# COMPUTATIONAL BLOCK-DIAGRAM EXECUTION OF MULTIPORT FIELDS MODELS WITH CAUSAL CONSIDERATIONS

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***Abstract.*** *This work is concerned with the modeling of dynamic systems based on the power transmition between system's elements. Complexes systems modeling demand a reasonable degree of condensation, often obtainned by using vetorials fields notation,i. e., generalized multiport elements with defined power flow and causality. A modular approach can be used to describe these systems. This paper considers a proceeding of modeling and simulating dynamic systems with multiport fields considered that are generally represented as block diagram models, i. e., blocks connected by directed lines, where each block represents data transformation. Block diagrams indicate input and output quantities for each block and, thus, are inherently causal. An automobile structural chassis model with some interacting subsystems is discussed as an example. Commertial simulation tools Simulink/Matlab is employed to obtain such model.*

***Keywords****: Bond Graphs, block diagram, multiport fields, dynamic systems, causality.*

## 1. Introduction

Nowadays the scientific and technological evolution demands from engineers and researchers a special attention to the area of systems dynamics. This demand is due to the high technology employed on the engineering projects that often involve subsystems from different physicals domains. In particular, bond graph technique (Rosenberg and Karnopp, 1983), (Karnopp, Margolis and Rosenberg, 1990) seems to be quite attractive for the solution of a wide kind of engineering problems, on a wide field of the knowledge even when dynamics subsystems of distinct natures interact to each other. This technique has been widely employed in academics and industrials means mainly due to its modular and multidisciplinary characteristics. Bond graph technique gives insight both to physics or equation level modeling, as it will be shown later in this text. However, it is noticed that physics level (physics based) modeling tools has been more and more disseminated than equation level modeling for many reasons: for equation level modeling, one must develop its own modeling equations, program these equations (either by hard coding, or constructing block diagrams based on these equations), debug them and can only then begin system analysis. Changes to the model often require major recoding of the model. Physics level (physics based) modeling tools contain predefined models of engineering elements (ex. a mass or hydraulic orifice). The user combines these elements like building blocks to approximate the real system. Constructing, expanding or changing the simulation model is usually quite simple. Some bond graph based tools for physical systems modeling are available. It is still possible to develop object–oriented physical-systems modeling by constructing a block diagram based on systems’ bond graph, which can be implemented by any signal processing packages (for instance, Matlab/ Simulink), (Mathworks Inc, 2000). In general, block diagrams are useful on the view of systems components constitutive functions and permits the study of signal-flow. This procedure, as it will be show further on, is very suitable for the vertical dynamics analysis of automotive systems that are considered to be composed by distributed (structural) and concentrated parameters subsystems.

This work aims to discuss the structural dynamic systems modeling and simulating procedure using multibond graph, defining a modular approach for mechanical systems modeling by using the generalized bond graph technique and the modularity of block diagrams. In such approach, the bond graph representative of a system is entirely converted into block diagram that keeps all bond graph properties, but without equating or writing a sing line of code to solve the system state-space equations. In case of problems involving multiport fields, it is necessary to equate only the fields’ characteristics matrices, usually in a very simple way. The work is structured in the following way: section 2 describes different form in which mathematical model of physical systems should appear; section 3 presents the elements of the modular procedure by block diagram, with some considerations about causality; on section 4, the multibond graph form is discussed for structural approach; and finally on section 5, an example of vertical dynamic problem in vehicle assembly is given. This example treats of a structural subsystem (chassis) with some others interacting subsystems (passengers, engine, etc.) that is good to demonstrate the modular methodology presented in this work.

## 2. Consideration on some forms to represent a model

In this part, some different forms in which a mathematical model may appear are described. For this purpose, consider the schematic diagram shown in Fig. 1, which represents, in this work, one passenger-seat subsystem of a vehicle. The subsystem is composed by a oscillating mass (passenger) supported by seat base with equivalent stiffness *k* and damper coefficient *b* and a prescript flux.

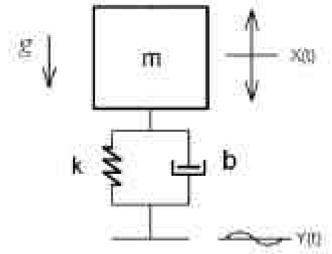


Figure 1. Schematic diagram of a vehicle passenger-seat subsystem.

Starting from the schematic diagram shown, it is possible to construct models that satisfactorily catch the dynamic behavior of the system for posterior computer simulation. For this, one can choose among some types of modeling, that using equation models, block diagram models, bond graph models or even iconic diagram models provided in some modeling tools (Controllab, 2004), (Lebrun and Richards, 1997). For equating the system model for dynamics, one may part from Newton's law to obtain the equation of motion as in Eq. 1, and then solve it in the form of Eq. 2 by computer coding or by constructing the block diagram representative of this equation, as seen in Fig. 2.

$$m\ddot{x}+b\dot{x}+kx=b\dot{y}+ky-mg \tag{1}$$

$$\ddot{x}=\frac{1}{m}\left[b\dot{y}+ky-mg-b\dot{x}-kx\right] \tag{2}$$

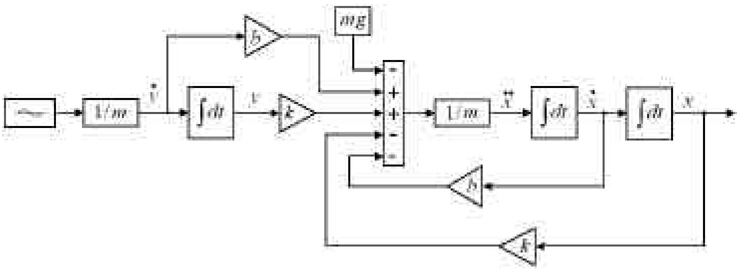

Figure 2. Subsystems' equation of motion in the block diagram form.

It can be noticed that the more the system gets increased the more equating becomes time-consuming, because of the increasing of the number of states equations and variables that must be considered. However the system of Fig 1 can still be represented in a more abstract form, as shown in Fig. 3, by using bond graphs. This generalized methodology permits to obtain systems state equations in a standard way, not just for mechanical systems but also hydraulic, thermal, electrical, combination systems, etc, from applying Kirchhoff's and Newton's laws, and by using a reduced number of elements that are the same for all applicable types of systems. Furthermore, bond graphs can also be processed in a standard way to produce block diagrams. It can also be considered to be a minimal representation of a block diagram.

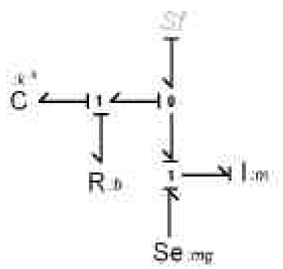


Figure 3. Subsystems' bond graph. *Sf* is prescript flux.

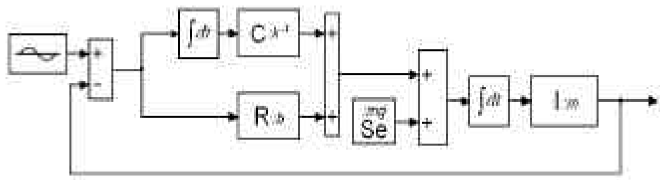


Figure 4. Subsystems' bond graph in the block diagram form.

Figure 4 shows the bond graph of Fig. 3 expanded into block diagrams without passing through the equation-formulation phase. Comparing the block diagrams of Fig 2 and Fig. 4, one may be able to see that the same relationship is implied but the bond-graph version is highly organized. Each line or bond in a bond graph implies the existence of a pair of signals whose flow is in opposite directions. These pairs in this case are force and velocity and the product of these signals is power.

This physical level modeling technique, with bond graph expressed in the block diagram form, may be quite interesting for increasing productivity in product development phase, because it eliminates major computer coding by cutting the necessity of formulate systems equations, although there are no problems to introduce some (purely) equation elements or subsystems in case of structural or nonlinear elements, control blocks, etc. Due to bond-graph/ block-diagram modularity, an engineer can create blocks libraries with many components common in its actuating branch, with theirs constitutive relations accurately determined (even by lab testing or measures), which contribute to draw the model nearest to the reality. Modeling can get even more sophisticated if it is applied masks to these blocks, which permits to customize the simulation and to reduce diagrams constructing time (Lebrun and Richards, 1997).

## 3. Block diagrams

### 3.1. Block diagram representation of physical models

A block diagram is a graphical way to represent equation information, where each directed line in the diagram represents a single variable, while each node or *block* shows a particular type of input-output relation between variables. Block diagram, obtained as discussed in the preview section, can often be condensate – by employing *transfer functions* or the so-called *block-diagram algebra*. The goal would be to reduce a large diagram into a single block with one input and one output. This procedure can be applied, but is not encouraged in this work because it eliminates the physical insight of bond-graph elements expressed by the blocks. Instead of that, one could group some subsystems into a major subsystem, regarding to the physics of the model and preserving the physical insight. As an example, consider that the block diagram of Fig. 4 with its elements (mass, equivalent stiffness and damping) could be condensate (by grouping or masking) into one subsystem that represents physically one passenger-seat subsystem on the vehicle model.

### 3.2. Causal considerations for systems' modeling and simulation

A characteristic of great importance of bond-graph technique is the possibility to represent, in a well defined form, the cause and effect relationship (causality) among power variables. In each passive connection there are present one variable of effort and one of flow, and it is physically possible to control just one of them, being the other resultant, function of the power.

In some systems, while assigning the causalities, one may often encounter a problem of ambiguity of causal structure when more than one causal form can be assigned. It may have in mind that the existence of differentiated causality storage elements or equivalent resistances indicate that a so—called *algebraic loop* is present in the graph. *Algebraic loops* and loops between a dependent and an independent storage element are called *zero-order causal paths* (*ZCPs*). This loop causes the resulting set differential equations to be implicit. Often this is an indication that a storage element was not modeled, which should be there from a physical systems viewpoint (Speranza Neto, M., 1999).

Integration has preference above a differentiation. At the integrating form, an initial condition must be specified. Besides, integration with respect to time is a process, which can be realized physically. Numerical differentiation is not physically realizable, since information at future time points is needed. Another drawback of differentiation occurs when the input contains a step function: the output will then become infinite.

As raised by Broenink (2000), since at causal analysis, one can decide whether or not to change the bond–graph model to obtain an explicit simulation model, it is useful to know about the consequences for simulation of relevant characteristics of the model. Implicit models (DAEs) can only be simulated with implicit integration methods. The iteration procedure of the implicit integration method is also used to calculate the implicit model. Explicit models (ODEs) can be simulated with both explicit as implicit integration methods. Sometimes, implicit integration methods need more computation time than explicit integration methods.

Therefore, when a block diagram is constructed from bond-graph models, differentiated causality storage elements are highly inadvisable, because of the undesirable differentiations involved. The application of causality inversion's elements or equation with intrinsic physical meaning should be done. Block diagrams containing *ZCPs* caused by equivalent resistors elements are generally satisfactorily solved by explicit solvers routines, and sometimes by simply adding a unit delay block with nonzero delay only to set the initial input signal back (Mathworks Inc, 2000).

## 4. Multibond graph for structures

In this work, bond graph are presented in a compact multibond graph form, which is a vetorial generalization of a simple bond graph. This notation proposed by Breedveld (1985) is more suitable for structural systems representation because in these cases, dynamical properties relate more than one pair of input-output variables what features, for linear models, a matrix constitutive relation.

On the structural systems modeling approach, it's common to associate bond-graph and finite-elements formulations, taking advantage of bond-graph characteristic of easily coupling continuous and discrete subsystems, and the fact of finite elements method (FEM) structure matrices are explicit fields for bond-graph technique as well.

The methodology employed here takes advantage of a structural system representation proposed by Da Silva and Speranza Neto (1993), where the structural model generated through FEM could be entirely used in bond graph technique. In such representation, FEM matrices of mass (M), stiffness (K) and damping (B) are directly related with bond-graph inertia (I), compliance (C) and resistive (R) fields, as shown in fig (5).

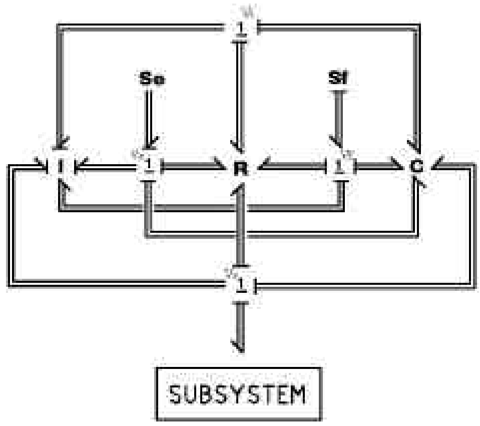


Figure 5. Generic multibond graph for structures.

On the bond graph of Fig. 5 the degrees-of-freedom (DOF) of the structure are divided in four kinds that are selected by each of the bond graph vetorial 1-junctions, which are identified by *Vi* when related with internal DOF; *Vs* when related with DOF that interact with external subsystems; *Ve* when related with DOF that receive prescribed effort; and *Vf* when related with DOF that receive prescribed flux (Da Silva, F. R., 1994). The FEM matrices must be arranged according to that topology to correctly represent the fields' transformations of power for all DOF of the system.

**5. Structural chassis model**

In this section, a vertical dynamics modeling of a vehicle with flexible chassis is to be discussed. The intention here is to demonstrate how powerful the modular procedure proposed previously can be, therefore regardless with profound analysis of the results. The vehicle proposed corresponds, in its characteristics, to a heavy truck, and the model aims to simulate the vehicle dynamical behavior due to road excitation and subsystems interactions.

**5.1. Vehicle model**

The physical model is constituted, in addition to the chassis, by three stiff-damper-mass subsystems as that shown in section 2, which represent the two passengers with seat and the engine, and also by four suspensions subsystems. The physical model so described is shown in Fig 6.

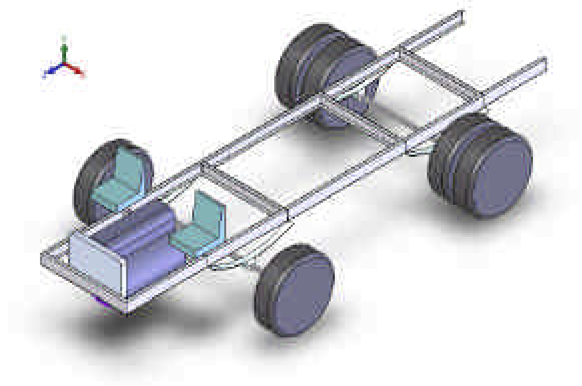

Figure 6. Physical model of the vehicle (passengers not shown).

The chassis is the only structural subsystem of the model, all the others components are attached on it. Its constitutive form is chosen to be similar to some real truck. Type stairway, straight stringer of profile "U" constant, with module of resistance of 108 cm³. It is conveniently divided into 30 elements with six DOF each, which are related with vertical displacement, flexural and torsional rotation by node, totaling 81 global DOF. Figure 7 presents an outline of the chassis' global model with node numbering, where front is to the left and model's horizontal plane is on sheet.

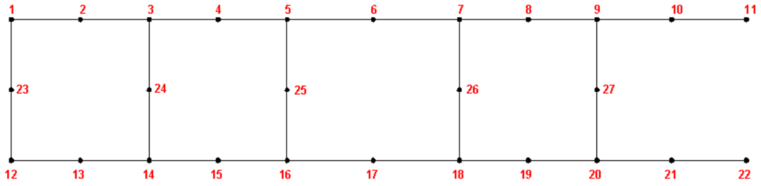


Figure 7. Global model of the structural chassis.

Chassis material is defined to be the LNE 38 alloy steel that meets several applications in automotives industries. Table 1 presents some physical and mechanical properties of LNE 38 alloy steel.

Table 1. Physical and mechanical properties of LNE 38 alloy steel

| **Yield stress** *(MPa)* | **Utimate stress** *(MPa)* | **% Stretching** ($L_0$ = 80 mm) | **Elasticity mod. (E)** *(GPa)* | **Shear mod. (G)** *(GPa)* | **Density** *(kg/m³)* |
|---|---|---|---|---|---|
| 380/520 | 460/610 | 18 | 200 | 80 | 7860 |

For the three stiff-damper-mass subsystems that represent the two passengers with seat and the engine, the parameters mass (*m*), stiffness coefficient (*k*) and damper coefficient (*b*) are set to be: *m* = 100kg, *k* = 19.620N/m and *b* = 1.400Ns/m for each passenger-seat subsystem; and *m* = 500kg, *k* = 122.625N/m and *b* = 24.761Ns/m for the engine.

Suspensions subsystems' physical model represents heavy duty suspensions and dampers with the following parameters: *k* = 193.256N/m, *b* = 12.872Ns/m (front); *k* = 313.595N/m, *b* = 19.828Ns/m (rear). Table 2 shows the weight of some parts of the vehicle that is to be simulated with critical weight condition. Critical loads for simulation are shown in tab. 3.

Table 2. Vehicle subsystems weight

| **Subsystems** | **Weight (kg)** |
|---|---|
| Passengers and seats | 200 |
| Engine | 500 |
| Front unsprung mass | 1.600 |
| Rear unsprung mass | 940 |
| Chassis | 950 |
| Total net weight | 7.700 |
| Critical weight | 10.500 |

Table 3. Axle's loads and critical load

| **Description** | **Load (N)** |
|---|---|
| Maximum front axle load (1) | 16.215,9 |
| Maximum rear axle load (1) | 26.457,6 |
| Critical load | 85.347,0 |

(1): Calculated for each wheel.

### 5.2. System multibond graph

The bond graph of the entire system is constructed by parts and assembled as in Fig 8, in a similar manner of that done by Da Rocha (1998). Notice that the structural bond graph of the chassis differs from the generic multibond graph for the absence of prescript flow on nodes what, for this case, eliminated mixed differential-integral bonds on I – field.

The lower-left part represents the attached subsystems on chassis, and is nothing else than three of the mass-spring-damper subsystem of Fig. 3 (that are two passengers and one engine) represented in a multiport notation. The lower-right subsystem represents the four suspensions in a multiport notation, where the module of the transformer (TF) relates the distance from the wheel axis to each spring fixation points. Horizontal line denotes a composition (not a summation) of the vectors contained in the three bonds from the subsystems (numbered by 19, 20 and 21) into the vectors (flow and effort) contained in the structure bond number 18.

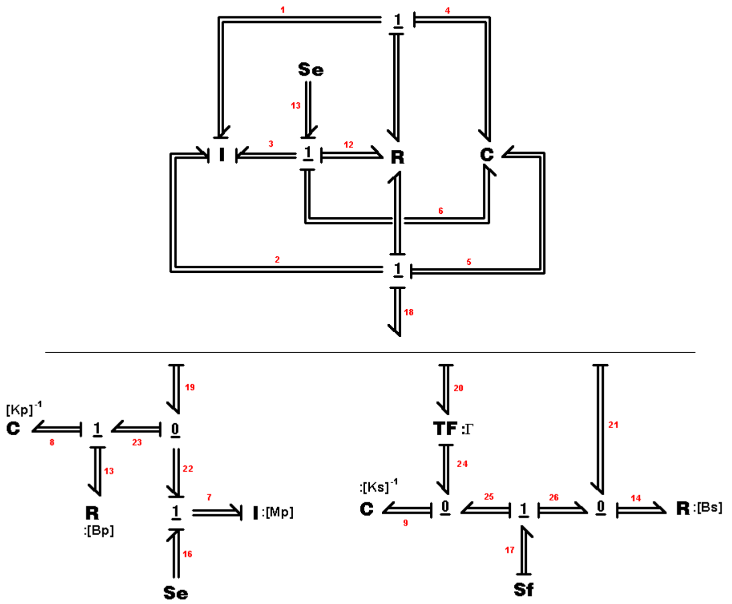


Figure 8. Global system causal multibond graph with bond numbering.

This multiport representation is suitable for the automobile system and also for a posterior simulation by block diagram exactly for the condensation it promotes and for its modular characteristic.

### 5.3. System's block diagram

Then, following the methodology proposed, a system block diagram is constructed, as in Fig. 9, from the bond-graph notation of the system seen in Fig. 8.

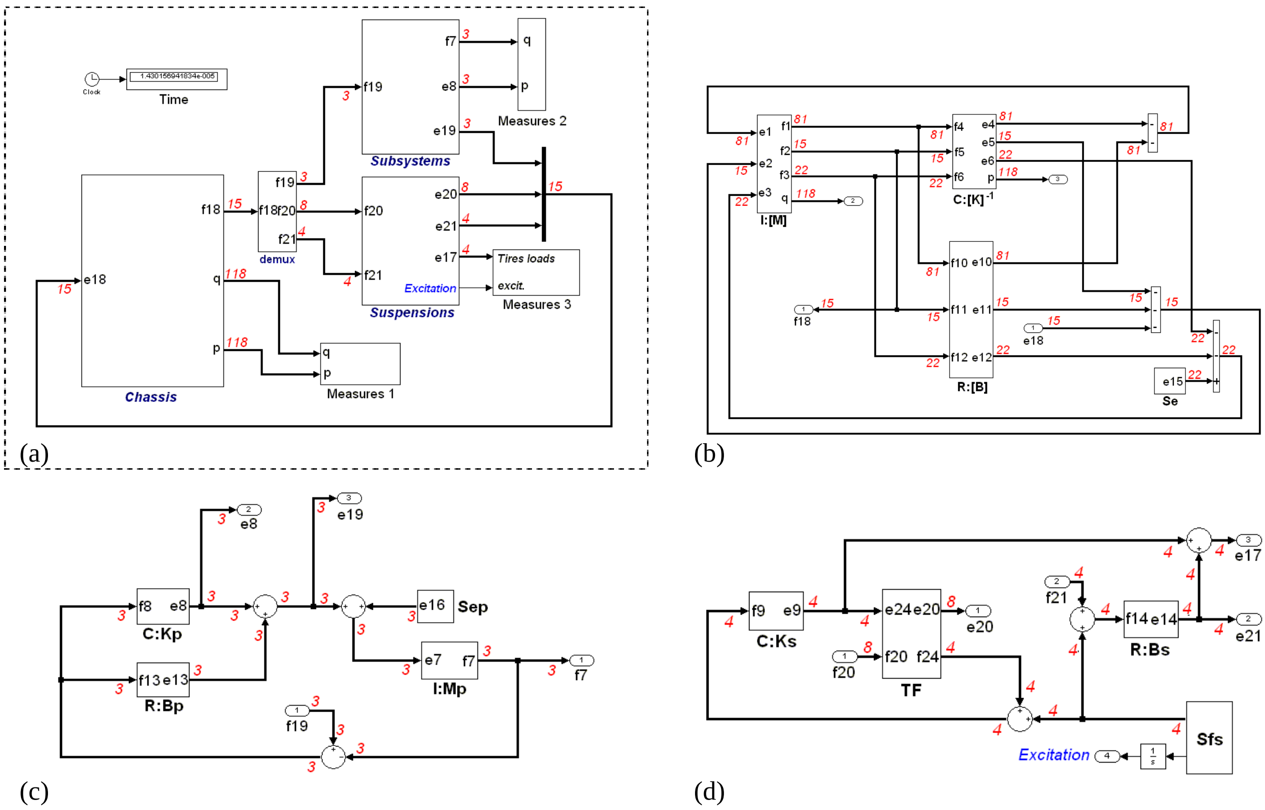


Figure 9. Block diagram of the model. (a) Entire system; (b) chassis subsystem; (c) attached subsystems; (d) suspensions subsystems. Wide lines represent vetorial signals. Numbers near each line indicate the signal dimension.

Figure 9(a) detaches the block diagram of the entire vehicle system, with its subsystems grouped in three categories: the structural chassis, the attached subsystems and the suspensions. Simulink signal conditioning blocks *mux* and *demux* have to be used to combine and split scalar or vector signal, respectively (Dabney and Harman, 2003). That's equivalent in the bond graph of Fig. 8 to the horizontal line composition of bonds *19*, *20* and *21* to form the bond *18*. In addition to the systems' block diagram, there are the measures blocks to plot and save system's responses.

The structural chassis subsystem of Fig. 9(b) is composed by the effort source (*Se*) that account nodal loads due to vehicle sprung mass; besides inertia (*I*), compliance (*C*) and resistive (*R*) blocks. Road excitation (*Sf*) in Fig 9(d) is modeled as sine wave with 0,1*m* of amplitude at a frequency of 80?*rad/s* with ?*rad* delay from the front to the rear axle.

One could notice that these elements and subsystems described above are very susceptible to be treated as standard physical elements or subsystems to be used for engineers or researchers and take part of theirs components library for posteriors re-uses.

**5.4. Results**

Some results are to be presented below which are obtained from simulating system's block diagram of Fig. 9. The simulation is considered to the steady-state phase, with initial state calculate from static.

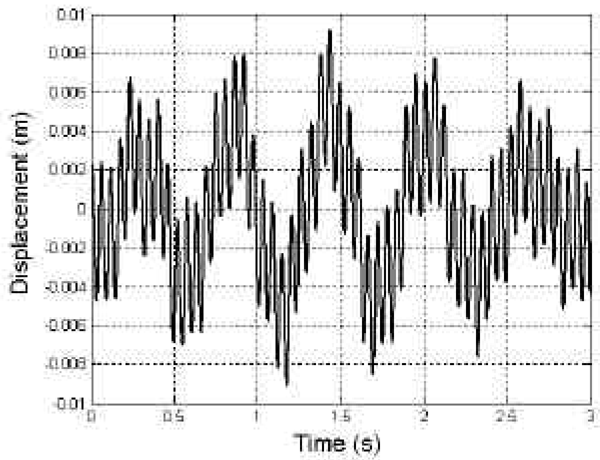


Figure 10. Vertical displacement on node 6.

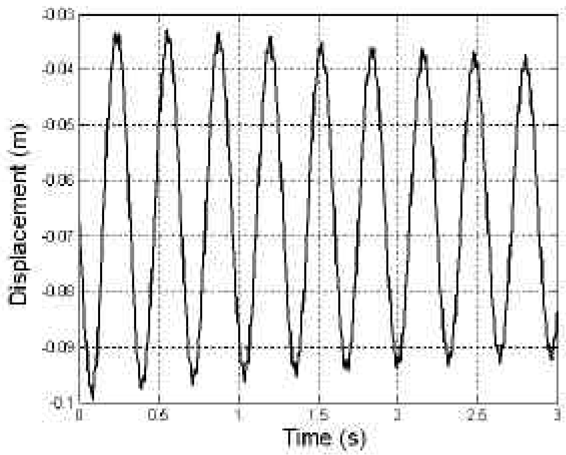


Figure 11. One seat vertical displacement.

Figure 10 shows to vertical displacement of the center of the right stringer (see Fig. 7), and in Fig. 11 an example of subsystem response in shown: the vertical displacement of one of the vehicle seat (with passenger).

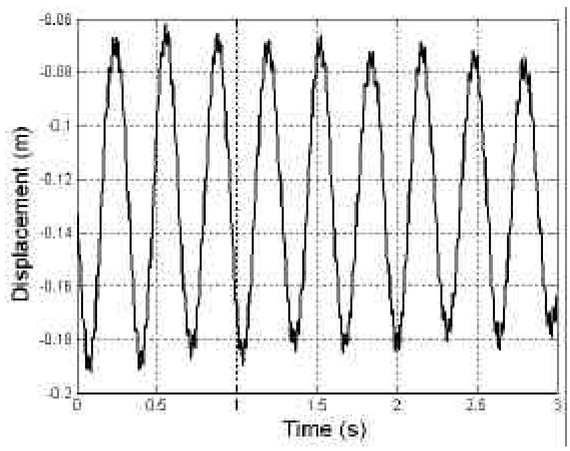


Figure 12. Vertical displacement of one rear suspension

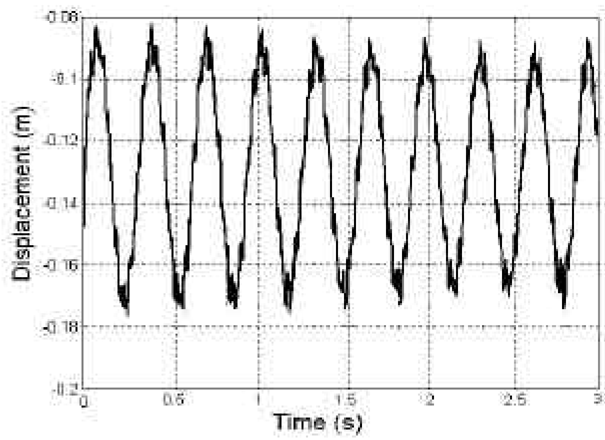


Figure 13. Vertical displacement of one front suspension

The graphic of Fig. 12 and Fig. 13 present the results of the vertical displacement of the rear and the front suspensions, respectively. In Fig. 14 on next page, another result from the simulation is plotted, that is a tire load on the rear wheel.

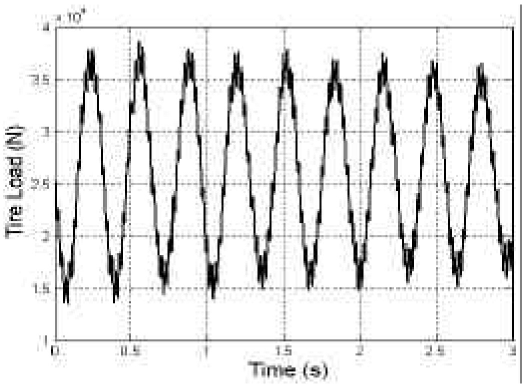


Figure 14. One rear tire load

## 6. Conclusions

In this work, a methodology of an object–oriented physical-systems modeling by constructing block diagrams based on systems' bond graphs was presented. Such methodology brings many advantages, like the possibility of implementation by general signal processing packages, reasonable time-earning, as model complexity grows, because of the cut of equation and writing computational codes for system solving, the upkeep of bond-graph physical insight from the real systems, and the easy adaptability to the user's needs. In addiction, a great condensation of the block diagrams can be obtained by using a multiport notation, as multiport fields can represent various similar bond-graph elements or even various element discretized portions in just one block. However, while the assignment of the causalities, some factors may lead into the apparition of *zero-order causal paths* (*ZCPs*) in the system and consequently the apparition of dependences among some system's elements. These dependences are translated in block diagrams as derivative or as *feedthrough* blocks, i.e. blocks with dependence between its input and output signal. As it has also been considered, the employment of special blocks should be required to bypass or "break" these *ZCPs.*

Finally, through the example discussed, a pertinent application on automotive industry is used to demonstrate the potential advantages of the methodology, which showed to be very suitable to model the dynamical behavior of modular (automotive) systems with interactions among several components, with both lumped and concentrated parameters.

## 7. Acknowledgements

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## 9. Responsibility notice