CONSTRUCTAL DESIGN OF A TRAPEZOIDAL ISOTHERMAL CAVITY INTO A SOLID CONDUCTING WALL

E. D. dos Santos
Dep. de Física, Fundação Universidade Federal do Rio Grande, Av. Itália, km 8, Cx. P. 474, Rio Grande, 96201-900, Brazil.

L. F. Scaglioni1
Dep. de Física, Fundação Universidade Federal do Rio Grande, Av. Itália, km 8, Cx. P. 474, Rio Grande, 96201-900, Brazil

J. A. Souza
Dep. de Física, Fundação Universidade Federal do Rio Grande, Av. Itália, km 8, Cx. P. 474, Rio Grande, 96201-900, Brazil

L. A. O. Rocha
Dep. de Física, Fundação Universidade Federal do Rio Grande, Av. Itália, km 8, Cx. P. 474, Rio Grande, 96201-900, Brazil
laorocha@brturbo.com.br

Abstract. *This paper reports, according to Bejan's constructal Theory, the geometric optimization of a trapezoidal cavity that intrudes into a solid conducting wall. The objective is to minimizing the global thermal resistance between the solid and the cavity. There is uniform heat generation on the solid wall. The trapezoidal cavity is isothermal and the solid conducting wall is isolated on the external perimeter. The total volume and the cavity volume are fixed with variable aspect ratios. One way to search for best shapes, according to Constructal Design, is increasing their complexity. In this sense, the trapezoidal cavity is optimized using three degrees of freedom. When compared to rectangular, triangular and elliptical cavities under the same thermal conditions the trapezoidal cavity performs much better.*

Keywords*: Constructal design, trapezoidal cavity, geometric optimization, heat conduction*

1. Introduction

Geometry matters. This is the lesson we have learned from Bejan's Constructal Theory (Bejan, 2000). The principle is the same in engineering and nature: the optimization of flow systems subjected to constraints generates shape and structure (Rosa *et al.*, 2004).

The field of heat transfer has demonstrated for many years how the principle of generating flow geometry works. The oldest and most clear illustration is the optimization of fins. Recent work on this subject account the optimization of the architecture of assemblies of fins (Kraus *et al.*, 2001). Many more examples are found in the growing volume of techniques for the cooling of compact and miniaturized packages of electronics (Peterson and Ortega, 1990, Kim and Lee, 1995, and Bar-Cohen and Kraus, 1998).

This paper proposes the study of the constructal design of another, equally basic feature of a solid wall with heat transfer: the open cavity. Open cavities, for example, are the regions formed between adjacent fins. They are essential promoters of nucleate boiling and condensation: see, for example, the vapotron effect (Biserni *et al.*, 2001, and Falter and Thompson, 1996). Open cavities are also important morphological features in physiology (Bejan, 2000).

We consider the optimization of the cavity shape in the most fundamental sense, without application to a particular device or field. We rely on the constructal method (Bejan, 2000, Biserni *et al.*, 2004, and Rocha *et al.*, 2005): the cavity shape is free to change subject to volume constraints, and in the pursuit of maximal global performance. The global performance indicator is the overall thermal resistance between the volume of the entire system (cavity and solid) and the surroundings.

This work considers the two-dimensional geometries where the conducting wall is a rectangular surface while the cavity is represented by a trapezoidal surface. The conducting wall and the trapezoidal cavity have variable aspect ratios which are the degrees of freedom to be studied and optimized. The performance of the optimized trapezoidal cavity is also compared with the performance of the optimized triangular, elliptical and rectangular cavities (Rocha *et al.*, 2004).

2. Trapezoidal Cavity

Consider the two-dimensional conducting body shown in Fig. (1). The external dimensions (H, L) vary. The third dimension, W, is perpendicular to the plane of the figure. The total volume occupied by this body is fixed,

$$V = HLW \tag{1}$$

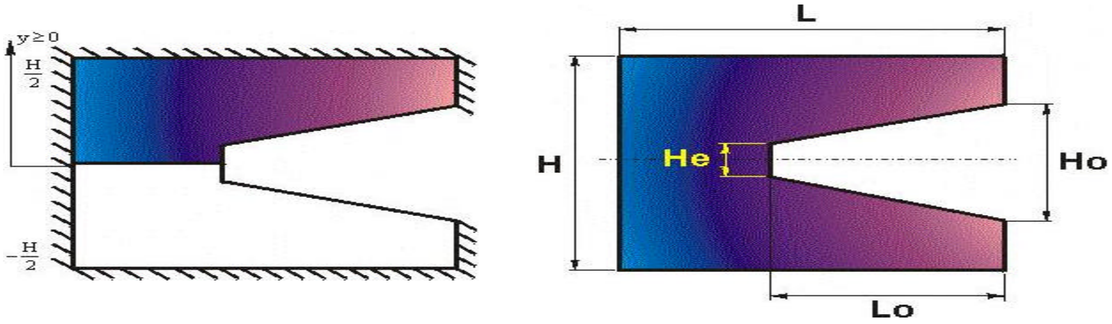

Figure 1. Isothermal trapezoidal cavity into a two-dimensional conducting body with uniform heat generation

Alternatively, the area $A = HL$ is fixed. The dimensions of the trapezoidal cavity (*He, H0, L0*) also vary. The trapezoidal cavity volume is fixed,

$$V_0 = L_0 W\left(\frac{H_e + H_0}{2}\right) \tag{2}$$

The constant volume constraint is justified in many applications: the cost of material and the weight and space of the heat transfer device make this constraint indispensable in design work. In constructal design, this constraint is part of the mechanism of generating the optimal geometric form that fills a given space. This constraint may be replaced by the statement that the volume fraction occupied by the cavity is fixed,

$$f = \frac{V_0}{V} = \frac{((H_e + H_0)/2)L_0}{HL} \tag{3}$$

The solid is isotropic with the constant thermal conductivity k. It generates heat uniformly at the volumetric rate q''' [W/m3]. The outer surfaces of the heat generating body are perfectly insulated. The generated heat current ($q'''A$) is removed by cooling the wall of the cavity. The cavity wall temperature is maintained at T_{min}. Temperatures in the solid are higher than T_{min}. The highest temperatures (the "hot spots") are registered at points on the adiabatic perimeter, for example, in the two corners (0, ±H/2) in Fig. (1).

The isothermal cavity wall assumption is made for simplicity in demonstrating the construction of optimal cavity shape. This assumption means that the heat transfer coefficient on the internal (exposed) surface of the cavity is sufficiently large, so that wall conduction poses a larger thermal resistance than convection. This assumption can be relaxed in future applications of this constructal method. In a more realistic model, the cavity wall temperature and heat flux would be related, and the wall temperature distribution would vary with the shape of the cavity.

An important thermal design constraint is the requirement that temperatures must not exceed a certain level. This makes T_{max} a constraint. The design also calls for installing a maximum of heat generation rate in the fixed volume, which, for example, corresponds to packing the most electronics into a device of fixed size. In the present problem statement, this design objective is represented by the maximization of the global thermal conductance $q'''A/(T_{\max} - T_{\min})$, or by the minimization of the global thermal resistance $(T_{\max} - T_{\min})/(q'''A)$.

The numerical optimization of the geometry consisted of simulating the temperature field in a large number of configurations, calculating the global thermal resistance for each configuration, and selecting the configuration with the smallest global resistance. The conduction equation for the solid region is

$$\frac{\partial^2 \tilde{T}}{\partial \tilde{x}^2} + \frac{\partial^2 \tilde{T}}{\partial \tilde{y}^2} + 1 = 0 \tag{4}$$

where the dimensionless variables are

$$\tilde{T} = \frac{T - T_{\min}}{q''''A/k} \tag{5}$$

$$\left(\tilde{x}, \tilde{y}, \tilde{H}, \tilde{L}, \tilde{H}_e, \tilde{H}_0 \tilde{L}_0\right) = \frac{(x, y, H, L, H_e H_0, L_0)}{A^{1/2}} \tag{6}$$

The boundary conditions are insulated perimeter of the conducting wall and isothermal trapezoidal cavity ($\tilde{T} = 0$). The maximal excess temperature, $\tilde{T}_{\max}$, is also the dimensionless global thermal resistance of the construct,

$$\tilde{T}_{\max} = \frac{T_{\max} - T_{\min}}{q''' A / k} \tag{7}$$

3. Numerical Method

The global objective function $\tilde{T}_{\max}$ can be determined numerically, by solving for the temperature field in every assumed configuration, and then calculating $\tilde{T}_{\max}$ to see whether $\tilde{T}_{\max}$ can be minimized by varying the configuration. In this sense, equation (4) was solved using a finite elements code, based on triangular elements, developed in MATLAB environment, precisely the pde (partial-differential-equations) toolbox. The grid was non-uniform in both $\tilde{x}$ and $\tilde{y}$, and varied from one geometry to the next. The appropriate mesh size was determined by successive refinements, increasing the number of elements four times from the current mesh size to the next mesh size, until the criterion $\left|\left(\tilde{T}^{j}_{i,\max} - \tilde{T}^{j+1}_{i,\max}\right)/\tilde{T}^{j}_{i,\max}\right| < 2\times10^{-4}$ is satisfied. Here $\tilde{T}^{j}_{i,\max}$ represents the maximum temperature calculated using the current mesh size, and $\tilde{T}^{j+1}_{i,\max}$ corresponds to the maximum temperature using the next mesh, where the number of elements was increased by four times. Table 1 gives an example of how grid independence was achieved. The following results were performed by using a range between 2,000 and 20,000 triangular elements.

Table 1. Numerical tests showing the achievement of grid independence (f = 0.1, H/L = 1, H_0/L_0 = 0.1, H_e/H_0 = 1.2).

Number of elements	$\tilde{T}^{j}_{i,\max}$	$\left\|\left(\tilde{T}^{j}_{i,\max} - \tilde{T}^{j+1}_{i,\max}\right)/\tilde{T}^{j}_{i,\max}\right\|$
172	0.101127	9.295×10^{-4}
688	0.101033	3.563×10^{-4}
2752	0.100997	1.188×10^{-4}
11008	0.100985	

To test the accuracy of the numerical code, the numerical results obtained using our code in Matlab pde have been compared with the numerical results obtained by Biserni *et al.* (2004). Table 2 shows that the two sets of results agree at least 0.5%.

Table 2. Comparison between the results obtained using our MATLAB partial-differential- equations (pde) toolbox code and the results obtained by Biserni *et al.* (2004) (*) (f = 0.3, H/L = 1, H_e/H_0 = 1)

H_0/L_0	$\tilde{T}_{\max}$	$\tilde{T}_{\max}$ (*)	$\left\|\left(\tilde{T}_{\max} - \tilde{T}_{\max}(*)\right)/\tilde{T}_{\max}\right\|$
1.875	0.1871	0.1873	1.068×10^{-3}
1.2	0.1432	0.1435	2.094×10^{-3}
0.8334	0.1083	0.1086	2.770×10^{-3}
0.4686	0.0654	0.0657	4.587×10^{-3}

4. Results

The numerical work consisted of determining the temperature field in a large number of configurations of the type shown in Fig.1.

Figure 2 shows that the thermal resistance can be minimized by selecting the aspect ratio H_0/L_0 of the cavity. The thermal resistance decreases when the aspect ratio H_0/L_0 also decreases and reaches its minimum when the cavity penetrates almost completely into the solid wall. This result agrees with the one reported by Biserni *et al.* (2004). The minimal thermal resistance reported in Fig. 2 is labeled $\left(\tilde{T}_{\max}\right)_m$ and its corresponding optimal aspect ratio is labeled $(H_0/L_0)_0$.

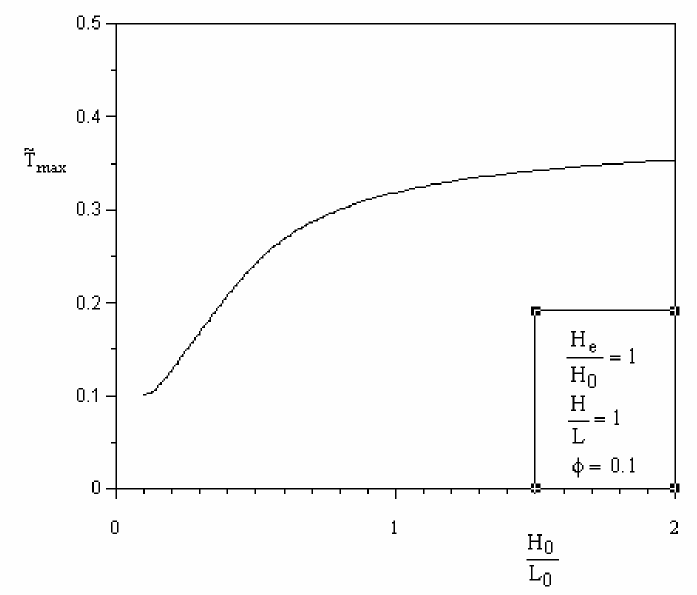

Figure 2. The minimization of the global thermal resistance when the external shape of the heat generation body is fixed and the cavity is rectangular

The second level of the numerical optimization scheme consisted of repeating the preceding work (Fig. 2) for many values of the second shape aspect ratio, H_e/H_0. Figure 3 shows that the thermal resistance also can be minimized with respect this aspect ratio. The optimal aspect ratio $(H_0/L_0)_0$ decreases when the aspect ratio H_e/H_0 increases and it is also reported in Fig. 3. The minimal thermal resistance reached in Fig. 3 is labeled $(\tilde{T}_{\max})_{mm}$ and the corresponding best shapes are labeled $(H_0/L_0)_\infty$ and $(H_e/L_0)_0$.

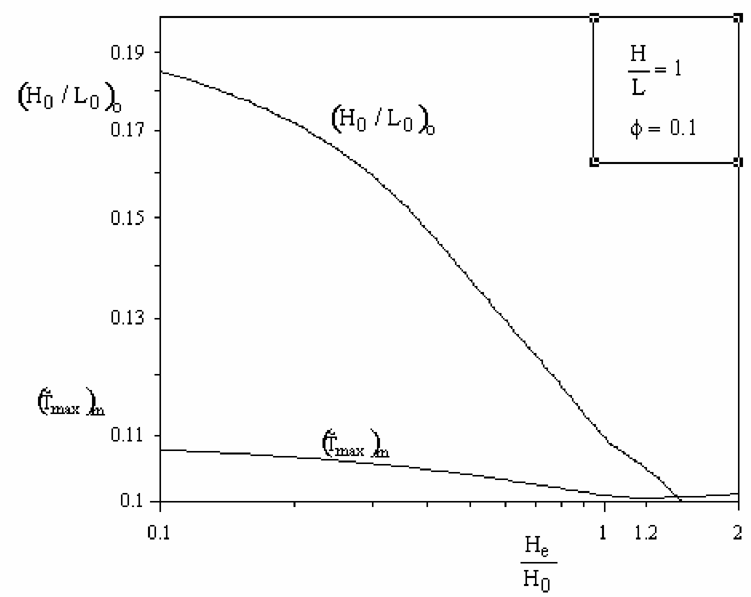

Figure 3. The minimized global thermal resistance and the optimized ratio $(H_0/L_0)_o$ as function of the ratio H_e/H_0 as the external shape of the cavity is fixed.

The last level of optimization is conducted varying the external shape of the cavity, *H/L*. The preceding work showed in Fig. 3 is repeated now and the optimal results for each *H/L* is reported in Fig. 4. The most important finding is that an optimal *H/L* ratio does not exist, i.e., the geometry of Fig. 1 can be optimized only with respect to two degrees of freedom. Figure 4 also shows that the doubled minimized thermal resistance reaches its maximum value when

H/L = 2. Performance increases as the value of *H/L* is increased or decreased. Some of the best shapes calculated in Fig. 4 are reported in scale in Fig. 5.

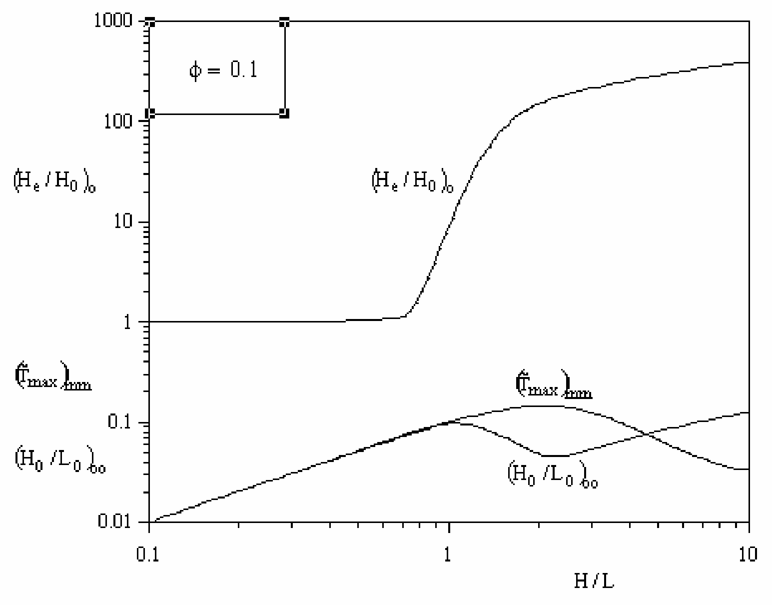

Figure 4 The behavior of the minimized global thermal resistance and the optimized geometry as the external shape of the cavity varies.

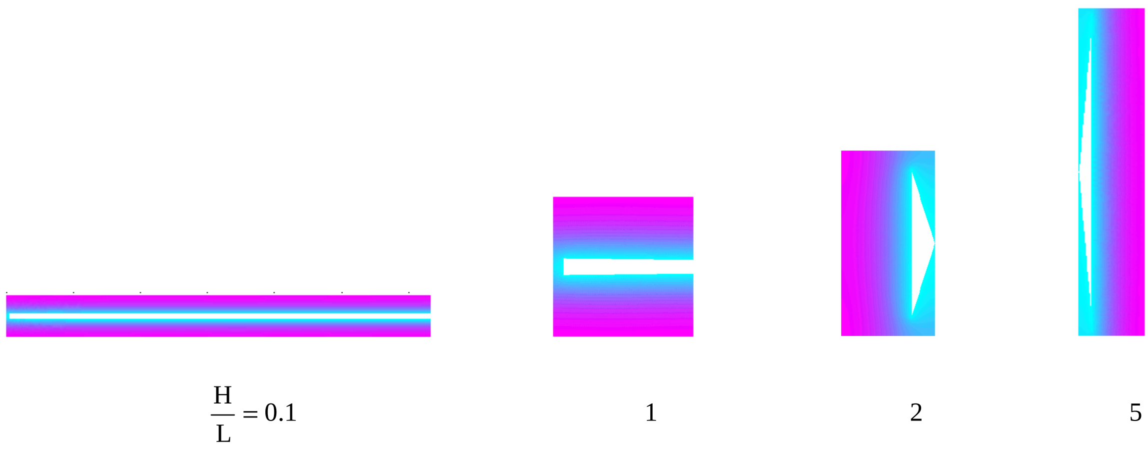

Figure 5. Best shapes reported in Fig. 4 (in scale).

5. Comparison among the rectangular, elliptical and triangular cavities

The minimal thermal resistance reported in Fig. 4 as function of the ratio *H/L* is compared in Fig. 6 with the ones reported by Rocha *et al.* (2004) namely, rectangular, triangular and elliptical cavities. Figure 6 shows that the trapezoidal cavity has better performance than the triangular and elliptical cavities. It has approximately the same performance when compared with the rectangular cavity when *H/L* is smaller than one, i.e., when the cavity is slender. This happens because the best cavity penetrates almost completely into the cavity in these cases. However, when *H/L* > 1, the trapezoidal cavity performs better than the rectangular and this effect is larger as the ratio *H/L* increases. We expected this trapezoidal cavity best performance because increasing the degrees of freedom, i.e., increasing the complexity is one of the directions of shape and structure can be improved (Bejan, 2000).

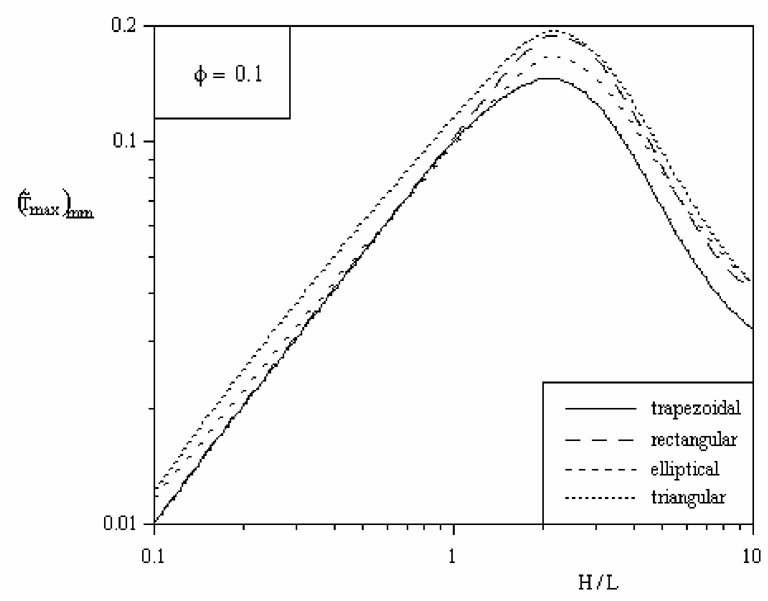

Figure 6. The effect of the external shape *H/L* on the global thermal resistance minimized as in Fig. 2, 4 and 6.

7. Conclusions

This work considers the two-dimensional geometries where the conducting wall is a rectangular surface while the cavity is represented by a trapezoidal surface. The results show that the thermal resistance can be minimized with respect the ratios H_0/L_0 and H_e/H_0. However, there is not an optimal *H/L* ratio. The doubled minimized thermal resistance reaches its maximum value when $H/L = 2$. Performance increases as the value of *H/L* is increased or decreased. This behavior is also presented approximately for all the studied shapes, i.e., this result is independent of the shape of the cavity.

We also show that the trapezoidal cavity performs better than the elliptical and triangular ones. The trapezoidal cavity has approximately the same performance than the rectangular one when $H/L < 1$, i.e., the cavity is slender. However, when $H/L > 1$, the trapezoidal cavity performs better than the rectangular and this effect is larger as the ratio *H/L* increases. The trapezoidal cavity performs better because it is more complex and this is one of the directions that shape and structure can be optimized.

Since we assume isothermal cavity, future work may extend this investigation to the case where the heat transfer on the internal surface of the cavity is accounted for by a constant heat transfer coefficient. We also can conduct this study exploring other values of the fraction volume f.

8. Acknowledgements

Mr. Santos' work was supported by CNPq, Brasilia, Brazil.

9. References

Bar-Cohen, A. and Kraus, A. D., 1998, “Advances in Thermal Modeling of Electronic Components and Systems”, Vol. 4, ASME Press Series.
Bejan, A., 2000, “Shape and Structure, from Engineering to Nature”, Cambridge University Press, Cambridge, UK.
Bejan, A., Rocha, L. A. O. and Lorente, S., 2002, “Thermodynamic optimization of geometry: T- and Y-shaped constructs of fluid streams”, Int. J. Thermal Sciences 39, pp. 949-960.
Biserni, C., Lorenzini, G. and Orlandelli, C. M., 2001, “Frequency analysis of the vapotron effect in forced convection: a challenge for numerical description of multiphase flows”, First Int. Conf. Computational Methods in Multiphase Flow, Multiphase Flow 2001, Orlando, FL, March 2001, WIT Press, pp. 173-182.
Biserni, C., Rocha, L. A. O. and Bejan, A., 2004, “Inverted fins: geometric optimisation of the intrusion into a conducting wall”, Int.J. Heat Mass Transfer 47, pp. 2577-2586
Falter, H. D. and Thompson, E., 1996, “Performance of hypervapotron beam-stopping elements at jet”, Fusion Technology 29, pp. 584-594.
Kim, S. J. and Lee, S. W., 1995, “Air Cooling Technology for Electronic Equipment”, CRC Press, Boca Raton, FL.
Kraus, A. D., Aziz, A. and Welty, J., 2001, “Extended Surface Heat Transfer”, Wiley, New York.
MATLAB, User's Guide, Version 6.0.088, release 12, 2000, The Mathworks, Inc.

Peterson, G. P. and Ortega, A., “Thermal control of electronic equipment and devices”, 1990, Adv. Heat Transfer 20, pp. 181-314.

Rocha, L. A. O., Montanari, G. C., Santos, E. D. dos and Rocha, A. da S., 2004, “Constructal Design applied to the study of cavities into a solid conducting wall”, Proceedings of 10th Brazilian Congress of Thermal Sciences and Engineering , Rio de Janeiro, Brazil.

Rocha, L. A. O., Lorenzini, E., and Biserni, C., 2005, “Geometric optimization of shapes on the basis of Bejan’s Constructal Theory”, International Communications Heat and Mass Transfer (accepted to publication).

Rosa, R.N., Reis, A.H., and Miguel A.F., 2004, “Bejan’s Constructal theory of Shape and Structure”, Evora Geophysics Center, University of Evora, Portugal.

10. Responsibility notice

The authors are the only responsible for the printed material included in this paper.