A SHAPE OPTIMIZATION PROCEDURE FOR FLUID FLOW PROBLEMS

Lindaura Maria Steffens
lindaura@emc.ufsc.br

Marcelo Krajnc Alves
krajnc@emc.ufsc.br

Hilbeth Azikri Parente de Deus
azikri@emc.ufsc.br

Universidade Federal de Santa Catarina – Departamento de Engenharia Mecânica.
Campus Universitário – Trindade – Caixa Postal 476 – 88040-900 – Florianópolis – SC – Brasil.

Abstract. *The objective of the work is to propose a shape optimization procedure for the minimization of the viscous dissipation of two-dimensional fluid flow problems. The fluid flow problem is assumed to be incompressible and steady state and is modeled by the Navier-Stokes equations. The approximate solution to the flow problem is obtained by applying the Galerkin Finite Element method. In order to satisfy the BB condition a T7/C3 interpolation scheme is employed (triangular element with seven nodes for the velocity and three for the pressure). The design variables of the problem of optimization are the position of the "Key-points", which describe the contour of the domain that is represented by cubic segments of "B-splines". The discrete optimization problem considers the minimization of the viscous dissipation of the flow and is subjected to volume and side constraints and is formulated through the application of the Augmented Lagrangian problem, reducing the problem to the solution of a sequence of box constrained optimization problems. The efficient solution of this class of optimal shape problems depends strongly on the strategy utilized in the sensitivity analysis of the fluid flow equation, which is necessary for the determination of the gradient of the objective function and constraints. Here, this is achieved with the use of an adjoint method for the determination of the sensitivity analysis and the computation of the required gradients. In order to investigate the performance of the proposed procedure, we solve some simple nozzle optimization problems.*

Keywords*: Navier-Stokes, Sensitivity Analysis, Shape Optimization.*

1. Introduction

Many are the applications of shape optimization of fluid problems and present in different engineering fields, are especially important in the aerospace and automotive engineering, and also in the design of valves and hydraulic pumping. In practice, engineering is interested in reducing the drag force in the wing of a plane or vehicle, or in reducing the dissipation in channels, hydraulic valves, etc…

This work proposes an integrated numerical procedure for problems that involve the shape optimization applied to fluid flow problems. The procedure is named integrated since it combines various distinct modules for the solution to the problem, such as: geometric modeling, mesh generation for finite elements, non-linear analysis of the fluid flow, sensitivity analysis, mathematical programming and shape optimization.

2. Equation of Navier-Stokes

• **Strong formulation:** The flow problem consists in solving the Navier-Stokes equations, (Temam, 1991), and can be formulated as: Let $\Omega \subset R^2$ be a fluid domain with boundary Γ. The problem consists in finding u (x) and p (x), $\forall x \in \Omega \cup \Gamma$, so that:

$$\begin{cases} \mathbf{u}(\mathbf{x}) \cdot (\nabla \mathbf{u}(\mathbf{x})) - 2\nu\nabla \cdot D(\mathbf{u}(\mathbf{x})) + \dfrac{1}{\rho}\nabla p(\mathbf{x}) = \mathbf{b}(\mathbf{x}) \text{ in } \Omega \text{ (conservation of linear momentum)} \\ \nabla \cdot \mathbf{u}(\mathbf{x}) = 0 \text{ in } \Omega \text{ (conservation of mass)} \end{cases} \quad (1)$$

in which **x** represent the design variables, **u** the velocity vector, p the fluid pressure, ν the dynamical viscosity, ρ the specific density, D the symmetric part of the velocity gradient, and **b** the body force vector.

The Equations in (1) are subjected to Dirichlet and Neumann boundary conditions, and are given by:

$$\begin{cases} \mathbf{u}(\mathbf{x}) = \mathbf{g}(\mathbf{x}) \text{ at } \Gamma_{\mathbf{u}} \\ 2\nu D(\mathbf{u}(\mathbf{x}))\cdot\mathbf{n} - \dfrac{1}{\rho} p(\mathbf{x})\mathbf{n} = \mathbf{h}(\mathbf{x}) \text{ at } \Gamma_{\mathbf{t}} \end{cases} \tag{2}$$

where $\Gamma_{\mathbf{u}}$ and $\Gamma_{\mathbf{t}}$ are the parts of the boundary subjected to a prescribed velocity and surface traction conditions respectively.

- **Weak formulation:** Let $\text{Kin}_{\text{u}} = \left\{\mathbf{u}(\mathbf{x}) \in [H^1(\Omega)]^2 \mid \mathbf{u}(\mathbf{x}) = \mathbf{g}(\mathbf{x}) \text{ at } \Gamma_{\mathbf{u}}\right\}$, $\text{Kin}p = \left\{p(\mathbf{x}) \in L^2(\Omega)\right\}$, $\text{Var}_{\text{p}} = \left\{\hat{p}(\mathbf{x}) \in L^2(\Omega)\right\}$ and $\text{Var}_{\text{u}} = \left\{\mathbf{v}(\mathbf{x}) \in [H^1(\Omega)]^2 \mid \mathbf{v}(\mathbf{x}) = \mathbf{0} \text{ at } \Gamma_{\mathbf{u}}\right\}$, where the linear spaces H^1 and L^2 are defined in Reddy (1997). The weak formulation can be stated as: Determine $(\mathbf{u}, p) \in \text{Kin}\mathbf{u} \times \text{Kin}p$, such that:

$$\iint_\Omega \left\{(\nabla\mathbf{u})\mathbf{u} - 2\nu\nabla\cdot D(\mathbf{u}) + \frac{1}{\rho}\nabla p - \mathbf{b}\right\}\cdot\mathbf{v}\, d\Omega + \iint_\Omega \left\{\frac{1}{\rho}\nabla\cdot\mathbf{u}\right\}\cdot\hat{p}\, d\Omega = 0,\ \forall\ (\mathbf{v},\hat{p}) \in \text{Var}\mathbf{u}\times\text{Var}p. \tag{3}$$

Now, adding a shock capturing term, applying the divergence theorem (Aris, 1989), considering the boundary conditions defined in Eq. (2), and doing some algebra, we are able to reformulate the weak form as follows: Determine $(\mathbf{u}, p) \in \text{Kin}\mathbf{u}\times\text{Kin}p$, such that:

$$\begin{gathered}\left\langle(\nabla\mathbf{u})\mathbf{u},\mathbf{v}\right\rangle_\Omega + 2\nu\left\langle D(\mathbf{u}),D(\mathbf{v})\right\rangle_\Omega - \frac{1}{\rho}\left\langle p, div(\mathbf{v})\right\rangle_\Omega - \frac{1}{\rho}\left\langle div(\mathbf{u}),\hat{p}\right\rangle_\Omega + \left\langle div(\mathbf{u}), \delta div(\mathbf{v})\right\rangle_\Omega = \left\langle\mathbf{b},\mathbf{v}\right\rangle_\Omega + \left\langle\mathbf{h},\mathbf{v}\right\rangle_{\Gamma_t}, \\ \forall\ (\mathbf{u},\hat{p}) \in \text{Var}\mathbf{u}\times\text{Var}p,\end{gathered} \tag{4}$$

with $\left\langle f(\mathbf{x}), g(\mathbf{x})\right\rangle_\Omega = \iint_\Omega \left\{f(\mathbf{x})\cdot g(\mathbf{x})\right\} d\Omega$, denoting the inner product of the arbitrary functions $f(\mathbf{x}),\ g(\mathbf{x}) \in L^2(\Omega)$. The parameter $\boldsymbol{\delta}$ in the shock capturing term is defined in Codina (2000).

- **Discretization of the problem (MEF):** A partition of the domain Ω is performed in elements Ω_e, in which the velocity and pressure fields are interpolated as:

$$\mathbf{u} = [\mathbb{N}_\mathbf{u}]\mathbf{q}_e^u,\ \mathbf{v} = [\mathbb{N}_\mathbf{u}]\mathbf{q}_e^v \ \text{ e } \ p = [\mathbb{N}_p]\mathbf{q}_e^p, \tag{5}$$

where $[\mathbb{N}_i]$ is the vector that contains the elementary interpolation functions, $\mathbf{q}_e^u$, $\mathbf{q}_e^v$ e $\mathbf{q}_e^p$ represent the vector of nodal velocities and pressures.

The discretization of Eq. (4) is done as:

$$\left\langle(\nabla\mathbf{u})\mathbf{u},\mathbf{v}\right\rangle_{\Omega_e} = \left[\iint_{\Omega_e}[\mathbb{N}^{disp}]^T[\mathbb{N}^{grad}]d\Omega\right]\mathbf{q}_e\cdot\hat{\mathbf{q}}_e = [\mathbb{k}_e^{grad}]\mathbf{q}_e\cdot\hat{\mathbf{q}}_e, \tag{6}$$

where

$$[\mathbb{N}^{disp}] = \begin{bmatrix}[\mathbb{N}_\mathbf{u}] & [0] & [0] \\ [0] & [\mathbb{N}_\mathbf{u}] & [0]\end{bmatrix}, \quad [\mathbb{N}^{grad}] = \begin{bmatrix}\mathbf{u}[\mathbb{N}_{\mathbf{u},x}] + \mathbf{v}[\mathbb{N}_{\mathbf{u},y}] & [0] & [0] \\ [0] & \mathbf{u}[\mathbb{N}_{\mathbf{u},x}] + \mathbf{v}[\mathbb{N}_{\mathbf{u},y}] & [0]\end{bmatrix}. \tag{7}$$

Also,

$$2\nu\left\langle D(\mathbf{u}),D(\mathbf{v})\right\rangle_{\Omega_e} = \left[\iint_{\Omega_e}[\mathbb{B}]^T[\mathbb{H}^\nu][\mathbb{B}]d\Omega\right]\mathbf{q}_e\cdot\hat{\mathbf{q}}_e = [\mathbb{k}_e^\nu]\mathbf{q}_e\cdot\hat{\mathbf{q}}_e, \tag{8}$$

with

$$[\mathbb{H}^{\nu}]=\begin{bmatrix}[2\nu] & [0] & [0]\\ [0] & [2\nu] & [0]\\ [0] & [0] & [\nu]\end{bmatrix},\qquad [\mathbb{B}]=\begin{bmatrix}[\mathbb{N}_{\mathbf{u},x}] & [0] & [0]\\ [0] & [\mathbb{N}_{\mathbf{u},y}] & [0]\\ [\mathbb{N}_{\mathbf{u},y}] & [\mathbb{N}_{\mathbf{u},x}] & [0]\end{bmatrix} \tag{9}$$

Moreover,

$$\frac{1}{\rho}\left\langle p, div(\mathbf{v})\right\rangle_{\Omega_e} = \frac{1}{\rho} = \left[\iint_{\Omega_e}[\mathbb{B}^{div}]\otimes[\mathbb{N}^{press}]d\Omega\right]\mathbf{q}_e\cdot\hat{\mathbf{q}}_e = [\mathbb{k}_e^{press}]\mathbf{q}_e\cdot\hat{\mathbf{q}}_e. \tag{10}$$

where

$$[\mathbb{N}^{press}]=\left[[0]\quad[0]\quad[\mathbb{N}_p]\right],\qquad [\mathbb{B}^{div}]=\left[[\mathbb{N}_{\mathbf{u},x}]\quad[\mathbb{N}_{\mathbf{u},y}]\quad[0]\right]. \tag{11}$$

We also have,

$$\frac{1}{\rho}\left\langle div(\mathbf{u}),\hat{p}\right\rangle_{\Omega_e} = \frac{1}{\rho} = \left[\iint_{\Omega_e}[\mathbb{N}^{press}]\otimes[\mathbb{B}^{div}]d\Omega\right]\mathbf{q}_e\cdot\hat{\mathbf{q}}_e = [\mathbb{k}_e^{\hat{p}}]\mathbf{q}_e\cdot\hat{\mathbf{q}}_e. \tag{12}$$

$$\left\langle div(\mathbf{u}),\delta div(\mathbf{v})\right\rangle_{\Omega_e} = \left[\iint_{\Omega_e}\delta[\mathbb{B}^{div}]\otimes[\mathbb{B}^{div}]d\Omega\right]\mathbf{q}_e\cdot\hat{\mathbf{q}}_e = [\mathbb{k}_e^{\delta}]\mathbf{q}_e\cdot\hat{\mathbf{q}}_e. \tag{13}$$

$$\left\langle \mathbf{b},\mathbf{v}\right\rangle_{\Omega_e} = \left[\iint_{\Omega_e}[\mathbb{N}^{disp}]^T\mathbf{b}d\Omega\right]\hat{\mathbf{q}}_e = [\mathbb{F}_e^{b}]\hat{\mathbf{q}}_e. \tag{14}$$

$$\left\langle \mathbf{h},\mathbf{v}\right\rangle_{\Gamma_t\cap\Omega_e} = \left[\iint_{\Gamma_t\cap\Omega_e}[\mathbb{N}^{disp}]^T\mathbf{h}d\Gamma\right]\hat{\mathbf{q}}_e = [\mathbb{F}_e^{t}]\hat{\mathbf{q}}_e. \tag{15}$$

Once determined the contribution of each element in the discretized weak formulation, we may assemble the element vectors and matrices into the global vectors and matrices and form the global system of nonlinear equations, given by:

$$\mathbf{q}\Leftarrow\bigcup_{e=1}^{ne}\mathbf{q}_e,\qquad [\mathbb{K}(\mathbf{q})]\Leftarrow\mathop{\mathrm{A}}_{e=1}^{ne}\{[\mathbb{k}_e^{grad}(\mathbf{u})]+[\mathbb{k}_e^{\nu}]-[\mathbb{k}_e^{press}]-[\mathbb{k}_e^{press}]^T+[\mathbb{k}_e^{\delta}]\},\qquad [\mathbb{F}^{ext}]\Leftarrow\mathop{\mathrm{A}}_{e=1}^{ne}\{[\mathbb{F}_e^{b}]+[\mathbb{F}_e^{t}]\}, \tag{16}$$

in which ne is the number of elements and **A** is an assemblage operator, responsible for the assemblage of the global equation system. Notice that, the term $[\mathbb{k}_e^{grad}(\mathbf{u})]$ is dependent of $(\mathbf{u})$, i.e., making the problem nonlinear. The global discrete formulation is given by:

$$[\mathbb{K}(\mathbf{q})]\mathbf{q}=[\mathbb{F}^{ext}] \tag{17}$$

In order to solve this set of nonlinear equations, we apply Newton's method.

3. Optimization

Contour definition: Different techniques may be used for the representation of the boundary of the domain to be optimized, such as the use of macro-elements, or polynomial functions. However, these techniques present some numerical problems when used with complex geometries. A considerable advance was obtained with the use of parametric *splines* representations of the boundary contours. They allow a local and require few input data. Among the different types of parametric *splines*, like the is the cubic B-*splines*, which are the most employed due to the satisfaction of the majority of practical requirements, see (Mortenson 1997 e Hinton e Sienz, 1997). Due to the simplicity and the versatility, this work adopted the parametric cubic B-*splines*, which may be represented as:

$$[\mathbf{p}(t)]^T = [p_x(t), p_y(t), p_z(t)]=[\mathbf{t}][\mathbb{R}_{\mathrm{BS}}][\mathbf{b}_{\mathrm{BS}}], \tag{18}$$

in which

$$[\mathbf{t}] = \{t^3\ t^2\ t\ 1\},\ 0 \le t \le 1, \qquad [\mathbb{R}_{BS}] = \frac{1}{6}\begin{bmatrix} -1 & 3 & -3 & 1 \\ 3 & -6 & 3 & 0 \\ -3 & 0 & 3 & 0 \\ 1 & 4 & 1 & 0 \end{bmatrix}, \qquad [\mathbf{b}_{BS}] = \begin{Bmatrix} b_{i-1} \\ b_i \\ b_{i+1} \\ b_{i+2} \end{Bmatrix},\ 1 \le i \le k-1, \tag{19}$$

where b_{i-1}, b_i, b_{i+1} e b_{i+2} are the position vectors of the vertices of control B_{i-1}, B_i, B_{i+1} e B_{i+2}, of the representative polygons that determine each curve segment and $(k-1)$ is the number of segment curves. (See Figure 1)

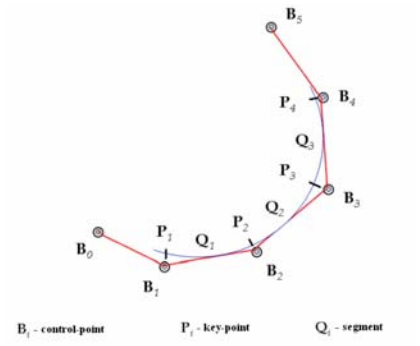

Figure 1. Cubic B-*spline* segment passing through a set of *key points* (P_i) .

Here, the design variables of the optimization problem are the position vectors of the *key points* (P_i) that define the curve segments, denoted by **s**, which describe the domain contour by the cubic B-*splines* segments.

• **Objective function:** is given by the dissipation of the viscous flow integrated in the entire domain Ω :

$$f(\mathbf{u}(\mathbf{x}(\mathbf{s})),\mathbf{s}) = 2\nu \left\langle D(\mathbf{u}(\mathbf{x}(\mathbf{s}))), D(\mathbf{u}(\mathbf{x}(\mathbf{s}))) \right\rangle_{\Omega(\mathbf{s})} = 2\nu \iint_{\Omega(\mathbf{s})} \{ D(\mathbf{u}(\mathbf{x}(\mathbf{s}))) \cdot D(\mathbf{u}(\mathbf{x}(\mathbf{s}))) \} d\Omega. \tag{20}$$

The finite element discretization of Eq. (20) is given by:

$$f(\mathbf{q}(\mathbf{s}),\mathbf{s}) = [\mathbb{K}^{\nu}(\mathbf{s})]\mathbf{q}(\mathbf{s}) \cdot \mathbf{q}(\mathbf{s}), \quad \text{where} \quad [\mathbb{K}^{\nu}(\mathbf{s})] = \sum_{e=1}^{ne} \left[\mathbb{k}_e^{\nu}(\mathbf{s}) \right] = \sum_{e=1}^{ne} \left[\iint_{\Omega_e} [\mathbb{B}(\mathbf{s})]^T [\mathbb{H}^{\nu}][\mathbb{B}(\mathbf{s})] d\Omega \right]. \tag{21}$$

• **Definition of the optimization problem:** the discrete optimization problem may be formulated as: Find $\mathbf{s} \in \mathbb{R}^n$ solution of

$$\begin{cases} \min\limits_{s/a} f(\mathbf{q}(\mathbf{s}),\mathbf{s}) \\ \mathbf{g}(\mathbf{s}) \le 0, \quad \forall \mathbf{s} \in S, \end{cases} \tag{22}$$

where $S = \left\{ \mathbf{s} \middle| s_i^{\inf} \le s_i \le s_i^{\sup},\ i = 1,..,n \right\}$ represents the set of lateral constraints, $\mathbf{g}(\mathbf{s}) = \Omega(\mathbf{s}) - \Omega^{\sup} \le 0$ a volumetric constraint and $\mathbf{q}(\mathbf{s})$ is the vector of nodal velocities and pressure associated with the finite element discretization of the flow problem.

Applying the Augmented Lagrangian method we are able to transform Eq.(22) into the solution of a sequence of bound constrained problems, formulated as:

Determine $\mathbf{s}^* \in \mathbb{R}^n$ such that $\mathbf{s}^* = \lim\limits_{k\to\infty} \mathbf{s}_k^*$ where $\mathbf{s}_k^*$ is the solution of the problem:

Given $\boldsymbol{\mu}^k \in \mathbb{R}^m$ and $\varepsilon > 0$ determine $\mathbf{s}_k^*$, which is the solution of:

$$\begin{cases} \min\limits_{s/a} \chi(\mathbf{s}, \boldsymbol{\mu}^k, \varepsilon^k) \\ \mathbf{s} \in S, \end{cases} \tag{23}$$

where

$$\chi(\mathbf{s},\mu^k,\varepsilon^k) = f(\mathbf{s}) + \frac{1}{2\varepsilon^k}\sum_{j=1}^{m}\Psi_j(g(\mathbf{s}),\varepsilon^k\mu_j^k) \text{ , with } \Psi_j(g_j(\mathbf{s}),\varepsilon^k\mu_j^k) = \begin{cases}[2\varepsilon^k\mu_j^k + g_j(\mathbf{s})]g_j(\mathbf{s}) \text{ if } g_j(\mathbf{s}) \geq -\varepsilon^k\mu_j^k \\ -(\varepsilon^k\mu_j^k)^2 \text{ if } g_j(\mathbf{s}) < -\varepsilon^k\mu_j^k\end{cases} \quad (24)$$

The Lagrange multiplier vector μ and the penalty parameters $\varepsilon > 0$, are then updated in order to completely define the iterative process. The actualization of these parameters is given in Steffens (2005). The convergence criterion used in the work is the Kuhn-Tucker optimality criterion, (Arora, 1989).

• **Sensitivity analysis:** The sensitivity analysis requires the determination of the gradient of some response function, with respect to the design variables. Generally, these functions are implicit and non linear with respect to the design variables, what make their gradient difficult to determine.

Consider the objective function, defined in Eq. (21), which depends explicitly and implicitly with respect to **s**. The partial derivative of f with respect to s_j is given by:

$$\frac{df}{ds_j} = \frac{\partial f}{\partial s_j} + \frac{\partial f}{\partial \mathbf{q}}\frac{d\mathbf{q}}{ds_j},\ j = 1,..,J = \text{number of design variables.} \quad (25)$$

The derivatives of f with respect to **q** and s_j are simple to be computed. In general, the determination of $d\mathbf{q}/ds_j$ is more complex. One way to circumvent the necessity of determining $d\mathbf{q}/ds_j$ is to apply the adjoint method. In this case, the objected function is redefined, and denoted by $\hat{f}$, by the introduction of the adjoint vector λ, and given by

$$\hat{f} = f + \langle \lambda, \mathbf{R} \rangle, \quad (26)$$

where $\mathbf{R}$ is the residual vector defined in the discrete form of the solution of the non-linear flow problem. Thus, since in the equilibrium condition $\mathbf{R} = \mathbf{0}$, we may choose λ arbitrarily and $\langle \lambda, \mathbf{R} \rangle = 0$. The gradient of the modified objective function is then given by:

$$\frac{d\hat{f}}{ds_j} = \frac{df}{ds_j} + \left\langle \lambda, \frac{d\mathbf{R}}{ds_j} \right\rangle = \frac{\partial f}{\partial s_j} + \left\langle \frac{\partial f}{\partial \mathbf{q}}, \frac{d\mathbf{q}}{ds_j} \right\rangle + \left\langle \lambda, \frac{\partial \mathbf{R}}{\partial s_j} + \frac{\partial \mathbf{R}}{\partial \mathbf{q}}\frac{d\mathbf{q}}{ds_j} \right\rangle = \frac{\partial f}{\partial s_j} + \left\langle \lambda, \frac{\partial \mathbf{R}}{\partial s_j} \right\rangle + \left\langle \frac{\partial f}{\partial \mathbf{q}} + \lambda[\mathbf{K_T}], \frac{d\mathbf{q}}{ds_j} \right\rangle, \quad (27)$$

in which $\partial \mathbf{R}/\partial \mathbf{q} = [\mathbf{K_T}]$, where $[\mathbf{K_T}]$ is the tangent matrix obtained in the solution of the fluid flow problem, when solved by Newton's method. Now, since λ can be chosen arbitrarily, it can be selected conveniently in order to eliminate the necessity of computing $d\mathbf{q}/ds_j$. Hence, λ is selected so that it is the solution of

$$[\mathbf{K_T}]^T \lambda = -\frac{\partial f}{\partial \mathbf{q}}. \quad (28)$$

Introducing the solution λ^* of Eq. (28) into Eq. (27), we derive:

$$\frac{d\hat{f}}{ds_j} = \frac{\partial f}{\partial s_j} + \left\langle \lambda^*, \frac{\partial \mathbf{R}}{\partial s_j} \right\rangle,\ j = 1,..,J. \quad (29)$$

Thus, in order to compute the gradient of the modified objective function, we compute the adjoint vector λ, solution of Eq. (28), and use the tangent matrix $[\mathbf{K_T}]$, since it has been already computed in the solution of the flow problem.

4. Problem cases

Consider the problems illustrated in Figures 2a and 2b, representing the flow of an incompressible fluid in a divergent channel, where are prescribed a prescribed flow velocity in the inlet, a traction free surface in the outlet, a no slip condition on the wall and a symmetry condition on the upper boundary. The cases to be considered are:

a) The optimization of a divergent channel;
b) The optimization of as divergent channel with an obstacle.

The problem consists in finding the optimal shape of for the domain in order to minimize the total dissipation of the flow, subjected to lateral constraints and a volume constraint. The Figures 2a and 2b show the geometries to be optimized, while Figures 3a and 3b illustrate the definition of the segments of the parameterized B-*splines* used to describe the domain contour.

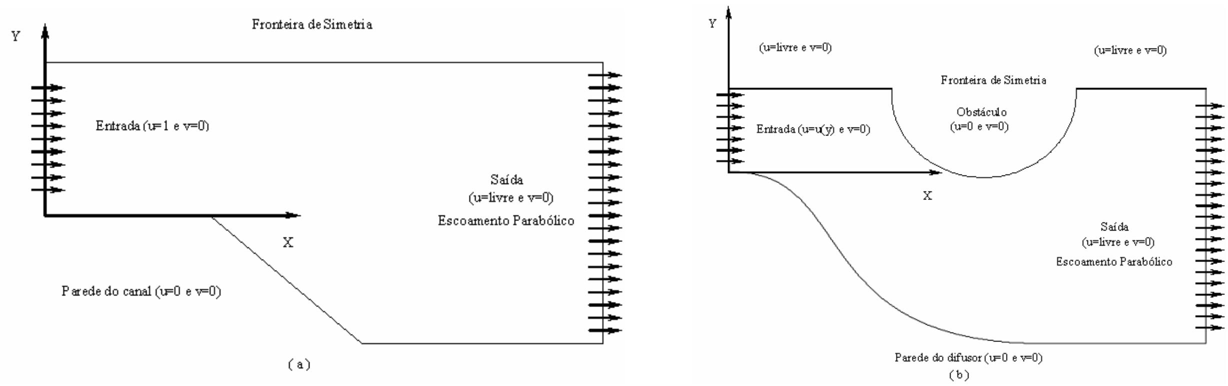

Figure 2. a) - Diffuser with a ramp to be optimized. b) - Diffuser with an obstacle to be optimized.

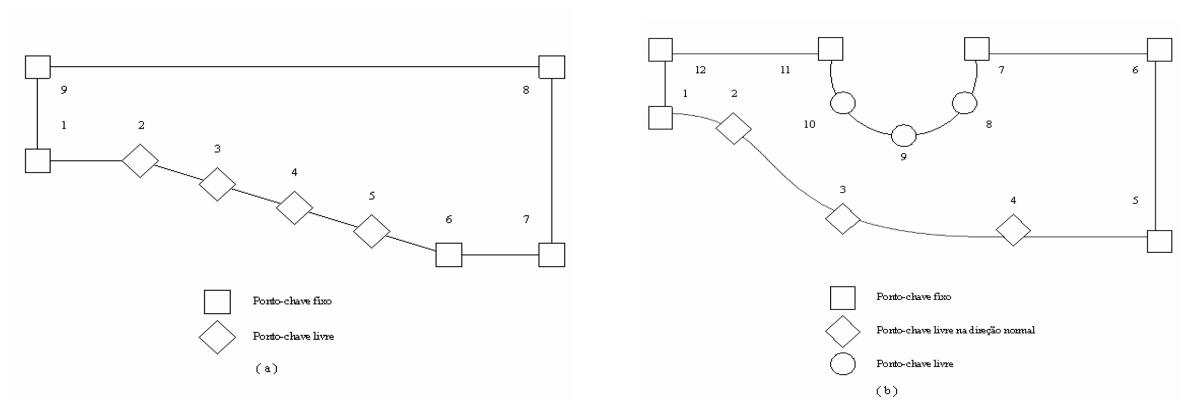

Figure 3. Definition of the *key points*.

In case a) the contour is described by 9 *key points*, 6 cubic B-*splines* segments, having 8 design variables defining the optimization problem and the corresponding degrees of freedom of each such key point, i.e., the form of their movement.

In case b) the boundary is described by 12 *key points*, 6 cubic B-*splines* segments, having 9 design variables. Notice that, the design variables are concentrated in the segments that represent the regions to be optimized.

The volume constraint imposed in both cases allows an increase of the initial volume to at most 10 %. The lateral constraints are given for case a) by:

$0,93 \leq x_2 \leq 1,07$; $-0,15 \leq y_2 \leq 0,1$; $1,8133 \leq x_3 \leq 1,8533$; $-0,4 \leq y_3 \leq -0,15$;
$2,6466 \leq x_4 \leq 2,6866$; $-0,65 \leq y_4 \leq -0,4$; $3,4799 \leq x_5 \leq 3,5199$; $-0,9 \leq y_5 \leq -0,65$;

and for case b):

$-0,8 \leq y_2 \leq 0$; $-2,5 \leq y_3 \leq -1,5$; $-3 \leq y_4 \leq -2,8$; $6,9 \leq x_8 \leq 7,1$;
$0,4 \leq y_8 \leq 0,7$; $5,4 \leq x_9 \leq 5,6$; $-0,7 \leq y_9 \leq -0,3$; $3,9 \leq x_{10} \leq 4,1$;
$0,4 \leq y_{10} \leq 0,7$;

The optimization analysis considered flows with a Reynolds of 100 and a tolerance of 10^{-4} for the Newton's iterative method. In cases a) and b) meshes with 1957 and 774 triangular elements. Here, we used a (Tri7/Tri3) element, using 7 points for the interpolation of the velocities field and 3 points for the interpolation of the pressure field. These meshes can be visualized in Figure 4.

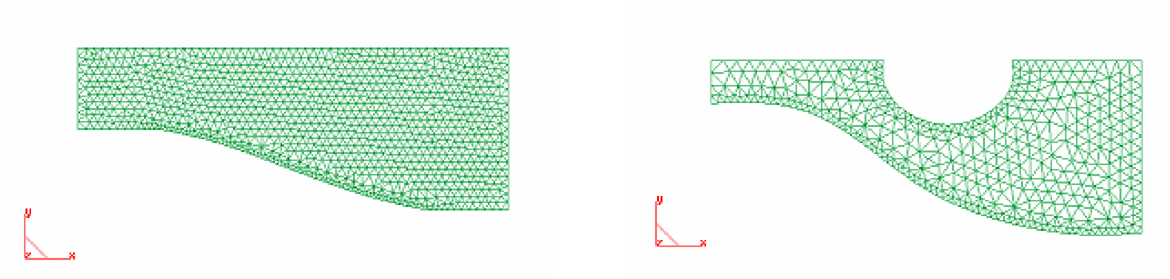

Figure 4. Meshes in the configuration initial.

Some of the results obtained in the flow analysis, obtained in the optimal shape configuration, are illustrated in Figure 5, for the fluid velocity field and Figure 6 for the fluid pressure results.

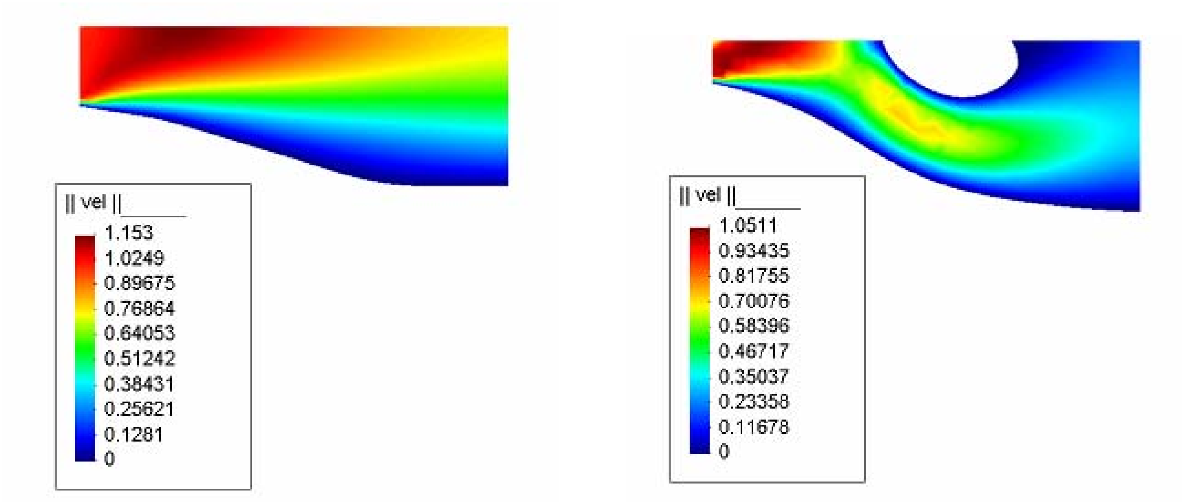

Figure 5. Euclidean norm of the velocity field.

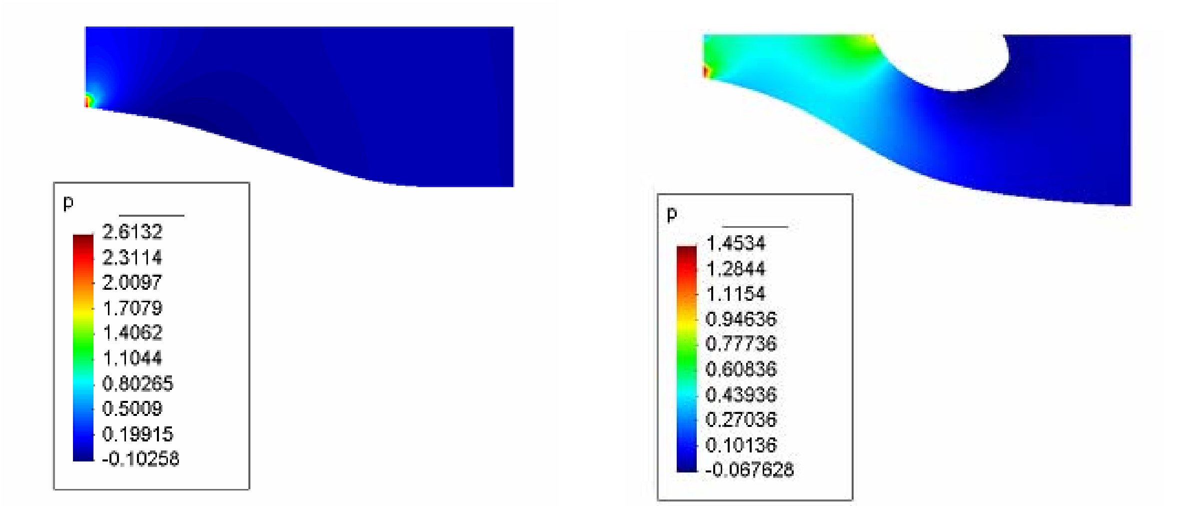

Figure 6. Pressure field.

The graphs in Figure 7 show the initial an optimal position of the boundary for both diffuser cases, enabling a proper comparison between the initial and the optimal final shape.

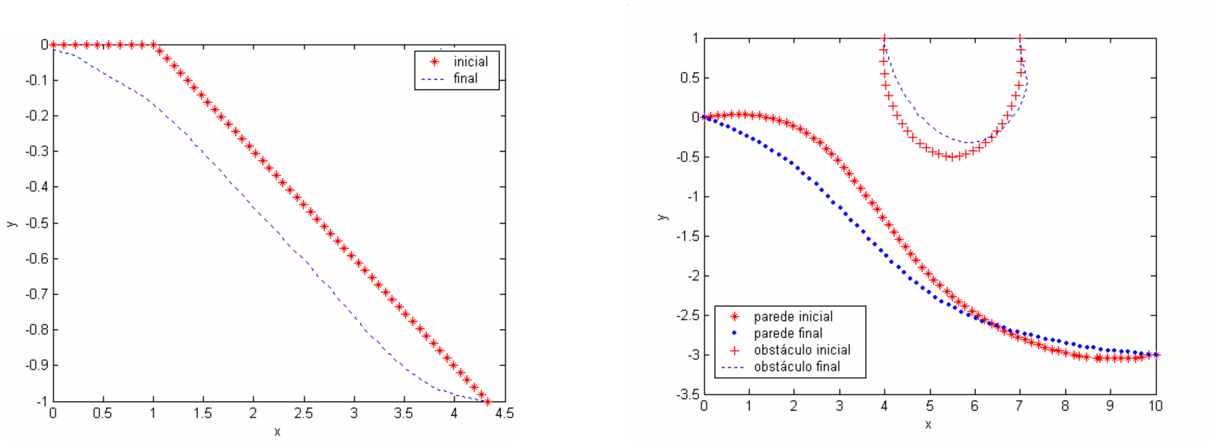

Figure 7. Initial and final position of the boundary at the wall and obstacle.

The results obtained for the reduction of the flow dissipations were significant. In case a) the reduction vas of approximately 28% (f_{initial} = 1.20751 e f_{otimal} = 0.79565), while in case b) was of approximately 55% (f_{initial} = 2.66943 and f_{otimal} = 1.21596).

From the figures (4), (5), and (6), we can observe the smoothening of the boundary due to the usage of the B-*splines* for the definition of the contour, what contributed for the considerable reduction of the flow dissipation.

Analysing the initial and final optimal shapes of the diffuser, we can notice a broadening of the channel in the ramp region causing a decrease of the flow velocity and of the velocity gradient reducing in this way the value of the objective function. In the case of a diffuser with an obstacle, the optimal shape presents an obstacle with a shape that seemed to be dragged by the flow, while satisfying the constraints, causing in this way a reduction of the influence of the obstacle to the flow, implying in a reduction of the objective function.

These results support the normal intuition that we would expect to have in these flow problems.

5. Conclusions

The discretization adopted by using the T7/C3 triangular element, for the solution of the flow problem, have shown to be adequate since they satisfy the Brezzi-Babuska (BB) condition with no need of additional terms. This simplifies considerably the work necessary to compute the sensitivity of the response of the flow problem with respect to the design variables.

The parametric representation of the boundary by cubic B-*splines* have shown to be very effective, since they not only allowed a significant reduction of the objective function but also, due to their simplicity of use and their ability to control locally the boundary, required a small number of design variables to obtain these results.

By using cubic B-*splines*, it's possible to represent smoothly any contour, with irregular boundaries and with complex obstacles, in a natural and simple way, requiring a few input data parameters, what simplifies the handling and control of the *key points* that are used to describe the countour.

6. References

Aris, R., 1989, "Vectors, Tensors, and the Basic Equations of Fluid Mechanics", Dover, New York.

Arora, J. S., 1989, "Introduction to Optimum Design", McGraw-Hill, New York.

Codina, R., 2000, "On Stabilized Finite Element Methods for Linear Sistems of Convection-Diffusion-Reaction Equations", Comp. Methods Appl. Mech. Eng., Vol. 188, pp. 61-82.

Hinton, E., Sienz J., 1997, "Reliable Structural Optimization with Error Estimation, Adaptivity and Robust Sensitivity Analysis", Computers & Structures, Vol. 4, pp. 31-63.

Mortenson, M.E., 1997, "Geometric Modeling", John Wiley & Sons, New York.

Reddy, B.D., 1997, "Introductory Functional Analysis with Applications to Boundary Value Problems Finite Elements", Spinger, New York.

Steffens, L.M., 2005, "Uma Metodologia Integrada para Otimização de Forma de Escoamentos de Fluidos", Dissertação de Mestrado, Dept. Engenharia Mecânica, Universidade Federal de Santa Catarina, Florianópolis, 2005.

Temam, R., 1991,"Navier-Stokes Equations", Elsevier Science Publishers Company, Inc., New York.

7. Responsibility notice

The authors Lindaura Maria Steffens, Marcelo Krajnc Alves and Hilbeth Azikri Parente de Deus, are the only responsible for the printed material included in this paper.