A NEW PROCEDURE FOR ROBUST CONTROL DESIGN OF COOLING MACHINES BASED ON VAPOR COMPRESSSION

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Abstract. *Cooling and air-conditioning systems are often poorly efficient concerning energy consumption. Their operation cycle often includes a thermostat that switches on and off a motor-compressor gear. Recently a great amount of resources are being applied in the research on non-polluting fluids, on the designing of new equipments and on the development of new controlling techniques. The benefits of the improvement on cooling systems control techniques include, besides the energy saving, a) the accurate and continuous and uniform temperature control in the target environment, b) the shortening of the superheating region, c) a lower number of starts-stops by time unit increasing in this way the equipment lifetime, d) faster cooling response, e) smaller and lighter compressors, etc. However, the simultaneous control of the freezing power and superheating degree is a challenging task. Besides the model uncertainties, the strong cross-coupling interaction that exists between the system inputs and outputs and the time delays in the control loop are some of the problems that might deteriorate the controller ideal performance. This paper presents a robust control design procedure as an alternative to more complex and expensive MIMO controllers. The objective is to independently control the comfort temperature and the superheating. The proposed technique allows robust control characteristics to be included naturally in the controller design.*

Keywords*: Robust Control, Model Uncertainties, Cooling Systems.*

1. Introduction

The improvement of power consumption efficiency of industrial devices is one of the main issues for the incoming century. In the first century of the industrial age, the world population has virtually exploded, nature has been almost devastated and energy resources have been depleted. In spite of that, the human living comfort has become essential for most of the whole world population, even for the third world people; and because of that energy per-capita consumption should continuously increase in the future. It is a fact that the next decades are going to testify a continuous and strenuous search for new devices and technologies to save energy resources. The energy consumption by heating and cooling systems in commercial and industrial buildings corresponds to 50% of the world energy consumption (Imbabi, 1990). Heating and cooling systems are high-energy consumption processes (Arguello-Serrano, 1999) and their operation in commercial and industrial buildings still are inefficient.

It is already known that the solution for an efficient operation of heating and cooling systems relies on the proper choice and design of an automatic control system. Low cost controllers such as On/Off control and PID control are currently used as the standard controllers in the heating, ventilation and air conditioning (HVAC) industry. However, their low energy efficiency causes an extra-undesired energy burning. An important fact that causes the low energy efficiency is that most control designs are only capable of dealing with constant thermal loads, which is not the general case; in practice, thermal loads are time varying. The temperature sensor location is another difficult in the control field of heating and cooling devices. The natural position for the temperature sensor is close to, or even inside, the target environment; in practice, to avoid the inclusion of time delays in the control loop, the temperature sensor is usually located close to the heat source (sink).

Furthermore, conventional single-input single-output (SISO) control of cooling machines is not capable of controlling the freezing power and super-heating independently (due to the I/O cross coupling). Several control strategies to deal with the control problem of time varying processes, time delays and I/O cross coupling have been proposed by the control community. Among them, robust control, adaptive control and intelligent control are the most important. A drawback of these sophisticated alternatives is that they are usually expensive and required advanced computational resources. To face time-varying thermal loads, time delays and I/O cross coupling, new-low cost multi-input multi-output (MIMO) control strategies must be explored. This paper introduces a MIMO control scheme that permits the independent control of the output variables of cooling machines based on the vapor compression cycle. Figure 1 shows the schematic diagram of a cooling machine of this type.

2. The MIMO Feedback Control

Output feedback has been the industrial standard for control purposes not only to shape the plant response, fulfilling performance specifications, but also to deal with output disturbances and model uncertainties.

Traditionally, the industrial control community has relied on the intrinsic robustness of output feedback controllers to face the control design problem for SISO plants. A diversity of controller tuning algorithms has been successfully developed and applied to SISO industrial plants. Behind this success there has always been a property that exists for all physical system, the dominance of the low-frequency poles in the system time response. This fact has been the background of nearly every robust control design technique. Considering this concept in the controller design, there is no need for solving the modeling problem as rigorously as it could be required without the pole dominance property.

Several attempts have been made to extend the SISO design techniques to the MIMO case. In this context, the usually strong input-output cross-coupling existing in MIMO systems becomes as important as the model uncertainties due to the size (order) of large-scale systems. With some exceptions, the success of MIMO control design also depends on the pole dominance property. In recent years, the research has been focused in new uncoupling techniques. It is worth to mention the pioneer contributions from Bristol (1966), Kouvaritakis (1979), Mees (1981), McAvoy, (1983) and Grosdidier and Morari (1986). Some characteristics of the use of these techniques are: The design procedure is usually carried out in the frequency domain. Model uncertainties are easily represented in the frequency domain (particularly, non-structural uncertainties). Low frequency models are, in general, accurate enough for control design in this environment. The standard PI controller (that responds for more de 90% of the industrial controllers) is designed to perform in the low frequency range. Output disturbances are usually low frequency signals.

3. The MIMO Control – A Brief Review

This section presents a brief review of the basic concepts on multivariable control systems. The following is based on the books from Maciejowski (1989) and Skogestad et al (1996). The system output, y(s), is given by

$$y(s) = T(s)P(s)r(s) + S(s)d(s) - T(s)m(s) \tag{1}$$

where *r(s)* is the reference input, *d(s)* represents the disturbances and *m(s)* is the measurement noise.

the function $S(s)$ is known as the output sensitivity function and is defined as

$$S(s) = [I + G(s)K(s)]^{-1} \tag{2}$$

and the system closed loop transfer function (or complementary sensitivity), *T(s),* is then given by

$$T(s) = S(s)G(s)K(s) \tag{3}$$

the input sensitivity function is defined as

$$S_i(s) = [I + K(s)G(s)]^{-1} \tag{4}$$

and its corresponding complementary function as

$$T_i(s) = K(s)G(s)S_i(s) \tag{5}$$

a multiplicative model for the plant uncertainty can be written as

$$G(s) = G_o(s)[I + W_i(s)] \tag{6}$$

Hence, the following criteria to assess the system performance and stability can be established:

a) The criterion for nominal performance is defined by

$$\left\| S(s)W_p(s) \right\|_\infty < 1 \tag{7}$$

where $W_p(s)$ is a performance weighting matrix and has the form

$$W_p(s) = w_p(s)\left[I\right] \tag{8}$$

thus, the nominal performance criterion can be re-written as

$$\bar{\sigma}[S(s)] < \frac{1}{w_p(s)} \tag{9}$$

where $\bar{\sigma}[.]$ is the greatest singular value of [.]

b) The criterion for robust performance (non structured uncertainty) is given by

$$\gamma\,\bar{\sigma}\big(W_p(s)S_i(s)\big) + \bar{\sigma}\big(W_i(s)T_i(s)\big) \le 1 \tag{10}$$

where γ = min(plant-controller condition number).

c) The criterion for robust stability (non structured uncertainty) is defined by

$$\|T(s)W_i(s)\|_\infty < 1 \tag{11}$$

where $W_i(s)$ is an uncertainty weighting matrix and has the form

$$W_i(s) = w_i(s)[I] \tag{12}$$

d) The criterion for robust stability can be re-written as

$$\bar{\sigma}[T(s)] < \frac{1}{w_i(s)} \tag{13}$$

e) The robustness analysis based on structured singular values was introduced by Doyle et al (1981). In this case, a robust performance condition for structured uncertainty can be established as

$$\mu\big(Q(s)\big) < 1 \qquad \forall\omega \tag{14a}$$

where, the matrix $Q(s)$ is defined as

$$Q(s) = \begin{bmatrix} Q_{11}(s) & Q_{12}(s) \\ Q_{21}(s) & Q_{22}(s) \end{bmatrix} = \begin{bmatrix} W_p(s)S_o(s) & W_p(s)S_o(s)G_o(s) \\ -W_i(s)K(s)S_o(s) & -W_i(s)K(s)S_o(s)G_o(s) \end{bmatrix} \tag{14b}$$

and

$$S_o(s) = \big(I + G_o(s)K(s)\big)^{-1} \tag{14c}$$

where $w_p(s)$ and $w_I(s)$ are defined in the frequency domain. Also, a robust stability condition for structured uncertainty can be written as

$$\mu\big(Q_{22}(s)\big) < 1 \qquad \forall\omega \tag{15}$$

Equations from (7) to (15) are applied in Section 6 to assess the closed loop system robustness and to validate the controller design.

4. The Cooling System

This paper is concerned with the control of a system constituted by an expansion valve, an evaporator and a compressor as shown in Fig.1. The system inputs are the expansion valve opening position, which defines the mass flow rate (MFR) and the compressor speed, which controls the volume flow rate (VFR). The system outputs are the super heating, ΔT, and the freezing power, Q_1, (Fig.2).

Ideally, the opening of the expansion valve and the velocity of the variable-speed compressor would be used to regulate the super heating and to control the generation of freezing power (Fig.3 with $G_{12}(s) = G_{21}(s) = 0$). Unfortunately, this is not the case. Actually, each of the outputs is a function of both inputs (the valve opening position and the compressor velocity) as shown in Fig.3. This means that $G_{12}(s)$ and $G_{21}(s)$ can not be neglected in practice.

The strong cross-coupling interaction among inputs and outputs characterizes this type of system. In this case, the system dynamics can be defined by a matrix transfer function of the form:

$$\begin{bmatrix} \Delta T(s) \\ Q_1(s) \end{bmatrix} = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} \begin{bmatrix} MFR(s) \\ VFR(s) \end{bmatrix} \quad \leftrightarrow \quad [Y(s)] = [G(s)][U(s)] \tag{16}$$

Several linear and non-linear computational models for cooling systems can be found in the technical literature (Koury, 1998; Machado, 1996; Rocha, 1995; Outtagarts, 1994). The model identification procedure (Machado, 1996) led to 2x2 matrix transfer function of the form:

$$[G(s)] = \begin{bmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{bmatrix} = \begin{bmatrix} \dfrac{-5.62}{(45s+1)} & \dfrac{-2.49(-70s+1)}{(59.52s+1)} \\ \dfrac{33.89(-36.37s+1)}{(25.65s+1)(67.79s+1)} & \dfrac{22.20(630s+1)}{(80s+1)(90s+1)} \end{bmatrix} \tag{17}$$

It should be notice that the plant is non-minimal phase and also non strictly proper.

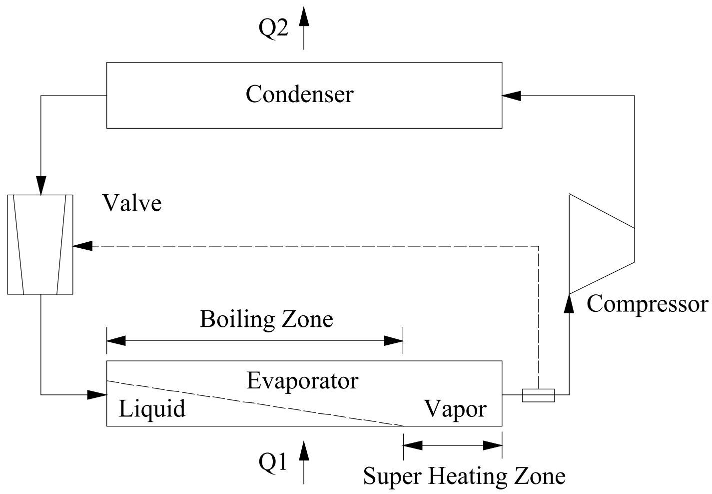

Figure 1. The Cooling System.

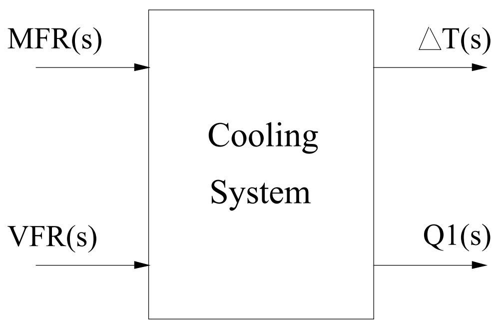

Figure 2. The Open Loop System.

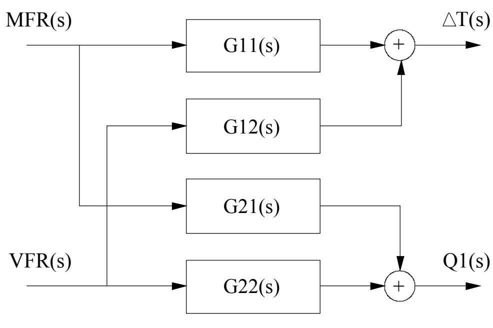

Figure 3. The Cooling System Cross Coupling.

5. The 2x2 MIMO Control Law.

The proposed strategy is basically a frequency-domain procedure. In this case, the MIMO controller design is carried out in two steps. First a MIMO pre-compensator, $K_1(s)$, is designed to scale the system and reach diagonal dominance at low frequencies and then a MIMO controller, $K_2(s)$, is designed to meet performance specifications. The advantage of this procedure it that for design purposes, $K_2(s)$ is diagonal and can be treated as a multiple SISO design since $G(s)K_1(s)$ is strongly diagonal dominant at low frequencies and diagonal at w = 0. Additionally, exact modeling is only required at steady state (w = 0) or at most at low frequencies. The MIMO control law has the form:

$$[U(s)] = [K_1(s)][K_2(s)][R(s) - Y(s)] = [K(s)][E(s)] \tag{18a}$$

where

$$[R(s)] = \begin{bmatrix} \Delta T(s)\ Setpoint \\ Q_1(s)\ \ Setpoint \end{bmatrix};\ [Y(s)] = \begin{bmatrix} \Delta T(s) \\ Q_1(s) \end{bmatrix};\ [E(s)] = \begin{bmatrix} \Delta T(s)\ Error \\ Q_1(s)\ \ Error \end{bmatrix};\ [U(s)] = \begin{bmatrix} MFR(s) \\ VFR(s) \end{bmatrix} \tag{18b}$$

and

$$[K(s)] = [K_1(s)][K_2(s)] = \begin{bmatrix} K_{11}(s) & K_{12}(s) \\ K_{21}(s) & K_{22}(s) \end{bmatrix} \tag{18c}$$

for $K_1(s) = K_1$ (decoupling at w = 0), the 2x2 MIMO PI controller becomes

$$[K(s)] = [K_1][K_2(s)] = [K_1]\begin{bmatrix} K_{P_{11}} + \dfrac{K_{I_{11}}}{s} & 0 \\ 0 & K_{P_{22}} + \dfrac{K_{I_{22}}}{s} \end{bmatrix} = [K_1]\begin{bmatrix} \dfrac{N_{11}(s)}{D_{11}(s)} & \dfrac{N_{12}(s)}{D_{12}(s)} \\ \dfrac{N_{21}(s)}{D_{21}(s)} & \dfrac{N_{22}(s)}{D_{22}(s)} \end{bmatrix} \tag{19}$$

It should be noticed that, all entries of *K(s)* have the general form of SISO PI controllers, however, the engineer only has to determine the diagonal elements of $K_2(s)$. The proposed designing technique leads to a closed loop transfer function that can be approximated (at low frequencies) by a diagonal matrix transfer function, Equation (20).

$$[Y(s)] \cong \begin{bmatrix} T_{11}(s) & 0 \\ 0 & T_{22}(s) \end{bmatrix} [R(s)] \quad \Leftrightarrow \quad \begin{bmatrix} \Delta T(s) \\ Q_1(s) \end{bmatrix} \cong \begin{bmatrix} T_{11}(s) & 0 \\ 0 & T_{22}(s) \end{bmatrix} \begin{bmatrix} \Delta T(s)\, Setpoint \\ Q_1(s)\ Setpoint \end{bmatrix} \tag{20}$$

Because of that the independent control of superheating and freezing power is tangible as it is shown in the next section. Figure 4 shows the MIMO controller structure. Figure 5 shows the block diagram for the closed loop system.

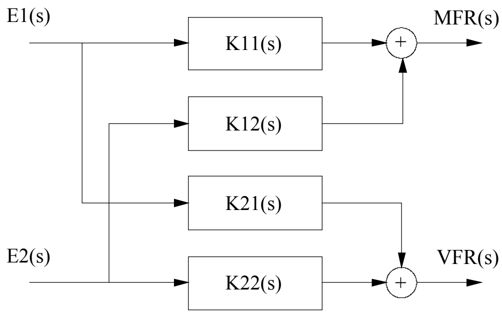

Figure 4. The MIMO Controller Scheme.

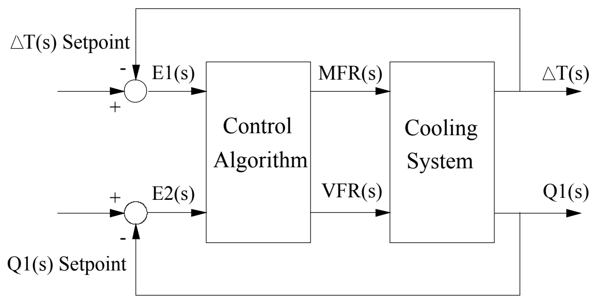

Figure 5. The Closed Loop System.

6. Assesing The Controller Performance.

In this work, the nominal performance criterion was specified as

$$\bar{\sigma}[S(s)] < \frac{1}{w_p(s)} = \frac{200\ s}{50\ s+1} \tag{21}$$

And the criterion for robust stability was chosen as

$$\bar{\sigma}[T(s)] < \frac{1}{w_i(s)} = \frac{10}{0.1s+1} \tag{22}$$

It is clear that in order to fulfill robust stability specifications the plant must be made strictly proper. This can be achieved by using a matrix low-pass filter of the form

$$[F(s)] = \begin{bmatrix} 1 & 0 \\ 0 & F_{22}(s) \end{bmatrix} \tag{23}$$

In this case the final controller equation becomes

$$[K_F(s)] = [F(s)][K_1(s)][K_2(s)] = [F(s)][K(s)] \tag{24}$$

The practical consequence of Eq. (24) is that with the inclusion of a filter in the control loop the compressor speed change is smother. This will cause an improved super heating transient profile. Several techniques for multivariable loop shaping can be found in the literature (Maciejowski, 1989, Skogestad, 1996, Ho & Xu, 1998). In this case, a robust performance was achieved using the following control law

$$[K_F(s)] = \begin{bmatrix} 1 & 0 \\ 0 & F_{22}(s) \end{bmatrix} \begin{bmatrix} K_{11}(s) & K_{12}(s) \\ K_{21}(s) & K_{22}(s) \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & \dfrac{1}{11s+1} \end{bmatrix} \begin{bmatrix} \dfrac{-1.0952s - 0.0219}{s} & \dfrac{-0.6133s - 0.0123}{s} \\ \dfrac{1.6719s + 0.0334}{s} & \dfrac{1.3867s + 0.0277}{s} \end{bmatrix} \tag{25}$$

Finally, the closed loop transfer function has the form

$$Y(s) = T(s)\ R(s) = [I + G(s)K(s)]^{-1} G(s) K_F(s)\ R(s) \tag{26}$$

7. Experimental Results.

Simulation results are presented here to illustrate the controller performance. Figures 6 to 9 graphically display the controller designing procedure. Figure 6 presents the system open loop responses. It shows the super-heating (quadrant II) and freezing power (quadrant IV) time responses. It can be observed (in quadrants I and III) the strong effect of the I/O cross coupling (in the ideal case, the time responses shown in quadrants I and III should remain at zero for all time or at least return to zero at steady state). Figure 7 shows the pre-filtered system, G(s) F(s), open loop responses. It should be noted that the inclusion of the pre-filter F(s) slows down the response of $G_{12}(s)$. Figure 8 presents the effects of the decoupling pre-compensator $K_1(s)$. It shows the super-heating (quadrant II) and freezing power (quadrant IV) open loop time responses to a unit step. It can be observed (in quadrants I and III) that the steady state effects of the I/O cross coupling were eliminated by the proposed control law. Finally, Figure 9 presents the system closed loop performance. Figures 10 to 12 present the effects of the pre-filter and the decoupling pre-compensator in the frequency domain. Figure 10 shows the non-strictly proper characteristic of the $G_{12}(s)$ transfer function (it is also non-minimal phase as shown by Eq.(17)). Figure11 presents the effects of the prefiltering on $G_{12}(s)$. Figure 12 shows the system decoupling in the frequency domain due to the inclusion of $K_1(s)$. Figures 13 and 14 display the robustness analysis corresponding to Equations (7) to (15).

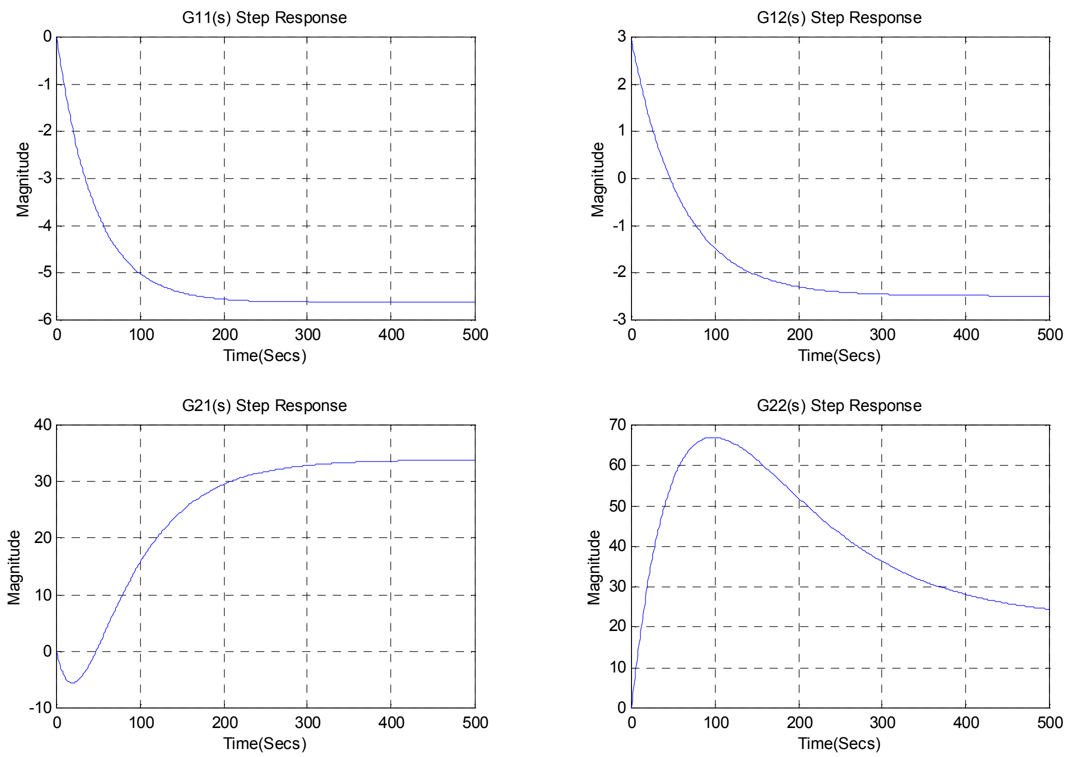

Figure 6. Open Loop Step Response – [G(s)].

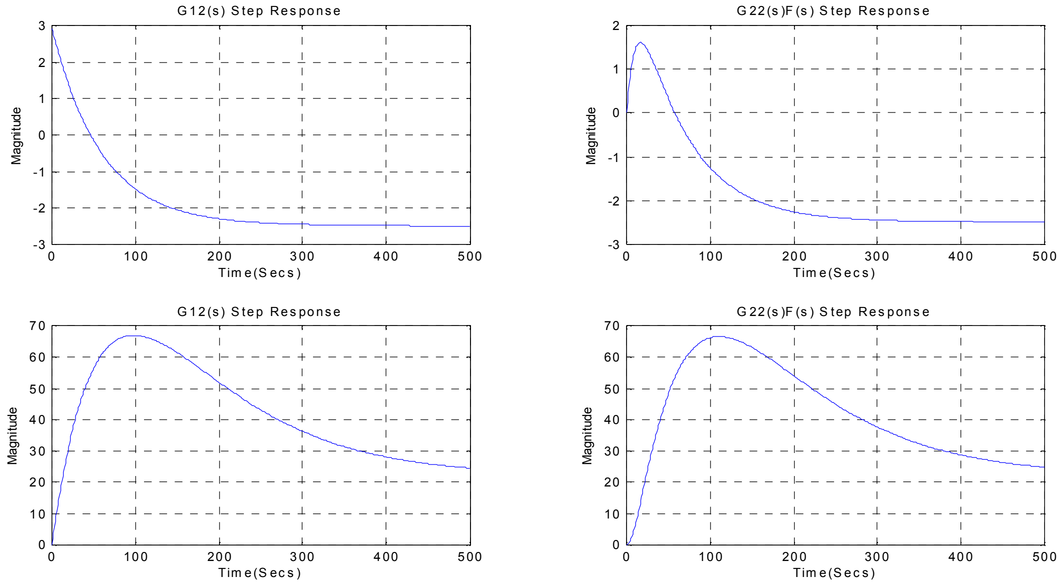

Figure 7. Comparison of the Open Loop Step Response – [G(s) and G(s) F(s)].

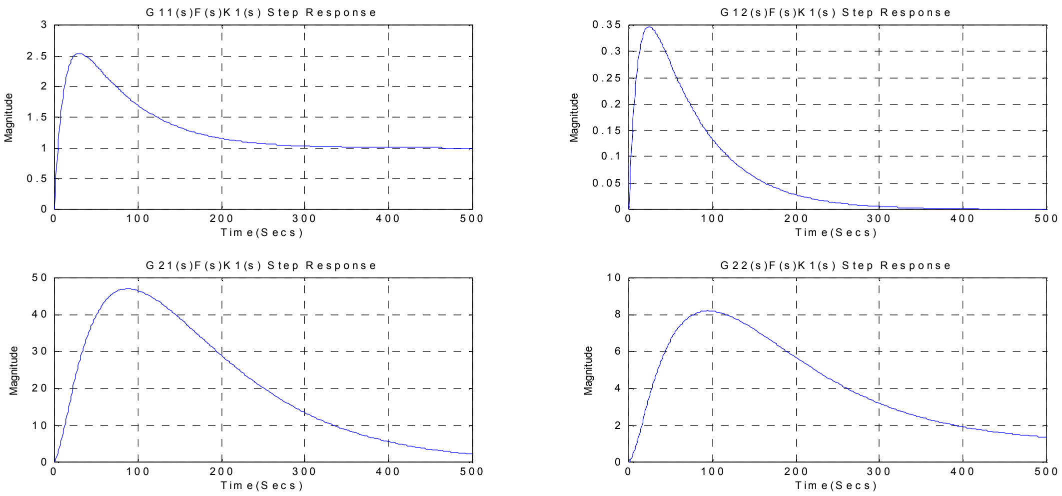

Figure 8. Open Loop Step Response – [$G(s)F(s)K_1(s)$].

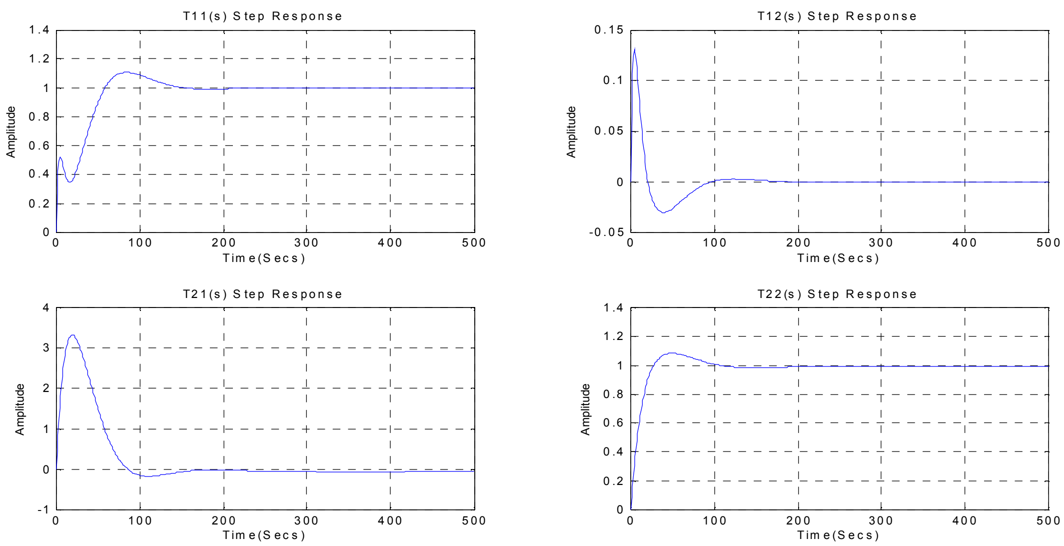

Figure 9. Closed Loop Response - T(s).

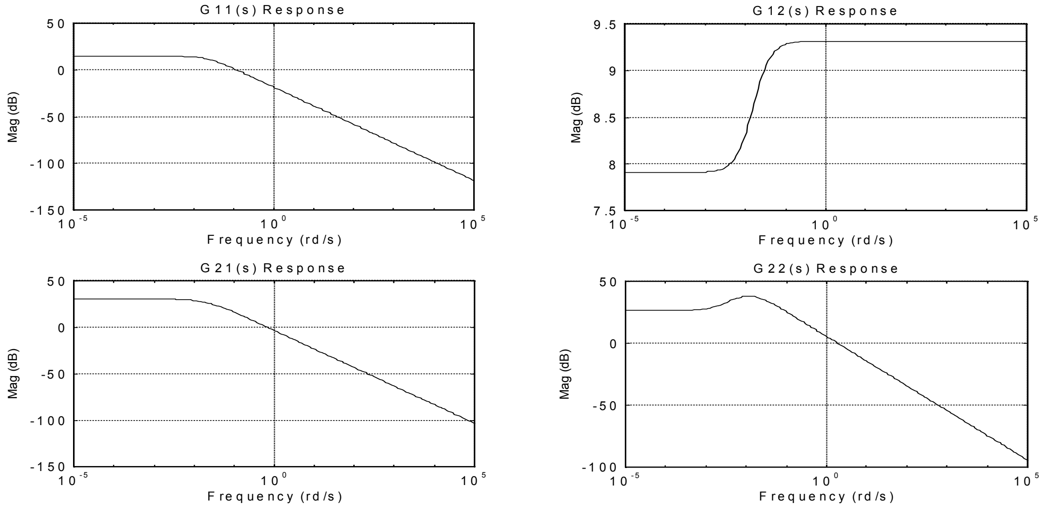

Figure 10. Nominal Plant Open Loop Frequency Response.

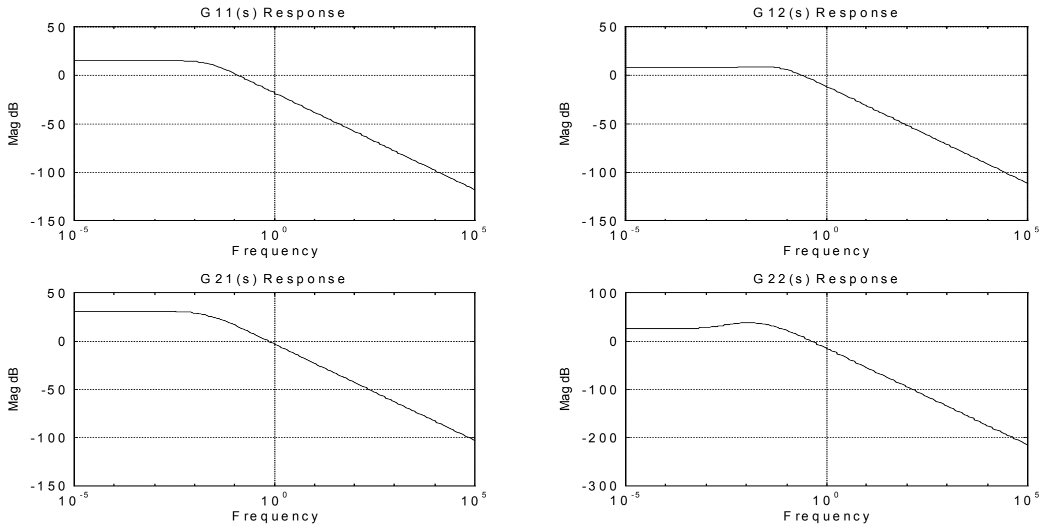

Figure 11. Open Loop Frequency Response of [G(s) F(s)].

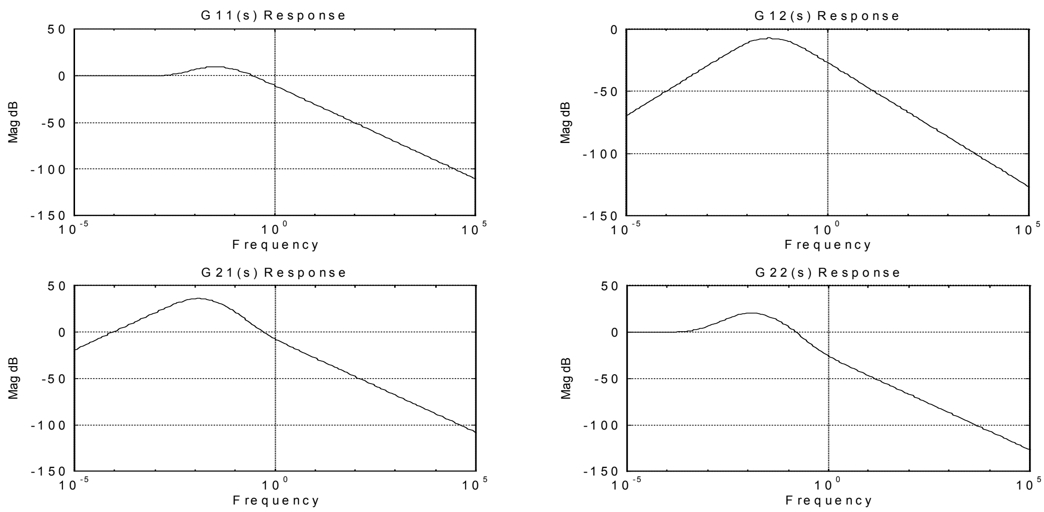

Figure 12. Open Loop Frequency Response of [G(s) F(s) K_1(s)].

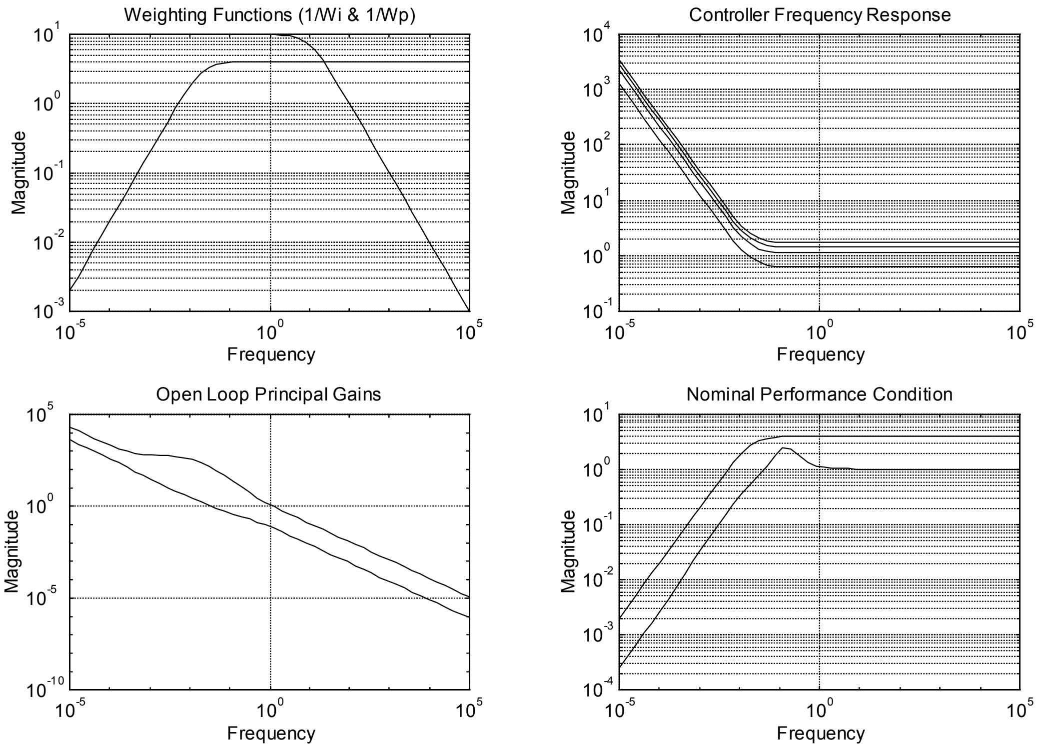

Figure 13. The Design Procedure.

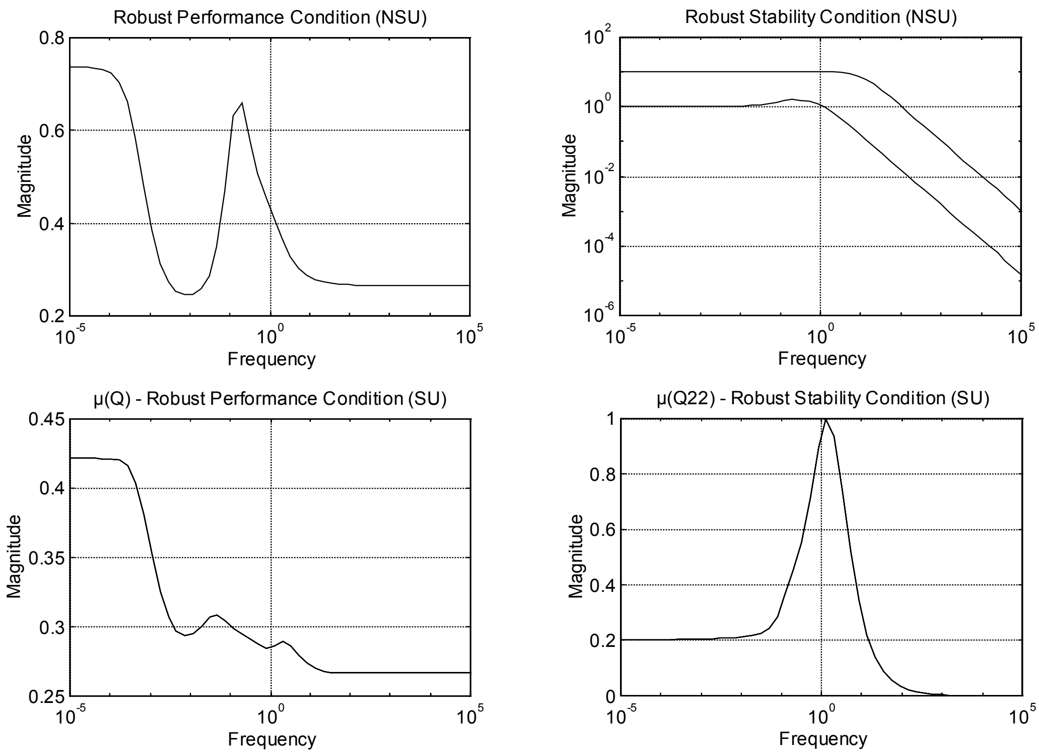

Figure 14. Controller Robustness Validation.

8. Final Comments.

Traditionally, classical on-off controllers for cooling machines have shown to be inefficient for energy saving purposes (Rocha, 1995; Machado, 1996). Variable compressor speed operation has recently emerged as the solution for the energy consumption minimization problem. The searching for an inexpensive compressor speed controller is currently on the focus of the attention of the control community and although some fine results can be found in the technical literature the final solution is still under investigation.

This paper shown that independent control of superheating and freezing power in a cooling system based on vapor compression is a feasible task. A MIMO controller for a cooling machine was designed and implemented in computer simulation. The potential of the proposed MIMO controller for saving energy and keeping comfort was verified through simulation. The main features of the proposed technique are:

a) As long as the controller be designed to work in the frequency range in which the plant is diagonal dominant, the design can be accomplished in a SISO environment. It should be noticed that this is the general case of PI control in industry which is designed to work in low frequency.
b) The implementation of the control law can be done as two independent single loop controls.
c) Due to the plant pre-compensator, model accuracy is in general required only at low frequencies.

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