901) and it was developed later by Hotteling (1933). Since then, it has been used in a variety of fields such as Psychology, Education, Chemistry, Engineering, Medicine, Geography and so on. In some situations Principal Components Analysis appears in conjunction with other statistical techniques such as regression analysis, analysis of variance and cluster analysis, for instance (Mingoti, 2001). This is exactly the case presented in the present work, where the PCA is combined with DOE strategies. In such application, a set of correlated geometric weld bead characteristics are taken as responses of a RSM arrange following the same proceedings described by Liao (2003), Rossi (2001), Su and Tong (1997), Ellekjaer et al. (1996) and Galopin et al. (1989).

The concept of Principal Components Analysis is based in the assumption that the correlated results of p characteristics are available for a certain number n of subjects, then one can estimate the covariance matrix by using the sample variances and co-variances of the p characteristics and decompose the matrix into a called eingenvalues and normalized eingenvectors. The principal components variables will be the product of the eingenvectors coefficients by the p observed characteristics (Johnson and Wichern, 1982). Considering $X = (X_1, X_2, ..., X_p)$ as the random vector containing the p characteristics of interest which where observed in p sampling units, p as the sample covariance matrix of the vector p the Principal Components denoted by p (p is p in p and p in p in p and p in p in

$$Y_{j} = \sum_{k=1}^{p} c_{jk} X_{k} \tag{3}$$

where the coefficients of the vector $c_j = (c_{j1}, c_{j2}, ..., c_{jp})$ must present maximal variance and null covariance between Y_j and Y_j and $C_j^T C_j = 1$ (Mingoti, 2001). It can be shown that the solution vector C_j is the normalized eigenvector corresponding to the eigenvalue λ_j of the sample covariance matrix. In this case, the variance of Y_j is equal to λ_j and the total variance of the vector X is given by $\sum_{i=1}^p \lambda_j$ and the principal components are uncorrelated.

Therefore, the variance and the covariance structure of the p characteristics in X are described by a new set of p uncorrelated linear combinations of these variables. The ratio between the variance of each component and the total variance is a measure of the importance of the component in the explanation of the variance and covariance structure of the p original characteristics (Jonhson and Wichern, 1982). The higher the value of this ratio the more important is the component. The same approach could be used if the p original characteristics are standardized by subtracting their means and dividing by their standard deviation. In this case the sample covariance matrix of the standardized variables is the sample correlation matrix of the original characteristics and the principal components would be given by Eq. (4):

$$Y_j = \sum_{k=l}^p c_{jk} Z_k \tag{4}$$

where Z_k is the standardized characteristic X_k and the vector c_j is the normalized eigenvector corresponding to eingenvalue λ_j of correlation matrix R_{pxp} of the original characteristics. The use of the correlation matrix is adequate when the variables are measured by different scales. This is exactly the case of the present work, where a PCA routine, available in the Minitab software, was used.

2.1. Desirability Function

The stringent global competition demands continuously elevating product quality. In this sense, the industrial processes have increasingly emphasized the development of procedures capable of optimizer a multi-response set of attributes simultaneously, with the objective to achieve the wide range of customers needs. To treat this kind of problem, Derringer and Suich (1980) created an efficient strategy of simultaneous optimization of several response variables. The algorithm created is called Desirability Function (D). The principle is very interesting: first, it must to establish a relationship between the several responses yi and each recognized predict variable Xi. This can be done using the regression techniques as RSM, least squares and so on. Then, using the desirability function the original set of response variable is transformed into a desirability value d_i , where $0 \le d_i \le 1$. The value of d_i increases as the desirability of the corresponding response increase. The individuals desirabilities are then combined using the geometric mean, as shown in Eq.(5):

$$D = (d_1(Y_1).d_2(Y_2)....d_k(Y_k))^{\frac{1}{k}}$$
(5)

This single value of D gives the overall assessment of the desirability of the combined response levels. The range of D will fall in the interval [0, 1] and will increase as the balance of the properties becomes more favorable (Derringer and Suich, 1980). If any d_i is equal to 0 (unacceptable condition), then the global desirability D is also zero. This is the reason why the geometric mean was employed in Eq. (5). In essence, this method of optimization condenses a multivariate optimization problem into an univariated one. The algorithm depends on the kind of response (maximize, minimize, target), the desired values for the responses and respective degree of importance and the defined bounds and its limits as described by Derringer and Suich (1980). In this work, to apply this procedure, the Minitab statistical software package was used.

2.2. Pulsed GMAW Process

Gas metal arc welding (GMAW) is widely used in industries for welding wide variety of ferrous and non-ferrous materials. GMAW achieves coalescence of metals by melting continuously fed current-carrying wire. However, its necessary a high-quality welding procedures to achieve good quality. This need is due to continuous control metal transfer that is necessary in GMAW.

The GMAW process when operates in low currents presents a globular metal transfer mode. If the current is increased, the process changes to spray mode. According to Praveen *et al.* (2005), the globular mode is characterized by periodic formation of big droplets at the extreme of electrodes, detaches due to gravitational force in to the weld pool and suffers from lack of control over molten droplets and arc instability due to formation of big droplets. The spray mode offers high deposition rate but minimum current for spray mode is too high for some materials, large heat input to workpiece, wide bead, and only downhand positional capability are some of its drawbacks (Praveen *et al.*, 2005).

An alternative transfer technique is the pulse gas metal arc welding (GMAW-P). This mode of metal transfer overcomes the drawbacks of the globular mode while achieving the benefits of spray transfer. The process is characterized by pulsing of current between low-level background current and high-level peak current in such a way that mean current is always below the threshold level of spray transfer. The background current is used to maintain arc where as peak currents are long enough to make sure detachment of the molten droplets (Praveen *et al.*, 2005).

The knowledge of the transition current zone between the globular and spray mode has great importance in the GMAW process. Hence, this paper presents a study of the main variables and levels that are responsible by the working conditions of the process.

2.3. Experimental Design

In order to comply with the objective of this work, it was used a power source working with an imposition current in the pulsed mode in order to get more flexibility in adjust the parameters and associated with the equipment a mechanical tractor that move the attached torch at the adjustable welding speed. All welding tests were performed using a weld bead on plate (BOP) technique. All welds were made using a wire AWS ER 70S-6, diameter of 1.2 mm, and base material of SAE 1045 with 120x40x6 mm dimension. The shielding gas used were a mixture of Argon + 25% CO₂ with a constant flow of 15 l/min. The welding speed were kept constant and fixed in 40 cm/min for all tests performed and the contact tip to plate

distance used was 22.5 mm. Table 1 shows the design of experiments used with all the parameters and its level designed according to a RSM methodology. In order to get a Duty Cycle (Ca), the peak time (tp) was kept fixed in 4 ms and the background time varied according to the desired level according to the Eq. (6):

$$Ca = \frac{tp}{tp + tb} \tag{6}$$

After the welding, all test specimens were cross-sectioned, polished and chemical attacked, and the penetration (p), reinforcement (r), width (w), and overall area (A) of the weld bead were determined. Also the convexity of the weld bead (CI) were determined by the relation between the reinforcement (r) and the width (w).

Table 1. Process pa	rameters
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Parameters	Unidade	Levels				
		-2	-1	0	+1	+2
Peak Current (Ip)	Amps	245	280	315	350	385
Background Current (Ib)	Amps	55	70	85	100	115
Duty Cycle (Ca)	%	35	40	45	50	55
Wire Feed Rate (Va)	m/min.	4,5	5,0	5,5	6,0	6,5

2.4. Results

The Response Surface design and the results generated by experimental trials are presented in Tab. 2. The method was used to adjust a second-order model with the five original responses of interest. The same table presents the PC scores. Table 3 presents the coefficients of the model obtained using the least square algorithm. In the same table, it can be noted the adjusted R-squared (R-Sq(adj)). Such expression takes into account the fact that R tends to overestimate the actual amount of variation accounted in the sample analysis. Using the regression equation derived from one sample to another independent sample, you will almost always get a smaller R in the new sample than in the original. If a set of responses exhibits a moderate-to-high covariance structure (as shown in Tab.4), the Principal component analysis (PCA) is adequated to summarize and represent the original data used. The new set, represented in the Tab. 2 by the columns PC1 e PC2, was also modeled by regression equations.

Table 5 presents the eigenanalysis of the correlation matrix presented in Tab. 4. The variance and the covariance structure of the p welding responses are described by a new set of p uncorrelated linear combinations of these variables, so called principal components. The ratio between the variance of each component and the total variance is a measure of the importance of the component in the explanation of the variance and covariance structure of the p original characteristics (Jonhson and Wichern, 1982). The higher the value of this ratio the more important is the component. The same approach could be used if the p original characteristics are standardized by subtracting their means and dividing by their standard deviation. In this case the sample covariance matrix of the standardized variables is the sample correlation matrix of the original characteristics and the principal components would be given. It was employed correlation matrix because the five original responses are in different units. Otherwise, according to Jonhson and Wichern (1982) is more appropriate. It can be noted that only the first and second eigenvalues are higher than one. This is the Kaiser's criteria to select the eigenvalues capable to replace the original correlated values (Johnson and Wichern, 1982). We can note in proportion line of Tab. 5 that the first eigenvalue explain 59,6% of all correlation structure, followed by the second eigenvalue, which explain 25,9%. In the desirability algorithm implemented in the Minitab software package, it was considered this degree of explanation as the importance of the transformed responses PC1 and PC2. The table 5 also show the associated eigenvectors for each eigenvalue. As mentioned in theory, the PC score is the product of original standardized responses by the respective eigenvectors.

The problem presented in this work used an approach to determine the desirable values of uncorrelated responses PC1 and PC2. However, it is difficult to determine the kind of optimization in the PC score system. In order to solve this gap in the proposed methodoly it was employed the loading plot, as shown in Fig. 1. The loading plot is capable to graphically demonstrate covariance clusters existent among the original responses. The line length indicates the larger-is-better criteria to the high correlation among variables. Hence, it can be noted that most of responses places in fourth quadrant, called "Cluster Factor Shape", are correlated with the shape of the weld bead. The values of PC1 and PC2 for this situation are defined in the following intervals: 0 < PC1 < 0.7 and -0.6 < PC2 < 0. These values will be used in desirability optimization algorithm (Tab. 6) of the transformed responses. The determination of the type of optimization can be found analyzing the cluster formed. As can be seen in Fig. 1, there is a high correlation among the width, penetration and area of the weld beads and this occurs where PC1 is at a maximum level and PC2 is at a minimum level. This criteria was adopted in this work.

Table 2. Response surface design and principal components scores.

		Fact	ors			Correlated Responses			Principal Components		
N	Ip	Ib	Ca	Va	р	r	W	CI	A	PC1	PC2
-	Amps	Amps	%	m/min	mm	mm	mm	%	mm ²	-	-
1	280	70	40	5	1,6	2,87	7,7	37,2	20,7	-1,0311	1,67741
2	350	70	40	5	1,6	2,9	6,6	44,4	19	-2,62731	1,0853
3	280	100	40	5	1,7	2,8	7	39,6	20,7	-1,55643	1,70348
4	350	100	40	5	1,87	3,1	6,3	43	18,7	-2,3394	0,36578
5	280	70	50	5	1,9	3	7,3	41,6	23,2	-0,92752	0,49137
6	350	70	50	5	1,66	3,7	6	52,5	20,9	-3,36591	-2,9408
7	280	100	50	5	1,96	2,9	8,1	35,5	21,5	-0,02091	1,56461
8	350	100	50	5	1,9	3,3	7,9	38	22,8	-0,25427	-0,33292
9	280	70	40	6	1,2	3,48	7,3	46	20,7	-2,61893	-1,36921
10	350	70	40	6	1,9	3,1	8,1	38,9	24,9	0,0645	0,19437
11	280	100	40	6	1,22	3,5	7,2	48,3	23	-2,54742	-1,85975
12	350	100	40	6	1,95	3,1	8,6	35,9	27	1,01958	0,25174
13	280	70	50	6	2,1	3,2	8,1	39,2	25,7	0,42666	-0,34166
14	350	70	50	6	2,08	3,2	8,7	36,4	25,1	0,94879	-0,025
15	280	100	50	6	1,96	3	8,4	36,4	25,5	0,64575	0,73517
16	350	100	50	6	2,3	3,2	9,2	28,5	28,4	2,77117	0,27662
17	245	85	45	5,5	1,85	3	8,8	40	29	0,8896	0,15847
18	385	85	45	5,5	2,29	3,12	8,1	38,1	27	0,96745	-0,10115
19	315	55	45	5,5	1,79	2,91	8,5	34,5	21,7	0,11669	1,64297
20	315	115	45	5,5	2,2	3	9,3	29,1	26	2,28851	1,26977
21	315	85	35	5,5	1,42	3	7,6	39,5	22,3	-1,32266	0,86187
22	315	85	55	5,5	2,1	3,4	9,2	32,7	28,3	2,09764	-0,79719
23	315	85	45	4,5	1,8	2,8	6	46,4	17,4	-3,14456	1,39711
24	315	85	45	6,5	2,02	3,2	8,1	39,5	26,8	0,4441	-0,45296
25	315	85	45	5,5	2,6	3,25	8,8	36,2	30,7	2,52657	-0,8404
26	315	85	45	5,5	2,1	3,3	8,1	40,7	27,6	0,55734	-1,03596
27	315	85	45	5,5	2,2	3,3	8,7	38,1	28,9	1,49074	-0,94046
28	315	85	45	5,5	2,5	3,1	8	38,8	26,9	1,10953	-0,11855
29	315	85	45	5,5	2,32	3,35	7,8	42	28	0,60846	-1,43704
30	315	85	45	5,5	2,4	3,3	8,3	39,8	30,1	1,52472	-1,25326
31	315	85	45	5,5	2,4	3,1	8,4	36,9	25,9	1,25863	0,17028

Table 3. Second-order model coefficients.

	р	r	W	A	CI	PC1	PC2
Constant	2,3600	3,2429	8,3000	38,9286	28,3000	1,2966	-0,7793
Ip	0,1042	0,0454	-0,0458	-0,4167	0,0750	0,1668	-0,1769
Ib	0,0683	-0,0154	0,1875	-1,7417	0,6667	0,4664	0,1328
Ca	0,1742	0,0604	0,3375	-1,6167	1,2667	0,7792	-0,2475
Va	0,0400	0,0838	0,5375	-1,5000	2,1500	0,8338	-0,3938
Ip ²	-0,0938	-0,0302	-0,0531	0,4658	-0,3792	-0,2596	0,1345
Ib ²	-0,1125	-0,0564	0,0594	-1,3467	-1,4167	-0,1911	0,4914
Ca ²	-0,1713	0,0048	-0,0656	-0,2717	-1,0542	-0,3948	0,1354
Va ²	-0,1338	-0,0452	-0,4031	1,4408	-1,8542	-0,8293	0,2453
Ip*Ib	0,0463	0,0094	0,1438	-1,4125	0,4125	0,3441	0,0351
Ip*Ca	-0,0988	0,1094	-0,0313	0,7250	-0,2000	-0,2435	-0,4511
Ip*Va	0,1175	-0,1256	0,4313	-3,3875	0,9500	0,8718	0,6745
Ib*Ca	-0,0038	-0,0531	0,2563	-1,9750	-0,0500	0,3294	0,3866
Ib*Va	-0,0325	0,0119	-0,0313	0,5125	0,4750	-0,0446	-0,1277
Ca*Va	0,0950	-0,1131	0,0938	-2,0000	-0,0125	0,3681	0,5924
$\mathbf{R}^2(\mathbf{adj.})$	83,20%	71,00%	72,20%	80,70%	75,50%	78,70%	73,60%

Table 4. Correlation table for the welding variables.

	0,019 ^(a)			
r	0,919 ^(b)			
***	0,558	0,023		
W	0,001	0,903		
IC	-0,533	0,337	-0,849	
IC.	0,002	0,064	0,000	
	0,722	0,331	0,792	-0,483
A	0,000	0,069	0,000	0,006
	p	r	W	IC

⁽a): Pearson correlation coefficient, (b): P-Value (There is a significant correlation if < 0,05)

Table 5. Eigenanalysis of the correlation matrix.

Eigenvalue	2,9775	1,2957	0,5165	0,1856	0,0246
Proportion	0,596	0,259	0,103	0,037	0,005
Cumulative	0,596	0,855	0,958	0,995	1
Variable	PC1	PC2	PC3	PC4	PC5
p	0,468	-0,073	0,79	-0,319	-0,225
r	0,014	-0,844	-0,263	-0,465	0,04
W	0,542	0,045	-0,459	0,134	-0,69
IC	-0,48	-0,404	0,306	0,503	-0,509
A	0,507	-0,341	0,059	0,64	0,461

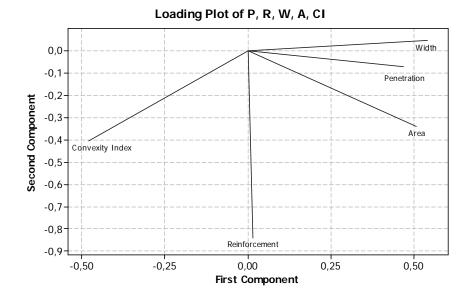


Figure 1 – Loading Plots in the plane of the first and second principal components.

Response	Goal	Lower Bound	Target	Upper Bound	Weight	Importance
Penetration	Maximum	2,1	2,6	2,6	1	10
Reinforcement	Target	2,5	3	3,5	1	1
Width	Target	8	8,5	9	1	5
Convexity Index	Target	20	30	40	1	0,1
Area	Maximum	25	31	31	1	7
PC1	Maximum	0,5	0,7	0,7	1	6
PC2	Minimum	-0,6	-0.6	-0.3	1	2

Table 6. Response optimization setup.

2.5 Analysis

Observing the results shown in Tab. 7 it can be noted that the procedure presented very good results. First, the original best levels are the same in the two approaches (Ip=315 A, Ib=85 A, Ca=45% and Va=5,5 m/min). Second, the overall desirability index is higher in the PCA approach. Comparing with the original case, D=0,54, when using the PCA procedure, the algorithm achieved a maximum desirability (D=1).

Predicted Resp	onses	Individual Desirability	Global Desirability
Penetration (p)	2,36	0,52	
Reinforcement (r)	3,24	0,514	
Width (w)	8,3	0,6	0,54
Convexity Index (CI)	38,9	0,107	
Área (A)	28,3	0,55	
PC1	1,297	1	1
PC2	-0,779	1	1

Table 7. Response optimization results.

Further works and discussions could explore the origin of this fact which probably is related with the covariance and correlation mechanisms. As referred in Tong et al. (2004), probably the original algorithm found a difficulty to converge,

mainly thanks to presence of a correlation among the several objective functions. Moreover, the individuals desirabilities had the same behavior. With the PCA approach, the algorithm achieved the target of the function while in the original scenario, this feature did not occur. Based on this fact, it is possible to conclude that the proposed procedure was capable to transform the correlated multiple responses into a set of uncorrelated components through PCA methodology, thereby simplifying the optimization process. It suggest that higher is the amount of responses presents, more efficiently this proposition can get.

3. Conclusions

This study used the PCA to simplify the multi-response optimization problems and to determine the optimal factor/level combination based in the DOE/Desirability strategy. A case study in which a pulsed GMAW was modeled by RSM to achieve the best properties and geometry of weld bead confirms the effectiveness of the proposed procedure. The PCA showed an adequate approach to reduce conflicts from the correlated structure of the original variables. In future works, it will be possible to study if the increase of the number of correlated responses results in a better setup to the variables. Nevertheless, we believe that the extension of this procedure to others applications needs a more elaborated study.

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5. Responsability notice

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