FINITE ELEMENT ANALYSIS OF THE THERMOMECHANICAL COUPLING EFFECTS ON DAMAGE LOCALIZATION IN ELASTOPLASTIC METALLIC BARS

Christian Nolte
CEFET/RJ - Department of Mechanical Engineering - Av. Maracanã, 229, 20271-110 - Rio de Janeiro - RJ - Brazil
e-mail: chnolte@superig.com.br r

Pedro Manuel Calas Lopes Pacheco
CEFET/RJ - Department of Mechanical Engineering - Av. Maracanã, 229, 20271-110 - Rio de Janeiro - RJ - Brazil
e-mail: calas@cefet-rj.br

Heraldo da Costa-Mattos
UFF - Department of Mechanical Engineering - Rua Passo da Pátria 156, 24210-240, Niterói, RJ, Brasil
e-mail: heraldo@mec.uff.br

Marcelo Amorim Savi
COPPE/UFRJ - Department of Mechanical Engineering – Cx Postal 68.503- 21.945.970 – Rio de Janeiro - RJ – Brazil
e-mail: savi@mecanica.coppe.ufrj.br

Abstract. *Thermomechanical coupling is an important phenomenon in different engineering problems. Inelastic cyclic strain promotes heating of metallic structural elements, and a considerable amount of heat can be generated in situations where high loading rates and/or high amplitudes of inelastic strain are of concern. The temperature rise of mechanical component depends on the loading amplitude, frequency and temperature boundary conditions. Although the thermomechanical phenomenon has a great influence in many situations, traditional design methodologies use isothermal models that do not consider the variation of the material temperature being possible unreal predictions. In this paper, a continuum damage mechanics model is proposed to study the thermomechanical coupling effects on the life prediction of metallic structures subjected to cyclic inelastic loadings. A thermodynamic approach permits a rational identification of the thermomechanical coupling in the mechanical and thermal equations. A numerical procedure is developed based on the operator split technique associated with an iterative numerical scheme in order to deal with the nonlinearities in the formulation. With this assumption, coupled governing equations are solved considering three uncoupled problems: thermal, thermoelastic and elastoplastic behaviors. Classical finite element method is employed for spatial discretization in the uncoupled problems. Numerical simulations of a plane steel truss structure subjected to cyclic loadings are presented and analyzed. Results suggest that the proposed model is capable of capturing important localization phenomena related to damage evolution.*

Keywords*: Thermomechanical Coupling, Modeling, Numerical Simulation, Elastoplasticity, Damage.*

1. Introduction

Thermomechanical coupling is an important phenomenon in different engineering problems. Inelastic cyclic strain promotes heating of metallic structural elements, and a considerable amount of heat can be generated in situations where high loading rates and/or high amplitudes of inelastic strain are of concern. (Simo and Miehe, 1992; Pacheco, 1994; Barbosa et al., 1995; Pacheco and Mattos, 1997). The temperature rise of mechanical component depends on the loading amplitude, frequency and temperature boundary conditions. However, in traditional low-cycle fatigue models, the variation of the material temperature due to thermomechanical coupling is not considered and unreal life predictions may be obtained. Indeed, there are situations where such coupling cannot be neglected and a physically more realistic model must take it into account.

Since temperature variation can interfere with the fatigue phenomenon and most classical low-cycle fatigue models only take into account isothermal processes, the ASTM standard for low-cycle fatigue testing (ASTM, 1992) establishes that the gradient of temperature during a testing program must not exceed the range of ±2°C. For situations with high inelastic amplitudes, the standard recommends the use of cooling devices and low frequency loadings to maintain the specimen temperature on the established range. However, this can be a difficult condition to achieve in operational real mechanical components.

In this paper, a continuum damage mechanics model with internal variables is proposed to study the thermomechanical coupling effects on the life prediction of metallic truss structures subjected to inelastic cyclic loadings (Pacheco, 1994; Lemaitre and Chaboche, 1990). A thermodynamic approach permits a rational identification of the thermomechanical coupling in mechanical and thermal equations. A numerical procedure is developed based on the operator split technique associated with an iterative numerical scheme in order to deal with the nonlinearities in the formulation. With this assumption, coupled governing equations are solved considering three uncoupled problems:

thermal, thermoelastic and elastoplastic. Classical finite element method is employed for spatial discretization in the uncoupled problems. Numerical simulations considering an austenitic stainless steel (AISI 316L) truss structure subjected to cyclic loadings are presented and analyzed. Results suggest that the proposed model is capable of capturing important localization phenomena related to damage evolution.

2. Constitutive Model

Constitutive equations may be formulated within the framework of continuum mechanics and the thermodynamics of irreversible processes, by considering thermodynamic forces, defined from the Helmholtz free energy, ψ, and thermodynamic fluxes, defined from the pseudo-potential of dissipation, ϕ (Lemaitre and Chaboche, 1990; Pacheco, 1994).

With this aim, a Helmholtz free energy is proposed as a function of observable variables, total strain, ε_{ij}, and temperature, T. Moreover, the following internal variables are considered: plastic strain, ε_{ij}^{p}, kinematic hardening, c_{ij}, isotropic hardening, p, and damage, D. The macroscopic quantity D ($0 \leq D \leq 1$) represents the material local degradation. When $D = 0$ the material is in a virgin state and when $D = 1$ the material is completely damaged. Therefore, the following free energy is proposed, employing indicial notation where summation convention ($i = 1,2,3$) is evoked (Eringen, 1967), except when indicated:

$$\rho\psi(\varepsilon_{ij}, \varepsilon_{ij}^{p}, c_{ij}, p, D, T) = (1-D)\left[W_e(\varepsilon_{ij} - \varepsilon_{ij}^{p}, T) + W_a(c_{ij}, p, T)\right] - W_T(T) \tag{1}$$

where ρ is the material density, W_e is the elastic energy density, W_a is the energy density associated to the hardening and W_T is the energy density associated with the temperature, defined as:

$$\begin{aligned}
&W_e(\varepsilon_{ij} - \varepsilon_{ij}^{p}, T) = \frac{E}{2(1+\nu)}\left[(\varepsilon_{ij} - \varepsilon_{ij}^{p})(\varepsilon_{ij} - \varepsilon_{ij}^{p}) + \frac{\nu}{1-2\nu}(\varepsilon_{jj} - \varepsilon_{jj}^{p})^2\right] - \frac{\alpha E}{1-2\nu}(\varepsilon_{jj} - \varepsilon_{jj}^{p}) \\
&W_a(c_{ij}, p, T) = \tfrac{1}{2} a\, c_{ij} c_{ij} + b\left[p + (1/d)\, e^{-dp}\right] \\
&\mathrm{W}_T(T) = \rho \int_{T_0}^{T} C_1 \log(\xi)\, \mathrm{d}\xi \; + \; \frac{\rho}{2} C_2 T^2
\end{aligned} \tag{2}$$

where T_0 is a reference temperature, E is the Young modulus, ν is the Poisson ratio, a is a material parameter associated with kinematic hardening, while b and d are material parameters associated with isotropic hardening. C_1 and C_2 are positive constants. The increment of elastic strain is defined as follows:

$$d\varepsilon_{ij}^{e} = d\varepsilon_{ij} - d\varepsilon_{ij}^{p} - \alpha_T dT \delta_{ij} \tag{3}$$

The last term is associated with thermal expansion and the parameter α_T is the coefficient of linear thermal expansion.

This model was developed in a general formulation and was previously applied to the study of various related problems (Pacheco, 1994; Pacheco and Mattos, 1997; Pacheco *et al.*, 2001; Oliveira *et al.*, 2003; Oliveira, 2004; Silva *et al.*, 2004). A detailed description of this constitutive model may be obtained in the cited references.

This contribution considers life prediction of metallic plane truss structures subjected to cyclic inelastic loadings. From the mechanical point of view it can be assumed that the truss elements experiments a uniaxial stress-sate, as the loadings are applied at the elements joints. On the other hand, for thermal characteristics it is assumed that the truss elements experiments a uniaxial heat flow conduction through the element length, as the truss elements cross-section dimensions are considerable smaller than its length. Under these assumptions, a one-dimensional model is formulated and tensor quantities presented in the general formulation may be replaced by scalar quantities. For this situation the thermodynamics forces (σ, P, B^c, B^p, B^D, s), respectively associated with state variables $(\varepsilon, \varepsilon^p, c, p, D, T)$, are defined as follows:

$$\begin{aligned}
&\sigma = \rho\frac{\partial\psi}{\partial\varepsilon} = (1-D)\left[\mathrm{E}(\varepsilon - \varepsilon^p) - E\alpha_T(T - T_0)\right] \quad ; \quad P = -\rho\frac{\partial\psi}{\partial\varepsilon^p} = \sigma \\
&B^c = -\rho\frac{\partial\psi}{\partial c} = -(2/3)\,X = -(1-D)\,a\,c \quad ; \quad B^p = -\rho\frac{\partial\psi}{\partial p} = -R = -(1-D)b\left[1 - e^{-dp}\right] \\
&B^D = -\rho\frac{\partial\psi}{\partial D} = W_e(\varepsilon_{ij} - \varepsilon_{ij}^{p}, T) + W_a(c_{ij}, p, T) \quad ; \quad \mathrm{s} = -\rho\frac{\partial\psi}{\partial T}
\end{aligned} \tag{4}$$

where X and R are auxiliary variables directly related to kinematic and isotropic hardenings, respectively. In order to describe dissipation processes, it is necessary to introduce a potential of dissipation $\phi(\dot{\varepsilon}^p, \dot{c}, \dot{p}, \dot{D}, q)$, which can be split into two parts: $\phi(\dot{\varepsilon}^p, \dot{c}, \dot{p}, \dot{D}, q) = \phi_I(\dot{\varepsilon}^p, \dot{c}, \dot{p}, \dot{D}) + \phi_T(q)$. This potential can be written through its dual $\phi^*(P, X, R, B^D, g) = \phi_I^*(P, X, R, B^D) + \phi_T^*(g)$, as follows:

$$\phi_I^* = I_f^*(P, X, R, B^D) \quad ; \quad \phi_T^* = \frac{T}{2}\Lambda\, g^2 \tag{5}$$

where $g = (1/T)\ \partial T/\partial x$ and Λ is the coefficient of thermal conductivity; $I_f^*(P, X, R, B^D)$ is the indicator function associated with elastic domain (Lemaitre and Chaboche, 1990),

$$f(\sigma, X, R) = \left|\sigma - X\right| - (S_Y + R) \le 0 \tag{6}$$

where S_Y is the material yield stress. A set of evolution laws obtained from ϕ^* characterizes dissipative processes,

$$\dot{\varepsilon}^p = \frac{\partial\phi^*}{\partial P} = \lambda\,\mathrm{sign}(\sigma - X) \quad ; \quad \dot{c} = \frac{\partial\phi^*}{\partial B^c} = \dot{\varepsilon}^p + \frac{\varphi}{a}B^c\dot{p} \quad ; \quad \dot{p} = \frac{\partial\phi^*}{\partial B^p} = \lambda \quad ; \quad \dot{D} = \frac{\partial\phi^*}{\partial B^D} = \frac{B^D}{S_0}\dot{p}$$
$$q = -\frac{\partial\phi^*}{\partial g} = -\Lambda T\, g = -\Lambda\frac{\partial T}{\partial x} \tag{7}$$

where λ is the plastic multiplier (Lemaitre and Chaboche, 1990) from the classical theory of plasticity, sign(x) = x / |x|, φ is a material parameter associated with kinematic hardening and q is the heat flow. By assuming that the specific heat is $c_p = -(T/\rho)\ \partial^2 W/\partial T^2$ and also considering the set of constitutive Eqs. (4) and (7), the energy equation can be written as (Pacheco, 1994):

$$\frac{\partial}{\partial x}\left(\Lambda\frac{\partial T}{\partial x}\right) - h\frac{Per}{A}(T - T_\infty) - \rho c_p\dot{T} = -a_I - a_T \qquad \text{where} \quad \begin{cases} a_I = \sigma\dot{\varepsilon}^p - X\dot{c} - R\dot{p} + B^D\dot{D} \\ a_T = T\left(\dfrac{\partial\sigma}{\partial T}\left(\dot{\varepsilon} - \dot{\varepsilon}^p\right) + \dfrac{\partial X}{\partial T}\dot{c} + \dfrac{\partial R}{\partial T}\dot{p} - \dfrac{\partial B^D}{\partial T}\dot{D}\right) \end{cases} \tag{8}$$

where h is the convection coefficient, T_∞ is the surrounding temperature, Per is the perimeter and A is the cross section area. Terms a_I and a_T are, respectively, internal and thermal coupling. The first one appears in the right hand side of the energy equation and is called internal coupling. It is always positive and has a role in the energy equation similar to a heat source in the classical heat equation for rigid bodies. The last term in the right hand side of the energy equation can be positive or negative and is called the thermal coupling.

3. Numerical Procedure

The numerical procedure here proposed is based on the operator split technique (Ortiz *et al.*, 1983; Pacheco, 1994) associated with an iterative numerical scheme in order to deal with nonlinearities in the formulation. With this assumption, coupled governing equations are solved from three uncoupled problems: thermal, thermo-elastic and elastoplastic. In this article, finite element method is employed to perform spatial discretization of governing equations. Therefore, the following moduli are considered:

Thermal Problem **-** Comprises a one-dimensional conduction problem with surface convection. Material properties depend on temperature and, therefore, the problem is governed by nonlinear parabolic equations. Classical finite element method is employed for spatial discretization while *Crank-Nicolson* method is used for time discretization (Lewis *et al.*, 1996; Segerlind, 1984).

Thermo-elastic Problem **-** Stress and displacement fields are evaluated from temperature distribution. Classical finite element method is employed for spatial discretization (Segerlind, 1984).

Elastoplastic Problem **-** Stress and strain fields are determined considering the plastic strain evolution in the process. Numerical solution is based on the classical return mapping algorithm (Simo and Miehe, 1992; Simo and Hughes, 1998).

As an application of the general procedure technique, plane FEM truss elements are considered. Linear shape functions are adopted for all finite element moduli (Segerlind, 1984).

4. Numerical Simulations

The proposed model is applied to the life prediction of a metallic (austenitic stainless steel AISI 316L) plane truss structure subjected to cyclic inelastic loadings. The plane truss structure shown in Fig. 1 with 10 mm diameter round truss elements is considered. Constant displacement boundary conditions ($u_x = u_y = 0$) and constant temperature boundary conditions (T = 20°C) are applied to nodes 2 and 4. A harmonic load (F) with amplitude of 31 kN and a period of 10 s is applied at node 1.

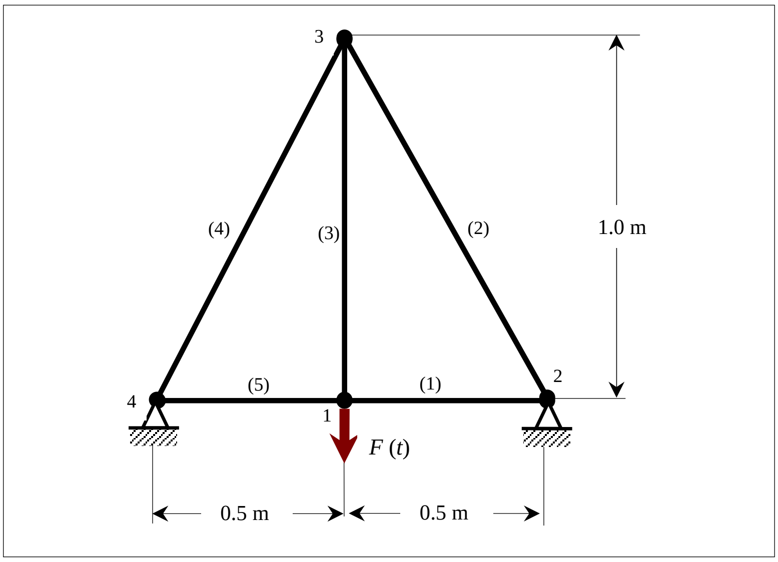


Figure 1. Plane truss structure.

Thermal and mechanical properties are temperature dependent and are fitted by linear equations from experimental data shown in Tab. 1 (Peckner and Bernstein, 1977; Pacheco, 1994). A linear kinematic hardening is considered ($\varphi = 0$). Moreover, the following constant parameters are used: convection coefficient (h) of 30 W/m^2, surroundings and initial structure temperature of 20°C, density (ρ) of 7800 kg/m^3 and a critical damage (D_{cr}) of 0.85.

Table 1. Material parameters for AISI 316L (Peckner and Bernstein, 1977; Pacheco,1994).

	Temperature	
Properties	20°C	600°C
E (GPa)	196	150
S_Y (MPa)	225	108
b (MPa)	60	80
d (-)	8	10
a (GPa)	108.3	17.5
α (1x10^{-6}/K)	15.4	18.0
c_p (J/Kg K)	454	584
Λ (W/m K)	13	21

Numerical simulations are performed with the aid of computational software developed in *C* programming language. In order to allow the evaluation of the thermomechanical effects in the damage localization of the truss, two distinct models are considered: *coupled* and *uncoupled*. The *coupled* model considers the thermomechanical coupling terms present in Eq. (8). On the other hand, the *uncoupled* model neglects these thermomechanical coupling terms and therefore, the thermal problem is solved as a rigid body. The heat flow removed by convection is calculated considering an average temperature obtained from the element nodal temperatures.

The low-cycle fatigue parameter (S_0) that appears in the Eq. (7) is adjusted through a process involving a direct comparison of the life predictions obtained with the *uncoupled* modelwith experimental data (Bathias and Bailon, 1980; Pacheco and Mattos, 1997). This procedure is, therefore, in accordance with the ASTM recommendations for low-cycle fatigue tests (ASTM, 1992). Figure 2 shows a comparison between a ε-N curve (Stephens, *et al.*, 2000) obtained from experimental data from a low-cycle fatigue test (Bathias and Bailon, 1980) and the model prediction, considering the *uncouple* model, after the parameter adjustment for a 316L stainless steel bar at room temperature (20 °C). The results indicate a good agreement between the experimental data with those predicted by fatigue life for a strain amplitude range from 2% to 5%. The coefficient S_0 of Eq. (7) presents a dependency with plastic deformation amplitude, and can be adequately represented by the following equation (in Pa) (Pacheco, 1994; Pacheco and Mattos, 1997):

$$S_0 = (546.3 \times 10^9)\, e^{-103.1\, \Delta\varepsilon^p} \tag{9}$$

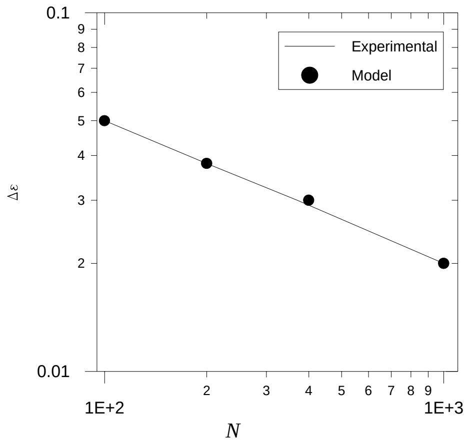
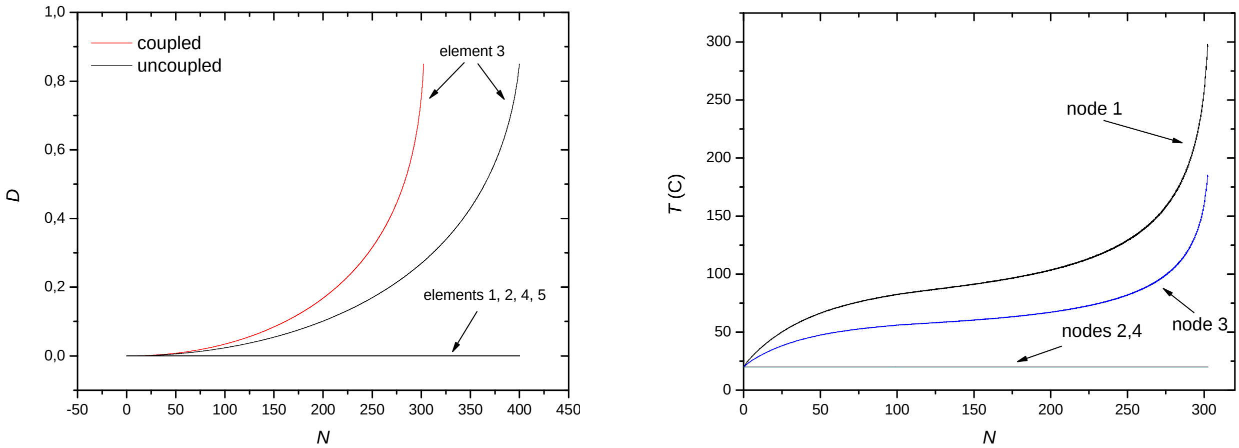


Figure 2. Proposed model fatigue life prediction and ε-N curve obtained from experimental data for stainless steel 316L bars at room temperature (Pacheco and Mattos, 1997).

The forthcoming analysis considers results obtained by both the *coupled* and *uncoupled* models applied for the truss shown in Fig.1. Figure 3a shows that both models indicate a damage localization process occurring in element 3, which is the first to reach the critical damage (D = 0.85). This element is the critical one since it experiments the higher levels of stress and temperature. Figure 3b present the temperature evolution for the truss nodes obtained with the *coupled* model (the *uncoupled* model has constant temperatures for all bars). This behavior may be understood as a two stage process. The first one occurs as a result of two opposite but "balanced" forces in the energy equation (8): the thermomechanical coupling terms (a_I and a_T) acts as a heat source and the conduction/convection terms remove the heat. In this stage, the structure experiments a controlled temperature rise. In the last stage, as the thermomechanical coupling terms dominates the energy equation, the temperature rises abruptly until the critical damage is reached.



(a) (b)

Figure 3. Damage evolution for *coupled* and *uncoupled* models (a). Temperature evolution for truss nodes for the *coupled* model (b).

Figure 4 shows the plastic strain, kinematic hardening, isotropic hardening and damage evolution of the critical truss element (element 3) for both models. Notice that *coupled* and *uncoupled* models predict a fatigue life of approximately 300 cycles and 400 cycles, respectively. Therefore, the *coupled* model fatigue life prediction is approximately 25%

lower. The thermomechanical coupling can be seen as a feedback phenomenon: the heat generated by the mechanical process causes an increase of temperature that promotes a decrease in the mechanical strength. As a consequence, the plastic strain amplitude tends to increase causing a greater temperature rise and so on. It is important to observe that the temperature boundary conditions influence the temperature evolution and therefore the mechanical behavior.

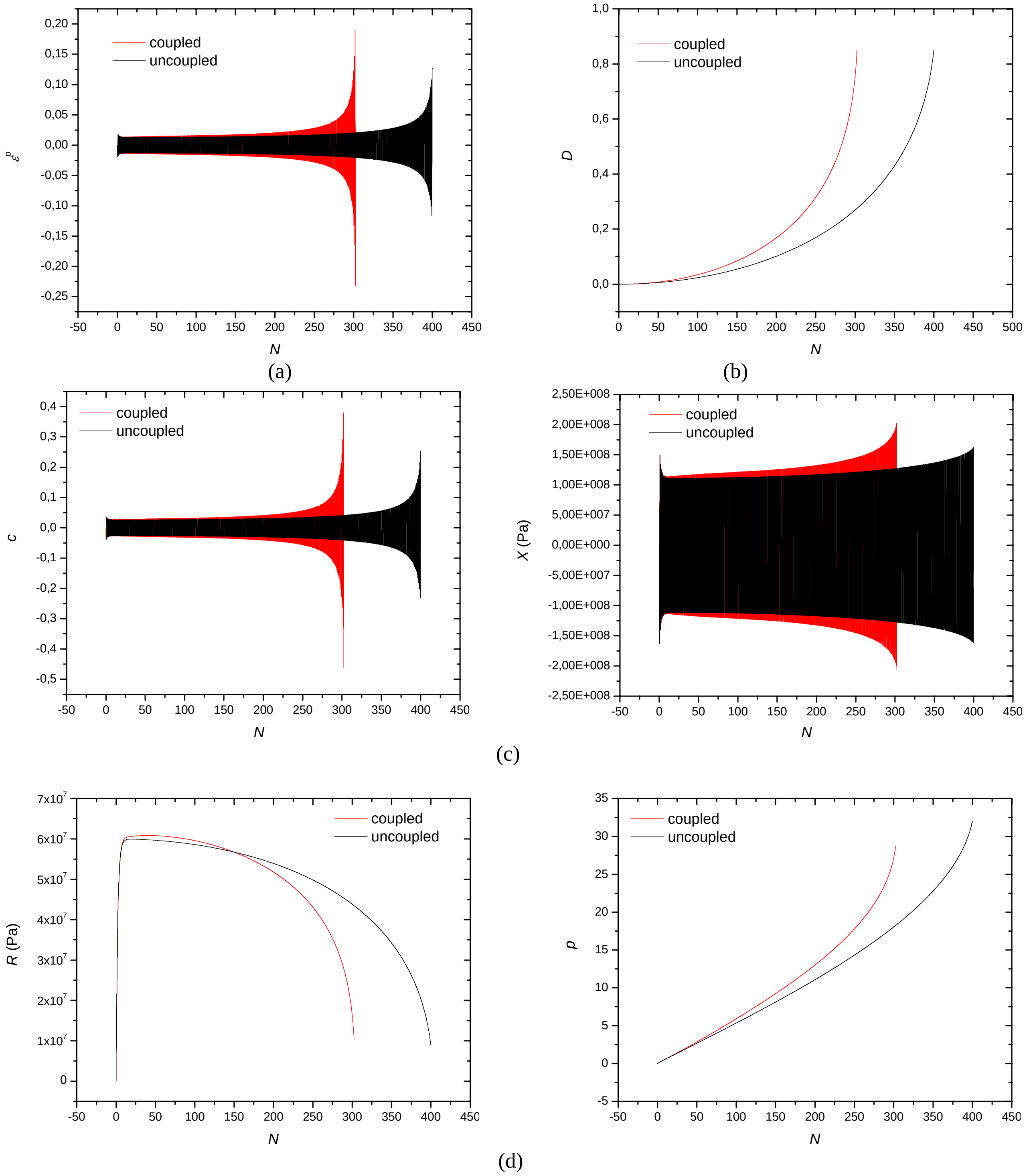


Figure 4. Plastic strain (a), damage (b), kinematic hardening (c), isotropic hardening (d) evolution for truss element 3. *Coupled* and *uncoupled* models.

Figures 5 and 6 present the stress-strain curves predicted by both models. These figures show different stress-strain hysteresis loops for *coupled* and *uncoupled* models, confirming that the thermomechanical coupling terms can affect considerably the material behavior. For truss element 3 (Fig. 5), both models show the damage evolution promoting the enlargement of the loop and the reduction of the stress-strain ratio (apparent Young modulus) in the elastic region (unloading linear region) as the damage directly affects the elastic energy density. For truss element 2 (Fig. 6), *coupled* model predicts the development of a stress-strain hysteresis loop with plastic whereas the *uncoupled* model predicts an elastic behavior.

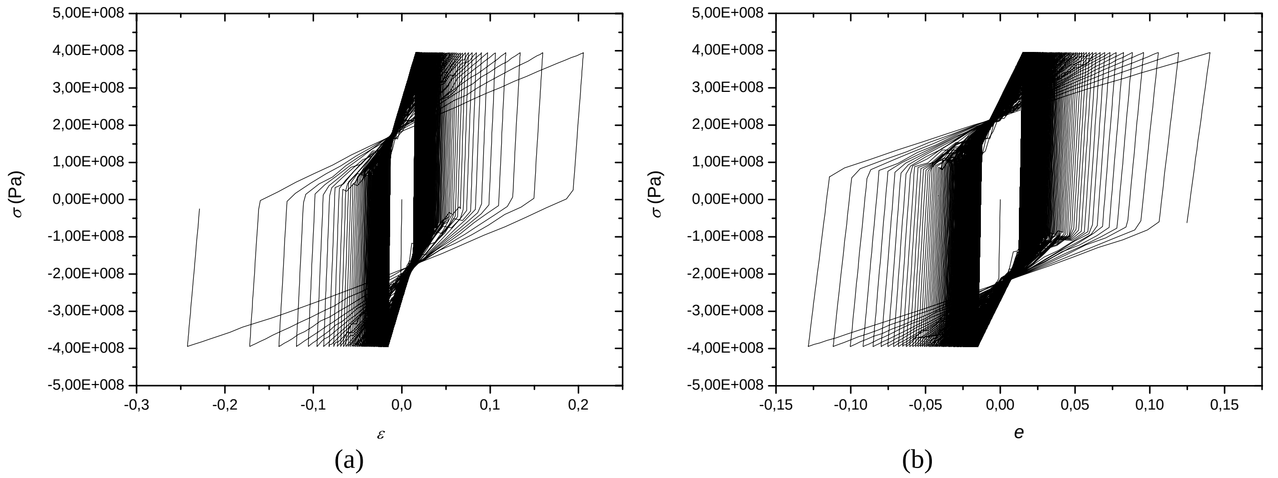


Figure 5. Stress-strain hysteresis loops for truss element 3. *Coupled* (a) and *uncoupled* (b) models.

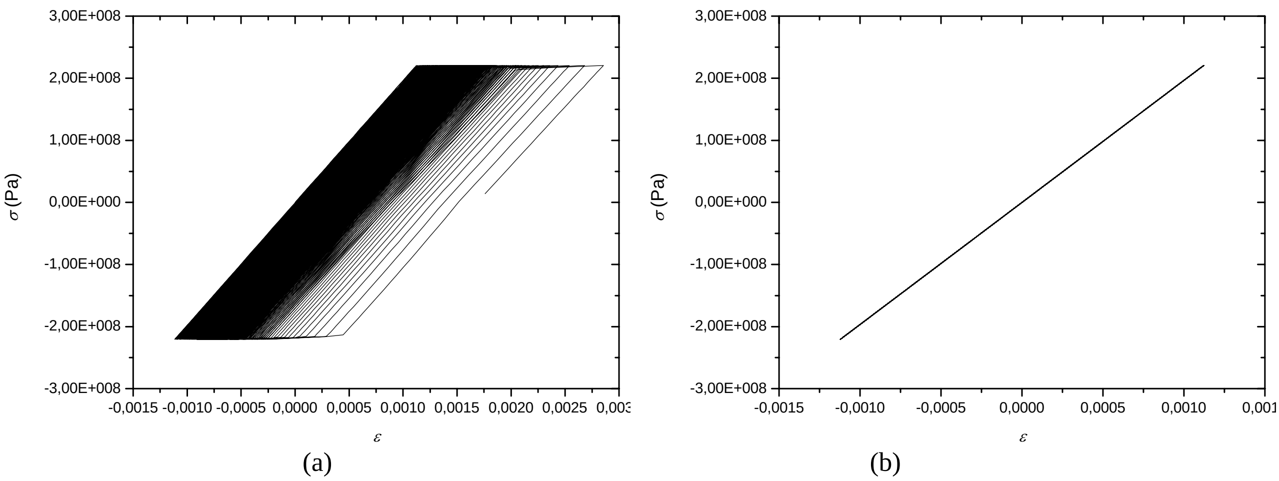


Figure 6. Stress-strain hysteresis loops for truss element 2. *Coupled* (a) and *uncoupled* (b) models.

5. Conclusion

In this paper an anisothermal constitutive model with internal variables based on continuum damage mechanics is proposed to study the thermomechanical coupling effects in elastoplastic truss bars subjected to inelastic cyclic mechanical loadings. This formulation provides a rational methodology to study complex phenomena like the amount of heat generated during plastic strain of metals and how it affects its structural integrity. The numerical procedure developed is based on the operator split technique and permits to deal with the nonlinearities in the formulation using traditional tested classical numerical methods, as the finite element method which is used for spatial discretization in the uncoupled problems. Numerical simulations considering an austenitic stainless steel (AISI 316L) truss structure subjected to cyclic loadings are presented and analyzed. Results suggest that the proposed model is capable of capturing important localization phenomena related to damage evolution. Numerical simulations show that is important to consider the thermomechanical coupling effects in low-cycle fatigue design of mechanical components, especially when high loading rates are involved. In these situations, if the thermomechanical effect is not included in the model wrong predictions can be obtained and unexpected failures may occur.

6. Acknowledgements

The authors would like to acknowledge the support of the Brazilian Research Agencies CNPq and CAPES.

7. References

ASTM E606-92, 1992; Standard Recommended Practice for Constant-Amplitude Low-Cycle Fatigue Testing, *ASTM Standards*, Vol. 03.01.

Barbosa, J.M.A., Pacheco, P.M.C.L. and Mattos, H.C., 1995; On the role of Temperature in the Mechanical Vibration of Elasto-Viscoplastic Bars, *COBEM-95/CIDIM-95, 13th Brazilian Congress of Mech. Engineering*, Belo Horizonte, Brazil, (in portuguese).

Bathias, C. and Bailon, J.P., 1980; *La Fatigue de Materiaux et des Structures*, Les Presses de L'Université de Montréal, Maloine S.A. Editeur, Paris.

Eringen, A.C., 1967, "*Mechanics of Continua*", John Wiley, New York.

Stephens, R.I., Fatemi, A., Stephens, R.R., and Fuchs, H.O., 2000; *"Metal Fatigue in Engineering"*, 2nd edition, John Wiley & Sons.

Lemaitre, J. and Chaboche, J.L., 1990, "*Mechanics of Solid Materials*", Cambridge Press.

Lewis, R.W., Morgan, K., Thomas, H.R. and Seethramu, K.N., 1996, "The Finite Element Method in Heat Transfer Analysis", John Wiley & Sons, USA.

Oliveira, W.P., Souza, L.F.G., Pacheco, P.M.C.L. e Savi, M.A., 2003; "Quenching Process Modeling in Steel Cylinders Using a Multi-Phase Constitutive Model", *COBEM-2003, 17th International Congress of Mechanical Engineering*, São Paulo.

Oliveira, W.P., 2004, *"Modeling the Quenching Process of Steel Cylinders using a Constitutive Multi-Phase Model"* (in portuguese), M.Sc. Dissertation, Mestrado em Tecnologia – CEFET/RJ.

Ortiz, M., Pinsky, P.M. and Taylor, R.L., 1983, Operator Split Methods for the Numerical Solution of the Elastoplastic Dynamic Problem, *Computer Methods of Applied Mechanics and Engineering*, v. 39, pp.137-157.

Pacheco, P.M.C.L., 1994, "*Analysis of the Thermomechanical Coupling in Elasto- Viscoplastic Materials*", Ph.D. Thesis, Department of Mechanical Engineering, PUC-Rio, in Portuguese.

Pacheco, P.M.C.L. and Mattos, H.C., 1997; "Modeling the Thermomechanical Coupling Effects on Low-Cycle Fatigue of Metallic Materials", 5th ICBMFF, 5th International Conference on Biaxial/Multiaxial Fatigue and Fracture, pp.291-301, Cracow, Poland.

Pacheco, P.M.C.L., Camarão, A.F. and Savi, M.A., 2001, "Analysis of Residual Stresses Generated by Progressive Induction Hardening of Steel Cylinders", *Journal of Strain Analysis for Engineering Design*, v.36, n.5, pp.507-516.

Peckner and Bernstein, 1977; *"Handbook of Stainless Steels"*, McGraw-Hill.

Segerlind, L.J., 1984, "Applied Finite Element Analysis", 2nd Edition, John Wiley & Sons, USA.

Silva, E.P., Pacheco, P.M.C.L. and Savi, M.A., 2004, "On the Thermo-Mechanical Coupling in Austenite-Martensite Phase Transformation Related to the Quenching Process", *International Journal of Solids and Structures*, ISSN 0020-7683, Vol. 41, pp. 1139-1155.

Simo, J.C. and Hughes, T.J.R., 1998, "*Computational Inelasticity*", Springer.

Simo, J.C. and Miehe, C., 1992, "Associative Coupled Thermoplasticity at Finite Strains: Formulation, Numerical Analysis and Implementation", *Computer Methods in Applied Mechanics and Engineering*, v.98, pp.41-104.

8. Responsibility notice