OUTPUT-ONLY MODAL ANALYSIS USING FREQUENCY DOMAIN DECOMPOSITION

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Abstract. *The advantage of this method is that no artificial excitation needs to be applied to the structure and if artificial loading is required, the force does not need to be measured. Then the excitation can be the own operation condition of the structure. All parameter estimation is based on the response signals only. The modal analysis using output-only focus this problem. In this case, the deterministic knowledge of the input it is replaced by the assumption that the input is a realization of a white noise process and the structural modal parameters of the model is evaluated such as a stochastic identification problem. The present paper discusses the use of frequency domain techniques for the modal identification of output-only mechanical structures. The study described in this paper presents the main results of the Experimental Modal Analysis using output-only. The problem is formulated from the output power spectrum density matrix, which is the only available measured data. The matrix is decomposed in a set of spectral density functions by Singular Value Decomposition (SVD) and the corresponding functions are treated as single degree of freedom system, in order to estimate the modal parameters of the model the implement approach is evaluated with experimental data.*

Keywords*: Output-Only Modal Analysis, Frequency Domain Decomposition, SVD, Spectral Density Function.*

1. Introduction

The Modal analysis techniques have been widely used to study the structural dynamic behaviour of a model through a matricial formulation, which allows an estimate of the vibration parameters of the model. This formulation leads to a mathematical model, that establishes a direct relationship between the excitation and the response of the structure, in terms of its modal parameters. In the experimental modal analysis this input-output relationship is obtained measuring the excitation and the response of the structure, in a predefine set of points. In this case, a set of complex functions establishing a direct relationship between the excitation force and the responses of the structure could be obtained. For a force applied in a point *j* and the corresponding response measured in a point *i* of the structure, one can define the called Frequency Response Function (FRF) of the model or its equivalent in the time domain Impulse Response Functions (IRF).

Generally in the experimental modal test, the modal parameters are derived from the FRF(s) measured in laboratory conditions well controlled. The excitation of the structure, in this case, is generally made using an hammer or a shaker. However, the behaviour vibro-acoustic of a structure in operation conditions (real conditions of load), they can be significantly different of a situation of a laboratory test. Most of the algorithms and methods of experimental modal analysis (Soneys, et al., 1992; Ewins, 1984; Juang and Pappa, 1985) are generally limited to controlled test using forced excitation.

Tests using a natural excitation condition due to the operation condition of the system (Bricker, 2000) are actually becoming an alternative to study the modal behaviour of structure in different areas. The tests and the algorithms used in the estimation of the parameters, they differ of those used in a laboratory condition. The tests based on the operation conditions also take into account, in the evaluation of the behaviour of the system, the environmental influences. Another aspect of interest in the modal analysis based in the operation conditions is the fact that, in some specific cases, to produce a forced excitation with standard equipment is very difficult or even impossible. Therefore, in these cases, the structure could be excited by a natural condition. Large engineering structures, for example, could use the wind or traffic as a condition of excitation for the evaluation of its dynamic properties.

The use of the operational conditions to excite a structure is likely to be a very convenient form to investigate the behaviour and the parameters of interest of the system in its own working environment. Another aspect of great relevance, is that the consolidation of those methods of modal parameters identification, based on the operation conditions, could guide an enormous advance in the area of healthy condition monitoring. Allowing the use of *in situ* diagnosis models for the monitoring and structural damage detection. It is also noticed that the modal analysis using output only presents a vast potential of application in different areas such as in large mechanical structures as in structures of civil engineering (bridges, viaducts, etc.) and others.

The experimental test using only output will be the main topic of this article and the identification of the modal parameters of the structure will be obtained using only the response data. The paper discusses the implementation of a methodology that uses a frequency domain decomposition method to calculate the modal parameters of the model. The implemented approach is evaluating with experimental data.

2. Modal Identification using only-output.

The Frequency Domain Decomposition technique proposed in this work it is an extension of the classic frequency domain approach, as it is discussed in Brincker, 2000; Brincker, 2001; Peeters and Roeck, 2001; Schwars and Richardson, 2001. In the classic approach, the parameters are estimated directly from the use of Discrete Fourier Transform, that allows an estimating of well separated modes directly from the analysis of the power spectrum density matrix, at the frequency peaks. This technique has some limitations concerning of its accuracy in the identification process. It gives a good estimating of natural frequencies and modes if the modes are very separated, the estimating of the damping it in difficulty in most cases.

The proposed technique removes some disadvantages associated with the classic approach such as the heavy dependency on the frequency resolution of the estimated power spectral density and difficulties of to work with close modes. Nevertheless, it keeps the important characteristic of being users friendly and it is still providing a physical understanding of the estimated parameters of the model if one observes the spectral density function as in the previous case.

In the Frequency Domain Decomposition method (FDD), the power spectral density matrix is decomposed in singular values and vectors by using the Singular Value Decomposition (SVD) for discreet frequency line of the spectrum in the line region of each mode. So, the spectral density matrix is decomposed in a set of spectral density function, that contain the contribution of that mode of the system. The first singular vector, calculated for each specific frequency peak, is the own estimating of that mode related with the peak. So, the spectral matrix is decomposed in a set of spectral density function, each one corresponding to an equivalent system of a single degree of freedom (SDOF), as will be discussed in next section.

3. Theoretical Background of Frequency Domain Decomposition

The FDD technique estimates the parameters using the Singular Value Decomposition (SVD) technique applied to output the spectral density matrix. The decomposition of the output spectrum separates the components of the modes and it allows to the definition of equivalents systems of Single Degree of Freedom (SDOF), associated with the frequency peaks. The number of lines that the calculation of the equivalents spectral density function is defined in terms of a coefficient of correlation that compares the identified mode and the singular vector obtained for each frequency line of the spectral density function in the region of the peak, as will be shown in this section.

The relationship between the input *x(t)* and the corresponding output *y(t)* for a invariant linear system, written as shown in equation (1) (Papoulis, 1991), will be used in the formulation of the method.

$$G_{yy}(jw) = \overline{H}(jw)\, G_{xx}(jw)\, H^T(jw) \tag{1}$$

$G_{xx}(jw)$ is the input Power Spectral Density Matrix (PSD) and $G_{yy}(jw)$ is the output PSD matrix (Bendat and Piersol, 1993; Maia, 1997). For r inputs, $G_{xx}(jw)$ is of order rxr and the output matrix $G_{yy}(jw)$ it is of order mxm, being m the number of measured responses. H(jω) it is the Frequency Response Function matrix (FRF) of order mxr. Assuming that the input is a perfect white noise, its spectral density is flat in the analysis range, that means, $G_{xx}(jw)$ is equal a constant C.

The FRF matrix now, can be written in partial fraction form, in terms of its poles and residues as discussed in classical Modal analysis, equation (2)

$$H(jw) = \sum_{K=1}^{n} \frac{R_K}{jw - l_K} + \frac{\overline{R}_K}{jw - \overline{l}_k} \tag{2}$$

n is the number of modes, $\lambda_{k's}$ are the poles and $R_{k's}$ are residues. The residue R_k, as in the convectional modal analysis theory, is defined in terms of the modal participation vector (Ewins, 1984), equation (3).

$$R_k = y_k g_k^T \tag{3}$$

y_k is the k[th] mode of vibration of the model and γ_k is the modal participation vector. Assuming that the input is a white noise, of power spectral density matrix, $G_{xx}(j\omega)$, equals constant C, the equation (1) can be re-defined as a sum, equation (4).

$$G_{yy}(jw) = \sum_{k=1}^{n}\sum_{s=1}^{n}\left[\frac{R_k}{jw - l_K} + \frac{\overline{R}_k}{jw - l_K}\right] C \left[\frac{R_s}{jw - l_s} + \frac{\overline{R}_s}{jw - \overline{l}_s}\right]^H \quad (4)$$

The subscript H denotes transpose complex conjugated. Multiplying the two partial fractions factors and making the use of the theorem of partial fraction of Heaviside, the output power spectral density matrix can be written as a sum of poles and residues, equation (5).

$$G_{yy}(jw) = \sum_{k=l}^{n} \frac{A_k}{jw - l_k} + \frac{\overline{A}_k}{jw - \overline{l}_k} + \frac{B_k}{-jw - l_k} + \frac{\overline{B}_k}{-jw - \overline{l}_k} \quad (5)$$

A_k is the k[th] residue matrix of the output PSD matrix. This residue matrix of order mxm is given by the expression (6).

$$A_k = R_K C\left(\sum_{s=1}^{n}\frac{\overline{R}_S^T}{-l_K - \overline{l}_S} + \frac{R_S^T}{-l_K - l_S}\right) \quad (6)$$

Taking the contribution of the k[th] mode, the residue is given by equation (7).

$$A_k = \frac{R_k C \overline{R}_K^T}{2a_K} \quad (7)$$

α_K is the real part of the pole $l_K = -a_K + jw_K$. This term becomes dominant to light damped systems, therefore the residue can be defined as proportional to the k[th] eigenvector.

$$A_k \propto R_K C \overline{R}_K \quad (8)$$

Re-defining the equation (8) in terms of the expression (3), gives:

$$A_k \propto y_K g_K^T C g_K y_K^T = d_K y_k y_k^T \quad (9)$$

d_k is a constant scalar. For a frequency w_k only a limited number of modes will contribute significantly for the residue, typically one or two modes. This permits one to fix the number of modes, which allows a definition a set of modes of interest, denoted by Sub (ω). Then, in the case of structures lightly damped the PSD density can be written like expression (10).

$$G_{yy}(jw) = \sum_{k=1}^{m} \frac{d_k y_k y_k^T}{jw - l_k} + \frac{\overline{d}_k \overline{y}_k \overline{y}_k^T}{jw - \overline{l}_k} = Sub(w) \quad (10)$$

Now the estimation of the modal parameters is defined by the Decomposition of the output spectral density matrx using the concept of the Singular Values Decomposition (SVD), of the matrix G_{yy}, results:

$$G_{yy}(jw_i) = U_i S_i U_i^H \quad (11)$$

The matrix $U_i = [u_{i1}, u_{i2}, \ldots\ldots, u_{im}]$ is an unitary matrix formed by singular vectors u_{ij} and S_i is a diagonal matrix contending the singular values s_{ij}.

The peak of amplitude corresponding to the k[th] mode will indicate, if there is not closed modes, that mode will be dominant and the first singular vector u_{i1} it is the estimating of the own mode,

$$\hat{y} = u_{i1} \tag{12}$$

The corresponding singular value is used to define the spectral density function of the equivalent single degree of freedom system (SDOF). The spectral density function is estimated comparing the identified mode, $\hat{y}$, with the singular vectors calculated at the frequency points in the right and left side of the peak. When these singular vectors present a high MAC-value with the mode $\hat{y}$, the corresponding singular value belongs to the corresponding density function of the system of one degree of freedom. If, after a certain frequency point, the singular vector does not present MAC values (Alemang, 82) above a predefined value, the search for parts of the estimated function that coincides with the spectral density function is finished. The MAC-value describes the degree of correlation between two vectors, values close to units means a good correlation of the vectors and value close to zero means absence correlation. The expression to calculate the Mac –value is given by the equation (13).

$$MAC(\{\hat{y}\},\{y\}) = \frac{\|\{y\}\{\hat{y}\}\|^2}{\|\{\hat{y}\}\|^2 \|\{y\}\|^2} \tag{13}$$

Identified the spectral density function of the equivalent system of one degree of freedom, it can be converted into the time domain, using Inverse Fourier Transform, to obtain the natural frequency and the damping of the mode. In this case, the concept of logarithmic decrement it is used for the calculation of the damping and the frequency (free decline). It is equivalent to the auto-correlation function of single degree of freedom system which will be used to estimate the parameters. Firstly all extreme k[th] corresponding those peaks of the function is found and the logarithmic decrement δ is obtained by equation. (14).

$$d = \frac{2}{k} \ln\left(\frac{r_0}{|r_k|}\right) \tag{14}$$

r_0 is the reference value of the auto-correlation function used to count the number of peaks and valley r_k. Thus, the logarithmic decrement and the initial value in the correlation function can be found by linear regression, the damping factor is calculated by equation (15).

$$V = \frac{d}{\sqrt{(d^2 + 4p^2)}} \tag{15}$$

4. Analysis of a Frame structure

The technique described above was implemented and evaluated for identification of natural frequencies, modes shapes and damping ratio of a frame structure. Two tests were conducted, using the classical modal analysis and the output-only modal analysis approach, the conventional modal analysis aims to provide a data base of reference to compare and evaluate the results obtained in the FDD procedure. The Figure (1) shows the structure and some details of the experimental setup used for the acquisition process. The responses of the structure are measured by accelerometers, placed in the connective of the bars. The test was conducted in a free-free condition. The structure was suspended by means of flexible cable (elastic) aiming to represent appropriately the suspension of the structure.

Figure 1a- Experimental Tests

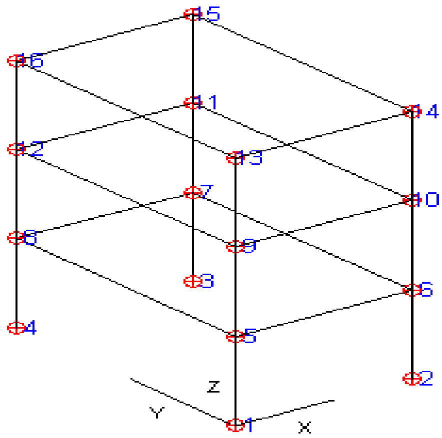


Figure 1b-Structure type "frame"

The excitation of the structure in this test was not exactly caused by a condition of operation the structure. To simulate a condition of operation it was introduced an unmeasured excitation force with random characteristics in the structure. And the responses were measures in all nodes in the x-direction, figure 1b. This unmeasured excitation is a white noise of spectral density G_{xx}.

The experimental tests, as discussed previously, were separated in two parts, a set of data containing the response and the excitation force and a set of data without the excitation force. In the first test, the input and the responses were captured, in order to accomplish the conventional modal analysis, the force applied in the point 5, x-direction, was measured by a force cell mounted between the shaker and structure. The obtained input-output relationship provides the FRF(s) used to estimate the modal parameters of the model. In the second test only the response of the model were measured and the parameters were estimated the by purposed frequency domain decomposition approach that takes only the output data.

The results of this conventional modal test, they will be used to confront with the results obtained from the Frequency Domain Decomposition, aiming a better evaluation of the approach as well as to certify the quality of the obtained results and to discus the validation of the implemented methodology. It seems that, in the stage a comparison with a well-defined data base, obtained from the use of consecrated techniques could give a better insite of the reliability of the results.

The identification of the modal parameters of the structure in the ordinary sense it was carried out by using the method of Ibraim (Ibraim, 1977), method of rational fraction polynomial (Richardson, et. al, 1982)) and method pick Peaking (Pendered, 1963). The results are shown in the table 1

The only-output modal analysis was conducted by using only the output spectral density matrix of the structure. Figure (2) shows typical measured spectral density functions, the measured range of frequency includes the first three modes.

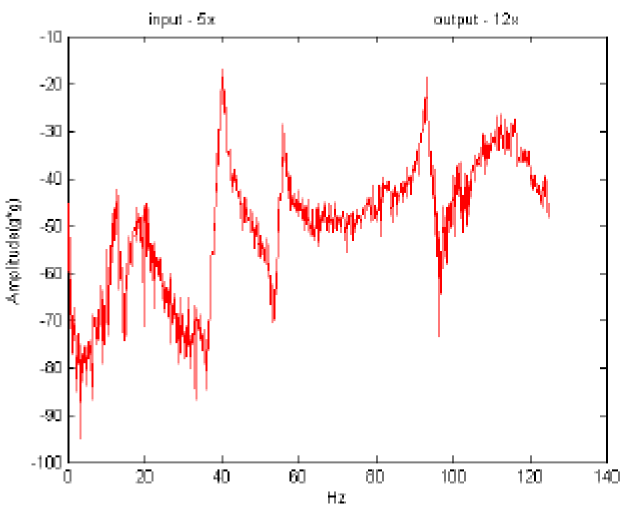


Figure 2 –Typical measured output spectral density function.

From the measured output, the spectral density matrix was defined and used estimate the modal parameters of the model by using the FDD technique. The decomposition of the output spectral density matrix, as discussed before, it is defined for a set of discreet frequencies lines near to the frequency peaks. The extension of the number of frequency line used to define the equivalent spectral density function of the equivalent single degree of freedom system, it is defined by MAC-values. This value is pre-defined by the analyst and, in this analysis, it was used all frequency lines presented MAC-values higher then 0.5. Figure 3a illustrates the estimated spectral density function for the first peak of frequency and its corresponding time representation. This new estimated spectral density function corresponds to a equivalent system of a single degree of freedom that will be used for estimating of the information of the corresponding mode shape table 1 shows the obtained results, the damping factor is estimated through the logarithmic decrement technique.

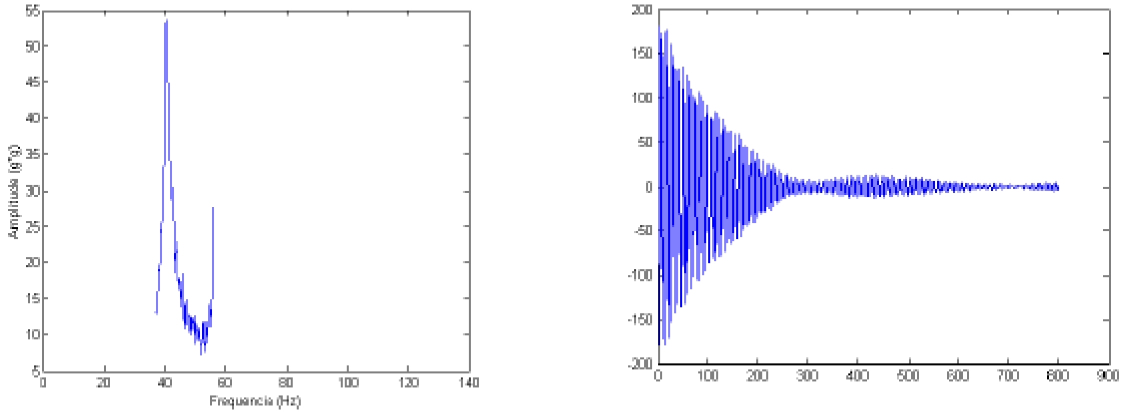


Figure 3a - Singular Values of the decomposition of spectral density matrix (1º mode).

The estimation of the parameters for second and third modes is analogous, figure 3b shows the spectral density function estimated in the second frequency peak and its corresponding time domain function.

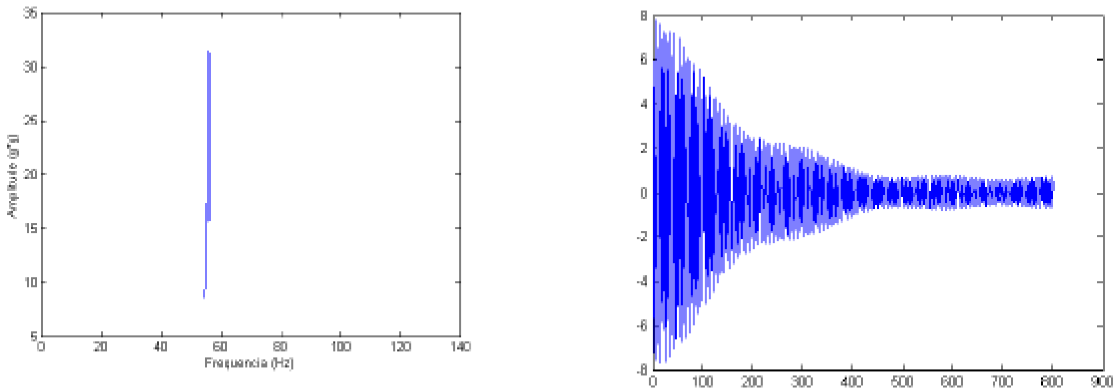


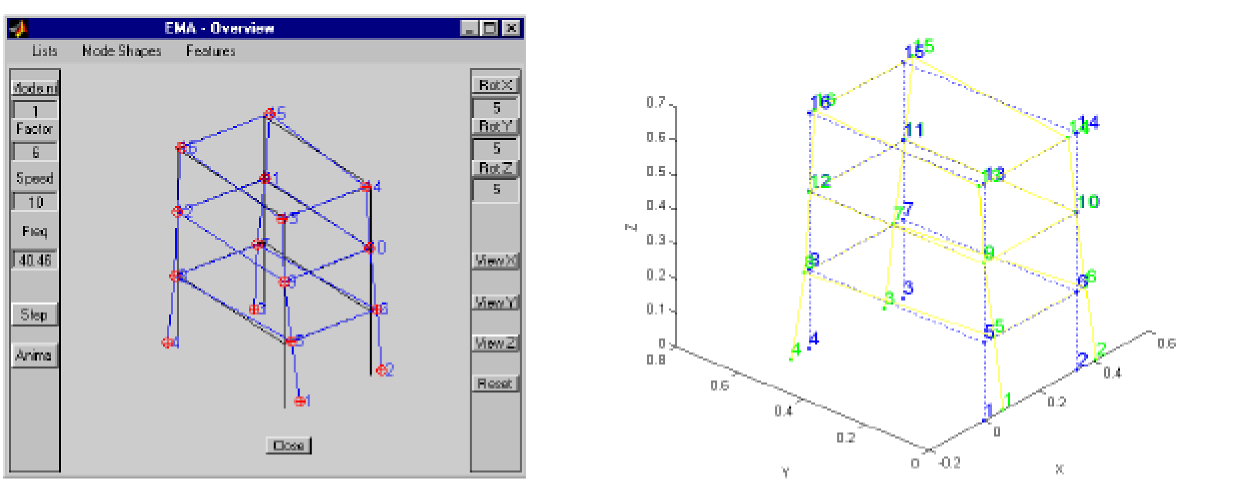
Figure 3b - Singular Values of the decomposition of spectral density matrix (2º mode).

The table 1 also shows a comparison of the results obtained by different estimation technique it presents the comparison, in terms of MAC-value of the modes shapes obtained by the Frequency Domain Decomposition technique and the values obtained with the classic modal analysis. In the Classic Modal Analysis technique the method of Ibraim, method pick peaking and rational polynomial were used to estimate the modes shapes of the structure.

Table 1 - Comparison of the modes shapes.

Modes	DDF	Pick peaking		Ibraim		Rational polynomial	
	ω (Hz)	ω (Hz)	MAC	ω (Hz)	MAC	ω (Hz)	MAC
1	40,46	40,46	0,9333	40,46	0,9758	40,52	0,9304
2	56,25	56,09	0,9049	56,24	0,9192	56,45	0,8902
3	92,96	93,12	0,8897	92,87	0,9643	92,89	0,9088

The implement approach contain also some graphics feature witch can be used to compare the results. An interface was created for better interaction of the users with the identification algorithm. The interface permits the animation visualising, step by step, the deformed and undeformed structure, rotation of the structure, numbering, etc. The Figure (4) shows the first two modes of vibrating of the system, obtained by the Frequency Domain Decomposition technique and fraction rational polynomial.



(a) (b)

Figure 4 - Original structure (back and blue) - (a) first mode (blue)-FDD
(b) First mode (yellow) –R.P.

Figure (5) presents the MAC matrix that shows the modes correlation, taking modes shapes obtained with the Frequency Domain Decomposition technique and the conventional techniques. It is noticed that all modes present a satisfactory correlation indices. An after observing of the figure 5 shows that the first and third modes presents a high degree of the correlation.

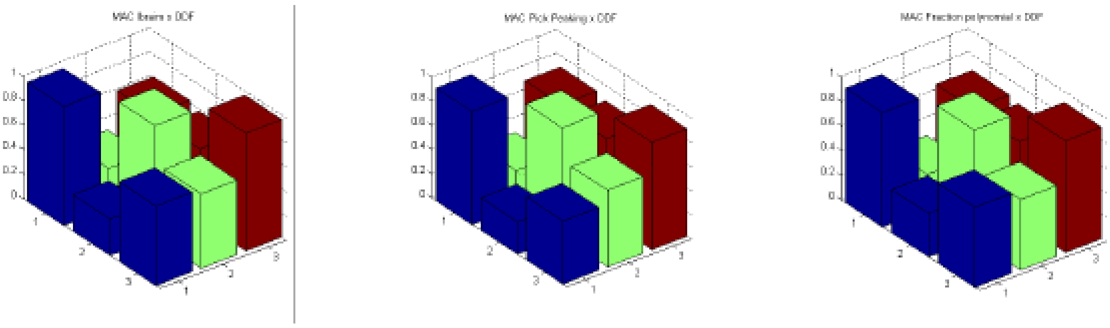


Figure 5 - Comparison of the modes shapes.

5. Conclusion

This paper discusses the identification of the modal para meters of mechanical structures using the frequency domain decomposition (FDD) method when only the output data are measured. The method is based in the SVD of the output spectral density matrix of the model. The methodology has been evaluated by means of experimental data of a laboratory frame Structure. The modal parameters of the structure were estimate using two different approach, the classic modal analysis and the output only modal analysis. The results obtained in the identification of the modal parameters using the classic modal analysis, in this case, were used as reference to evaluate the quality of the estimated results by using the FDD method.

The obtained results have been shown promising. The modal parameters of the system were estimated with the same order of precision as compared with the results obtained in the classic modal analysis. This shows that the methodology could be applied in the identification of real structures, using only the output data, it will be next step of the work.

6. References

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