UNDERWATER VEHICLE DYNAMIC MODELING

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Abstract. *This paper presents an approach to develop dynamic models of underwater vehicles which have open frame architecture and symmetrical planes. A ROV (Remotely Operated Vehicle) was especially constructed to perform experimental results to validate the used dynamic model approach. This dynamic model validation was performed through confrontation between open loop experiments and simulation results. One of the motivations to develop this research is that the underwater robotics in Brazil is a recent scientific domain, mainly in experimental aspects. It was verified that it is very difficult to obtain two identical open loop responses in two experiments made under the same conditions. This happens because it is difficult to reproduce the initial state of the ROV and the water conditions. Comparisons between experimental and simulation results in open loop showed that the dynamic model reproduce relatively well the experiments, indicating that the model approach can be used to ROVs or AUVs(Autonomous Underwater Vehicles) with symmetrical planes and open frame architecture.*
***Keywords**: Underwater vehicles, kinematics, dynamics, robotics, ROV.*

1. Introduction

The main objective of the present work is to validate a dynamic model of an underwater vehicle, actively controlled in four degrees of freedom and having an open frame architecture and well approximately three symmetrical planes. The vehicle used to obtain the experimental results is a ROV (Remotely Operated Vehicle), constructed specially to validate experimentally the dynamic modeling theory (Moraes *et al*., 2005).

Most of the bibliography divides the underwater robotic vehicles in two groups (Yuh, 2000): Remotely Operated Vehicles (ROV) and Autonomous Underwater Vehicles (AUV). A ROV receives energy and changes information with the panel of control placed at the surface through an umbilical cable. From the control panel the operator can plan tasks or use one joystick to maneuver the vehicle directly. An AUV does not suffer the intervention from the human operator during the mission and also it does not possess umbilical cable. The power plant is onboard the vehicle, as well as the central processing unit. Due to the inexistence of handle the vehicles, they are independents and have greater freedom of movement and its use grows up because of the advances in the processors and in the ways of energy storage, that guarantee a bigger autonomy of them.

The domain of underwater robotics is not yet very developed in Brazil, compared with international recent works. One of the first works was the master dissertation of Dominguez (1989), in which it was developed a ROV dynamic model software simulation. After this work, Cunha (1992) proposed an adaptive control to track the position of a ROV. In a more recent work, Hsu *et al.*(2000) proposed a procedure to dynamic model identification of actuators used in ROVs and AUVs (motor with helices). Barros and Soares (2002) showed a proposition of a low cost vehicle which may be in ROV or AUV format. Souza and Maruyama (2002) investigated the performance of some position control laws applied to underwater vehicles. Tavares (2003) presented a basic review work in dynamic modeling and control of underwater vehicles, restricted to simulation results.

In the world-wide level there are a great number of published works on the domain of underwater vehicles. An important work was developed by Fossen (1994), in which there are concepts of kinematics, dynamics and control. Fossen and Fjellstad (1995) and Ridao *et al.* (2001) worked with modeling of the interaction between fluid and structure. There are many vehicles developed mainly by universities or research centers of the USA and Europe (Aoki *et al*., 1999, Chardard and Copros, 2002, Koh *et al*., 2002, Liddle, 1986, Newman e Stakes, 1994). Observing the international references on underwater robotics, it can be seen that, even out of Brazil, the experimental research increased intensively in the Nineties.

2. Experimental setup

A ROV was constructed at the Applied Mathematics and Control Laboratory of the Federal University of Rio Grande (FURG, RS, Brasil), actively controlled in four degrees of freedom and having two degrees of freedom with passive control. A photograph of this vehicle can be seen in Fig. 1. A few project specifications were established, and the actuators, sensors, structure and onboard electronics were projected and constructed. The ROV developed, named ROVFURG-I, has four actuators (motor with helices) and three sensors: two accelerometers (each one working in two axis) and one gyroscope. The two degrees of freedom with passive control are the motions in roll and pitch angles. It was identified the current versus actuator force for each one of the four actuators (Moraes *et al.* (2005)).

Fig. 1. A photograph of the ROVFURG-I.

2. Kinematic Model

A rigid body in motion in space can be studied through two systems of reference: one inertial (system inertial, fixed to the Earth) and another fixed to the body (body system). These two systems are represented in Fig. 2. The body frame system was considered as placed with origin coinciding with the mass center of the vehicle, as showed in Fig. 3. The vector of linear velocity ($\boldsymbol{\nu}_1$) and the vector of angular velocity ($\boldsymbol{\nu}_2$) can be written on the body system as:

$$\boldsymbol{\nu}_1 = [u, v, w]^T,\ \boldsymbol{\nu}_2 = [\ p, q, r\ ]^T \tag{1}$$

These two vectors can be put in a unique body velocity vector, in the form:

$$\boldsymbol{\nu} = [\boldsymbol{\nu}_1^T, \boldsymbol{\nu}_2^{\ T}]^T \tag{2}$$

In analogous way, the vectors of linear and angular inertial velocities can be written as:

$$\dot{\boldsymbol{\eta}}_1 = [\dot{x}, \dot{y}, \dot{z}]^T, \dot{\boldsymbol{\eta}}_2 = [\dot{\phi}, \dot{\theta}, \dot{\psi}]^T, \tag{3}$$

These two vectors form the following vector of inertial velocities:

$$\dot{\boldsymbol{\eta}} = [\dot{\boldsymbol{\eta}}_1^T, \dot{\boldsymbol{\eta}}_2^T]^T \tag{4}$$

Integrating the inertial velocities one get the vectors of position ($\boldsymbol{\eta}_1$) and orientation ($\boldsymbol{\eta}_2$) of the vehicle:

$$\boldsymbol{\eta}_1 = [x, y, z]^T,\ \boldsymbol{\eta}_2 = [\phi, \theta, \psi]^T \tag{5}$$

and

$$\boldsymbol{\eta} = [\boldsymbol{\eta}_1^T, \boldsymbol{\eta}_2^T]^T, \tag{6}$$

where $\boldsymbol{\eta}_1 = [x, y, z]^T$ is the vector with the inertial position of the mass center, and $\boldsymbol{\eta}_2 = [\phi, \theta, \psi]^T$ is the vector with the Euler angles.

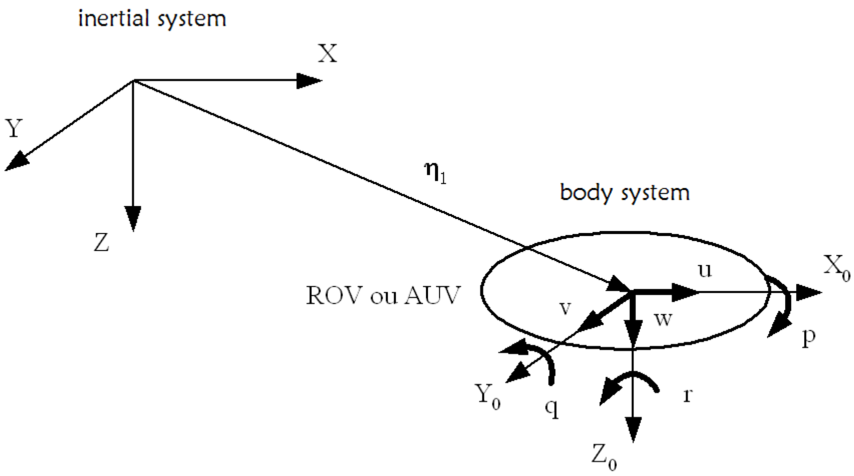

Figure 2. Reference systems.

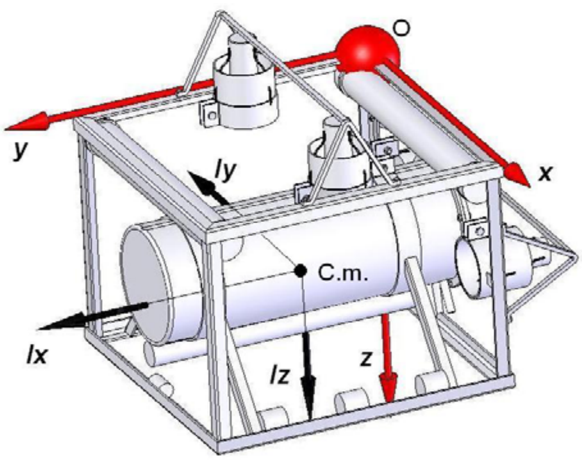

Figure 3. The underwater vehicle representation and the body frame system.

The force vector (τ_1) and the torque vector (τ_2) applied to the vehicle written on the body system are:

$$\boldsymbol{\tau_1} = [\Sigma X, \Sigma Y, \Sigma Z]^T, \quad \boldsymbol{\tau_2} = [\Sigma K, \Sigma M, \Sigma N]^T \tag{7}$$

and

$$\boldsymbol{\tau} = [\boldsymbol{\tau_1}^T, \boldsymbol{\tau_2}^T]^T \tag{8}$$

ΣX, ΣY and ΣZ represent the addition of the applied forces respectively in the directions X_0, Y_0 e Z_0 of the body frame. The addition of the moments applied about the axis X_0, Y_0 e Z_0 are represented by ΣK, ΣM e ΣN, respectively.

Using the Euler angles formulation, the transformation between the linear velocity body frame to the linear velocity inertial frame can be performed with the equation:

$$\dot{\boldsymbol{\eta}}_1 = \mathbf{J}_1(\boldsymbol{\eta}_2)\mathbf{v}_1 \tag{9}$$

where the transformation matrix has the form:

$$\mathbf{J}_1(\boldsymbol{\eta}_2)=\begin{bmatrix} c\psi c\theta & -s\psi c\phi + c\psi s\theta s\phi & s\psi s\phi + c\psi c\phi s\theta \\ s\psi c\theta & c\psi c\phi + s\phi s\theta s\psi & -c\psi s\phi + s\theta s\psi c\phi \\ -s\theta & c\theta s\phi & c\theta c\phi \end{bmatrix} \tag{10}$$

In the case of angular velocity, the transformation between the body frame to the inertial frame is performed with the equation:

$$\dot{\boldsymbol{\eta}}_2 = \mathbf{J}_2(\boldsymbol{\eta}_2)\mathbf{v}_2 \tag{11}$$

where the transformation matrix has the form:

$$\mathbf{J}_2(\boldsymbol{\eta}_2)=\begin{bmatrix} 1 & s\phi t\theta & c\phi t\theta \\ 0 & c\phi & -s\phi \\ 0 & s\phi / c\theta & c\phi / c\theta \end{bmatrix} \tag{12}$$

It can be seen that, with the Euler formulation, the angular velocity transformation matrix is not defined at θ = 90°. However, this position (pith angle = 90°) is very difficult to happen in practical motions of underwater vehicles.

The linear and angular velocities transformations may be defined with the following unique equation:

$$\dot{\boldsymbol{\eta}} = \mathbf{J}(\boldsymbol{\eta}_2)\mathbf{v} \tag{13}$$

where

$$\dot{\boldsymbol{\eta}} = \begin{bmatrix} \dot{\boldsymbol{\eta}}_1 \\ \dot{\boldsymbol{\eta}}_2 \end{bmatrix} \qquad \mathbf{J}(\boldsymbol{\eta}_2) = \begin{bmatrix} \mathbf{J}_1(\boldsymbol{\eta}_2) & \mathbf{0}_{3x3} \\ \mathbf{0}_{3x3} & \mathbf{J}_2(\boldsymbol{\eta}_2) \end{bmatrix} \qquad \mathbf{v} = \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix} \tag{14}$$

3. Dynamic Model

The equations representing the dynamic equilibrium in forces and torques may be written in the following form:

$$\mathbf{M}_{RB}\dot{\mathbf{v}} + \mathbf{C}_{RB}(\mathbf{v})\mathbf{v} = \boldsymbol{\tau}_{RB} \tag{15}$$

In this equation, $\mathbf{M}_{RB}$ is the inertia matrix, $\mathbf{C}_{RB}(\mathbf{v})$ is the Coriolis and centrifuge matrix, $\mathbf{v} = [u, v, w, p, q, r]^T$ is the velocity vector and $\boldsymbol{\tau}_{RB} = [X_{RB}, Y_{RB}, Z_{RB}, K_{RB}, M_{RB}, N_{RB}]^T$ is the resulting vector of all forces and moments applied to the vehicle. All terms in equation (15) are related to the body frame system.

The inertia matrix is constituted by the mass (m), inertia moments (I_x, I_y, I_z), inertia products (I_{xy}, I_{yz}, I_{xz}) and center mass coordinates ($\mathbf{r}_G = [x_G, y_G, z_G]^T$) (Fjellstad, 1994):

$$\mathbf{M}_{RB} = \begin{bmatrix} m & 0 & 0 & 0 & mz_G & -my_G \\ 0 & m & 0 & -mz_G & 0 & mx_G \\ 0 & 0 & m & my_G & -mx_G & 0 \\ 0 & -mz_G & my_G & I_x & -I_{xy} & -I_{xz} \\ mz_G & 0 & -mx_G & -I_{xy} & I_y & -I_{yz} \\ -my_G & mx_G & 0 & -I_{xz} & -I_{yz} & I_z \end{bmatrix} \tag{16}$$

The Coriolis centrifuge matrix has the form (Tavares, 2003):

$$\mathbf{C}_{RB}(\mathbf{v}) = \left[\begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -m(y_G q + z_G r) & m(y_G p + w) & m(z_G p - v) \\ m(x_G q - w) & -m(z_G r + x_G p) & m(z_G q + u) \\ m(x_G r + v) & m(y_G r - u) & -m(x_G p + y_G q) \end{matrix} \right. \tag{17}$$

$$\left. \begin{matrix} m(y_G q + z_G r) & -m(x_G q - w) & -m(x_G r + v) \\ -m(y_G p + w) & m(z_G r + x_G p) & -m(y_G r - u) \\ -m(z_G p - v) & -m(z_G q + u) & m(x_G p + y_G q) \\ 0 & -I_{yz} q - I_{xz} p + I_z r & I_{yz} r + I_{xy} p - I_y q \\ I_{yx} q + I_{xz} p - I_z r & 0 & -I_{xz} r - I_{xy} q + I_x p \\ -I_{yz} r - I_{xy} p + I_y q & I_{xz} r + I_{xy} q - I_x p & 0 \end{matrix} \right]$$

The force of additional mass is due to the inertia of the volume of the fluid that involves the vehicle. Therefore the vehicle speeds up in relation to the fluid and it also must win its own inertia and the inertia of the fluid. The vehicle seems to have a mass (inside the water) greater than its actual one. The dynamic contribution, in terms of inertial and coriolis centrifuge forces and torques, of the additional mass is expressed by the following equation:

$$\boldsymbol{\tau}_A = -\mathbf{M}_A \dot{\mathbf{v}} - \mathbf{C}_A(\mathbf{v})\mathbf{v} \tag{18}$$

If the vehicle has three symmetrical planes (that is the case of the vehicle of the present work), the matrices of the equation (18) have the forms (Fossen, 1994):

$$\mathbf{M}_A = \begin{bmatrix} A_{11} & 0 & 0 & 0 & 0 & 0 \\ 0 & A_{22} & 0 & 0 & 0 & 0 \\ 0 & 0 & A_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & A_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & A_{55} & 0 \\ 0 & 0 & 0 & 0 & 0 & A_{66} \end{bmatrix} \tag{19}$$

$$\mathbf{C}_A = \begin{bmatrix} 0 & 0 & 0 & 0 & A_{33}w & -A_{22}v \\ 0 & 0 & 0 & -A_{33}w & 0 & A_{11}u \\ 0 & 0 & 0 & A_{22}v & -A_{11}u & 0 \\ 0 & A_{33}w & -A_{22}v & 0 & A_{66}r & -A_{55}q \\ -A_{33}w & 0 & A_{11}u & -A_{66}r & 0 & A_{44}p \\ A_{22}v & -A_{11}u & 0 & A_{55}q & -A_{44}p & 0 \end{bmatrix} \quad (20)$$

The matrix of the equations (19) and (20) are the inertia and coriolis centrifuge due to the mass additional, respectively.

In this work it was used the model introduced by Conte and Serrani (1996) to the drag and sustentation efforts:

$$\boldsymbol{\tau}_{DL} = -\mathbf{D}(\mathbf{v})\mathbf{v} \quad (21)$$

$$\mathbf{D}(\mathbf{v}) = -\begin{bmatrix} X_{|u|u}|u| & 0 & 0 & 0 & 0 & 0 \\ 0 & Y_{|v|v}|v| & 0 & 0 & 0 & 0 \\ 0 & 0 & Z_{|w|w}|w| & 0 & 0 & 0 \\ 0 & 0 & 0 & K_{|p|p}|p| & 0 & 0 \\ 0 & 0 & 0 & 0 & M_{|q|q}|q| & 0 \\ 0 & 0 & 0 & 0 & 0 & N_{|r|r}|r| \end{bmatrix} \quad (22)$$

$$\begin{aligned} X_{|u|u} &= \frac{\rho}{2}\nabla^{2/3}C_x(0°,0°) & K_{|p|p} &= \frac{\rho}{2}\nabla^{5/3}C_p \\ Y_{|v|v} &= \frac{\rho}{2}\nabla^{2/3}C_y(90°,0°) & M_{|q|q} &= \frac{\rho}{2}\nabla^{5/3}C_q \\ Z_{|w|w} &= \frac{\rho}{2}\nabla^{2/3}C_z(90°,90°) & N_{|r|r} &= \frac{\rho}{2}\nabla^{5/3}C_r \end{aligned} \quad (23)$$

Where ρ is the specific mass of the water and ∇ is the volume of the vehicle. The terms C indices (x, y, z, p, q, r) are coefficients, constants in the present case. This means that the coupling between the movements of the vehicle and the angles of attack is neglected, resulting in a diagonal matrix as showed in equation (22).

The forces of weight (W) and fluid push (B) (Archimedes principle) depend on the mass (m) and the volume of the vehicle (∇), of the specific mass of the fluid (ρ) and the gravity acceleration (g). Its modules are express as:

$$\begin{aligned} W &= mg \\ B &= \rho g \nabla \end{aligned} \quad (24)$$

The forces and the moments are expressed by the following forms:

$$\begin{aligned} \mathbf{f}_W &= \mathbf{J}_1^{-1}[0,0,W]^T \\ \mathbf{f}_B &= -\mathbf{J}_1^{-1}[0,0,B]^T \end{aligned} \quad (25)$$

$$\begin{aligned} \mathbf{M}_W &= \mathbf{r}_G \times \mathbf{f}_W \\ \mathbf{M}_B &= \mathbf{r}_B \times \mathbf{f}_B \end{aligned} \quad (26)$$

$$\boldsymbol{\tau}_{WB} = \begin{bmatrix} \mathbf{f}_W + \mathbf{f}_B \\ \mathbf{M}_W + \mathbf{M}_B \end{bmatrix} = -\begin{bmatrix} (W-B)s\theta \\ -(W-B)c\theta s\phi \\ -(W-B)c\theta c\phi \\ -(y_G W - y_B B)c\theta c\phi + (z_G W - z_B B)c\theta s\phi \\ (z_G W - z_B B)s\theta + (x_G W - x_B B)c\theta c\phi \\ -(x_G W - x_B B)c\theta s\phi - (y_G W - y_B B)s\theta \end{bmatrix} \tag{27}$$

The equation (15) is rewritten as:

$$\mathbf{M}_{RB}\dot{\mathbf{v}} + \mathbf{C}_{RB}(\mathbf{v})\mathbf{v} = \boldsymbol{\tau}_A + \boldsymbol{\tau}_{DL} + \boldsymbol{\tau}_{WB} + \boldsymbol{\tau} \tag{28}$$

where $\boldsymbol{\tau}$ is the vector with the forces and the torques control. Defining $\boldsymbol{\tau}_{WB} = -\mathbf{g}$ and considering the equations (18) and (21), the differential equations of dynamic system, written on the body frame, become:

$$(\mathbf{M}_{RB} + \mathbf{M}_A)\dot{\mathbf{v}} + (\mathbf{C}_{RB}(\mathbf{v}) + \mathbf{C}_A(\mathbf{v}))\mathbf{v} + \mathbf{D}(\mathbf{v})\mathbf{v} + \mathbf{g} = \boldsymbol{\tau} \tag{29}$$

Defining $\mathbf{M} = \mathbf{M}_{RB} + \mathbf{M}_A$ and $\mathbf{C}(\mathbf{v}) = \mathbf{C}_{RB}(\mathbf{v}) + \mathbf{C}_A(\mathbf{v})$ and considering the kinematics relation of the equation (13), the model of the vehicle can be written with the following form:

$$\begin{aligned} \mathbf{M}\dot{\mathbf{v}} + \mathbf{C}(\mathbf{v})\mathbf{v} + \mathbf{D}(\mathbf{v})\mathbf{v} + \mathbf{g}(\boldsymbol{\eta}) &= \boldsymbol{\tau} \\ \dot{\boldsymbol{\eta}} &= \mathbf{J}(\boldsymbol{\eta})\mathbf{v} \end{aligned} \tag{30}$$

4. Experimental and simulation results

Many experiments were made in order to identify parameters of the dynamic model. At first, the main objective was to identify the parameters of the additional mass matrix (equation (19)) and the coefficients of the drag matrix (C indices (x,y,z,p,q,r) in equation (23)). There are three symmetrical planes and this adds facilities because the parameters are approximately constants. The parameters identification process was made by tries and errors. The two horizontal actuators were turned on with 1, 2 and 3*A* each one, and it was observed the open loop time response. Many simulations with the same conditions of the experiments were made. Changes in parameters were performed until the simulations reach the equivalent experiments in open loop. Despite of being hard, this approach is facilitated because the two matrices that need the coefficient identifications are diagonals (equations (19) and (22)).

Figure 4 shows two open loop experimental results, obtained with 3*A* and 2*A* applied to the left horizontal actuator and to the right one, respectively. In these figures it can be seen the horizontal plane trajectories, obtained through the accelerometer and gyroscope measures, transformed to the inertial frame. The time of trajectory was 35*s*. Experiments like that were repeated six times, and these two ones showed below were the most different among all of them. The simulation made with the same conditions is showed in Fig. 5. It can be seen that the simulation is close to the experimental trajectory 1, but more different from the experimental trajectory 2.

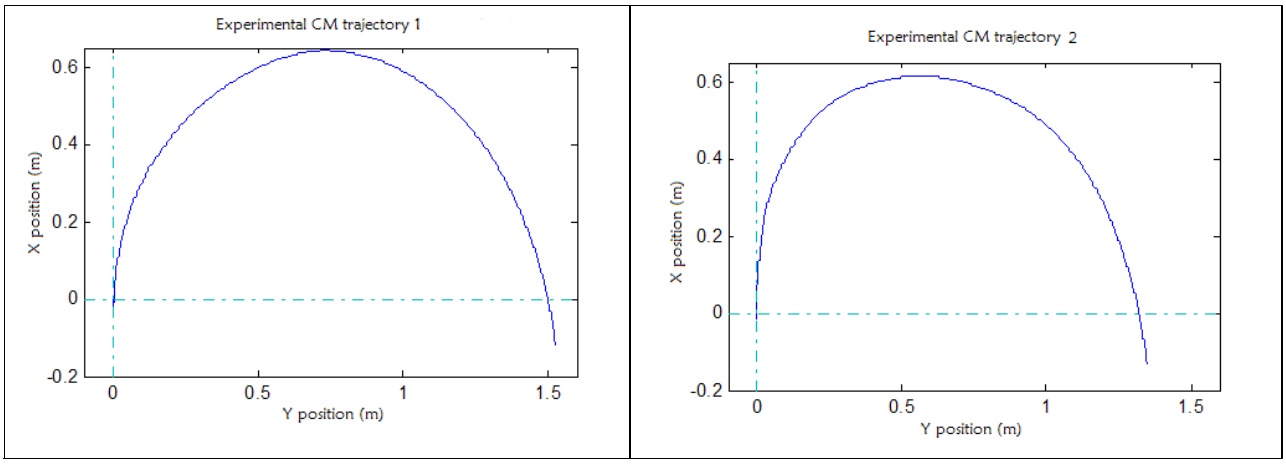

Fig. 4 Open loop experimental results (inertial horizontal plane trajectory).

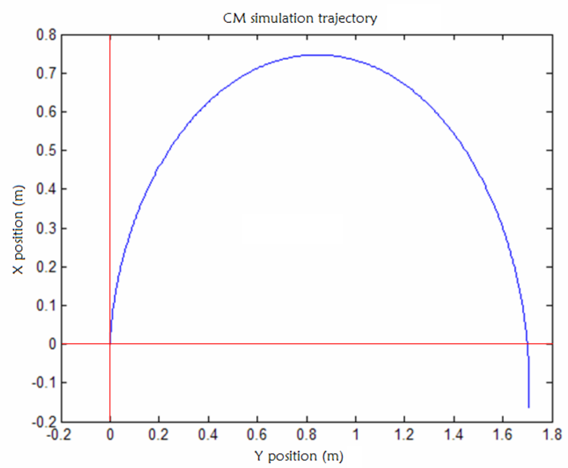

Fig. 6 Open loop simulation (inertial horizontal plane trajectory).

5. Conclusions

In this work it was applied a modeling techniques to a ROV, controlled passively in two degrees of freedom. The ROV was projected and constructed having three symmetrical planes in open frame architecture and this fact aids to parameters modeling identification. Tests in the laboratory (out of the water) revelead that the integration of data sensors (accelerometers and gyroscope) are reliable to use in time history trajectory below 40s, approximately. It was observed that it is very difficult to repeat exactly two experimental results made under the same conditions. This happens because it is difficult to reproduce the initial state of the ROV and the water conditions. Despite of this actual situation, the model reproduced relatively well the experiments in open loop, indicating that the model approach can be used to ROVs or AUVs with symmetrical planes and open frame architecture. In future works it will be investigated the introduction of improvements in the sensor signal filtering process to generate more precise inertial space trajectories, which could be used, at least, for some minutes.

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