COMPARATIVE ANALYSIS OF METHODS FOR REDUNDANCY SOLUTION OF UNDERWATER VEHICLE-MANIPULATOR SYSTEMS

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Abstract. *This paper presents a comparative analysis between some conventional methods to redundancy resolution of Underwater Vehicle-Manipulator Systems (UVMS) and a new approach based on introducing kinematical constraints. In this approach the screw representation of movements is used and incorporated on the so-called Davies method to solve the kinematics of closed kinematic chains. The UVMS is described as an open-loop chain and presents a virtual kinematic chain concept. This concept allows to close this chain and so, to apply the Davies method to solve the direct kinematics. The paper outlines that the Davies method constitutes a systematic way to express the joint rates of passive joints as functions of the joint rates of the actuated joints in closed kinematics chains and that, combined with the virtual chain concept, it constitutes an useful approach to solve the direct and the inverse kinematics of the UVMS. The proposed approach is compared to others methods and simulations confirm its efficiency.*

Keywords*:* *inverse kinematics, comparative analysis, UVMS.*

1. Introduction

Currently, the use of underwater robotic systems is common to accomplish missions as sea bottom and pipeline surveys, cable maintenance, offshore structures monitoring and maintenance, and collect/release of biological surveys. In this framework, the use of a manipulator mounted on an autonomous underwater vehicle plays an important role. Such systems are commonly named Underwater Vehicle-Manipulator Systems (UVMS).

In this paper we deal with the kinematic control of the UVMS, in which the goal is to find a suitable vehicle/joint trajectories that correspond to a desired end-effector trajectory. The output of this kinematic control provides the reference values to the dynamic control law of this UVMS. We consider the kinematic control independent of the dynamic control as in (Antonelli and Chiaverini, 2002, Antonelli, 2003).

An UVMS is always kinematically redundant because, due to the degrees of freedom (dofs) provided by the vehicle itself, it possesses more dofs than those required to execute a given task. However, it is not always efficient to use vehicle thrusters to move the manipulator end effector because of the difficulty of controlling the vehicle in hovering. Moreover, due to the different inertia between vehicle and manipulator, movement of the latter is energetically more efficient. On the other hand, reconfiguration of the whole system is required when the manipulator is working at the boundaries of its workspace or close to a kinematic singularity. Thus, motion of the sole manipulator is not always possible or efficient.

Therefore, a redundancy resolution technique might be useful to achieve system coordination in such a way as to guarantee end-effector tracking accuracy and, at same time, additional control objectives, e.g., energy savings, increase of system manipulability, or obstacles avoidance. For real-time solutions the coordination algorithm must be computationally efficient.

The simplest way to obtain the redundancy resolution is to use the pseudoinverse of the Jacobian matrix. With this approach, however, the problem of handling kinematic singularities is not addressed and their avoidance cannot be guaranteed. This problem has been minimized using other techniques based on the use of the pseudoinverse as the Damped Least Square (Nakamura and Hanafusa, 1986) and the Singular Robust Task Priority (Antonelli, 1998). In any way, the use of the pseudoinverse introduces dimensional difficulties (Campos, 2004). Although Chung (1994) demonstrates the repeatability of the movement in the joints space, by hypothetical restoring springs included at each joint, this technique is not advisable in static problems (Davidson, Hunt, 2004).

In this paper, we compare a new approach to solve the redundancy of UVMS to usual methods. This approach is based on introducing kinematic constraints to the UVMS movements to obtain a square matrix, which may be inverted, without introducing algorithmic singularities. A virtual kinematic chain that closes the UVMS kinematic chain introduces additional kinematic constraints. The inverse kinematic of this closed chain is solved using the Davies method. The proposed approach also addresses additional control objectives, here focused in energy savings. Simulation results show the method efficienty under the point of view of energy consumption, when compared with other methods based in Pseudoinverse solution.

The paper is organized as follows. In the section 2 we describe the kinematics of the UVMS as a manipulator with a mobile base using screws to represent its differential movement. The Davies method, which is used to relate the joint velocities in closed kinematic chains, is presented in section 3 and the virtual kinematic chain concept is introduced in section 4. In the section 5 we discuss the inverse kinematics of the UVMS. Some methods are briefly described in the section 6 with the objective to compare with our approach. The simulations are shown and analyzed in the section 7. The paper conclusions are presented in section 8.

2. Kinematics of Manipulators with Mobile Base

In this section, we describe the movement of an UVMS. More specifically, we obtain the description of the UVMS manipulator end effector motion with respect to an inertial frame, i.e., the kinematics of manipulators with mobile base. In order to introduce a new approach to solve the inverse kinematics to this class of systems, the model of an UVMS using screws to represent its differential motion is presented in this section (see Guenther et al. 2005, for more details about screw representation).

2.1 Serial manipulator kinematics

For a serial manipulator, we may consider the movement of the end effector as being twisted instantaneously about the joint axes of an open-loop chain (Tsai, 1999). These instantaneous twists may be added linearly to give the resulting motion of the end effector. Consider, for example, the manipulator with six rotational joints. For this manipulator the differential kinematics may be written as

$$ {}^{B}\$_{E} = \sum_{i=1}^{6} {}^{B}\hat{\$}_{mi}\,\Psi_{mi} \tag{1} $$

where ${}^{B}\hat{\$}_{mi}$ is the manipulator i-th normalized screw described in the base frame (B-frame), Ψ_{vi} is the correspondent i-th magnitude and ${}^{B}\$_{E}$ is the screw that represents the end effector's motion in the B-frame.

With the objective to complete the manipulator's model with mobile base, the next section describes the differential kinematics of this base applied in underwater environments, represented by a vehicle.

2.2 Vehicle differential kinematics

We may consider the motion of a rigid body as being twisted instantaneously about several screw axes. These screws form a screw system whose order is defined by the number of linearly independent screws that span the system (Tsai, 1999). A vehicle has, in general, six degrees of freedom thus; its motion may be represented by a screw belonging to a six-th order screw system ($\$_V$). In other words, six independent screws may span the vehicle motion, i.e.,

$$ \$_{V} = \sum_{i=1}^{6} \hat{\$}_{vi}\,\Psi_{vi} \tag{2} $$

where $\hat{\$}_{vi}$ is the i-th normalized screw and Ψ_{vi} is the i-th amplitude. It should be outlined that, in case the set of normalized screws $\{\Psi_{vi}, i = 1{,}6\}$ is linearly independent, Eq.(2) represents the general vehicle motion as well as describes the motion of an open-loop chain with six links and six kinematic pairs (joints) each one with only one degree-of-freedom.

Using Eq.(2) to describe the vehicle's motion, we may choose an open-loop chain with three orthogonal prismatic (P) joints and a spherical (S) one (Fig.2) in order to represent the motion in a Cartesian reference frame. The first joint of this chain is prismatic and allows the motion between the first link and the base along the x-axis. It is named px and its motion is represented by the normalized screw $\hat{\$}_{px}$. The second and the third joints are prismatic, allow the motion between the second and the first, and between the third and the second links, along the y-axis (py) and the z-axis (pz), respectively, and are represented by the normalized screws $\hat{\$}_{py}$ and $\hat{\$}_{pz}$. The spherical joint is instantaneously substituted by three rotational joints in the x, y and z directions (rx, ry, rz) and its motions are represented by the normalized screws $\hat{\$}_{rx}$, $\hat{\$}_{ry}$ and $\hat{\$}_{rz}$. Due to its architecture this chain is often named PPPS.

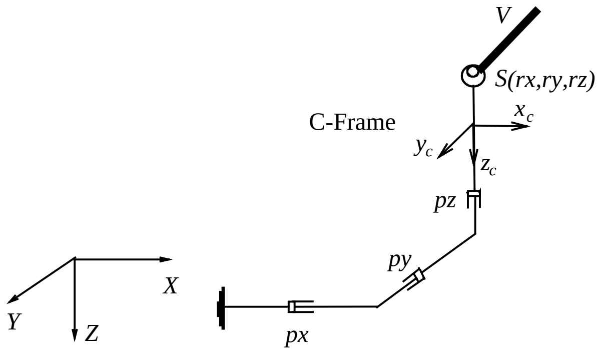


Figure 1 – Vehicle movement represented by an (PPPS) open kinematic chain.

The screws representing the PPPS chain independent movements are expressed in it's simplest form if we choose a reference frame attached to the link that connects joints pz and rx (the first rotational joint of the spherical joint), designated as C-frame in Fig. 2, to denote a suitable Cartesian reference frame. Using the normalized screw, we obtain (Campos, 2004):

$$
\begin{aligned}
&{}^{C}\hat{\$}_{rx} = \begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\end{bmatrix}^T ;\ {}^{C}\hat{\$}_{ry} = \begin{bmatrix}0 & 1 & 0 & 0 & 0 & 0\end{bmatrix}^T ;\\
&{}^{C}\hat{\$}_{rz} = \begin{bmatrix}0 & 0 & 1 & 0 & 0 & 0\end{bmatrix}^T ;\ {}^{C}\hat{\$}_{px} = \begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0\end{bmatrix}^T ;\\
&{}^{C}\hat{\$}_{py} = \begin{bmatrix}0 & 0 & 0 & 0 & 1 & 0\end{bmatrix}^T ;{}^{C}\hat{\$}_{pz} = \begin{bmatrix}0 & 0 & 0 & 0 & 0 & 1\end{bmatrix}^T .
\end{aligned} \tag{3}
$$

This normalized screws set is even linearly independent and represents the vehicle motion in the C-frame. In practice the vehicle motion is usually described in a frame located at the vehicle gravity center with the axes directed according to its principal inertia directions, here designed by vehicle frame (V-frame). To obtain the vehicle motion description in this V-frame, we may use the matrix of screws transformation between the C-frame and the V-frame, i.e., ${}^{V}\$_V = {}^{V}T_C\, {}^{C}\$_V$, where

$$
{}^{i}T_j = \begin{bmatrix} {}^{i}R_j & 0 \\ {}^{i}W_j\, {}^{i}R_j & {}^{i}R_j \end{bmatrix} \tag{4}
$$

$$
{}^{i}W_j = \begin{bmatrix} 0 & -p_z & p_y \\ p_z & 0 & -p_x \\ -p_y & p_x & 0 \end{bmatrix} \tag{5}
$$

$$
{}^{C}\$_V = \sum_{i=1}^{6} {}^{C}\hat{\$}_{vi}\,\Psi_{vi} \tag{6}
$$

where ${}^{i}R_j$ is the rotational matrix between the frames *i* and *j*, ${}^{i}W_j$ is the 3 x 3 skew-symmetric matrix representing the vector ${}^{i}p = [p_x, p_y, p_z]$ (expressed in the *i*th frame) and ${}^{C}\hat{\$}_{v1} = {}^{C}\hat{\$}_{px}$, ${}^{C}\hat{\$}_{v2} = {}^{C}\hat{\$}_{py}$, ${}^{C}\hat{\$}_{v3} = {}^{C}\hat{\$}_{pz}$, ${}^{C}\hat{\$}_{v4} = {}^{C}\hat{\$}_{rx}$, ${}^{C}\hat{\$}_{v5} = {}^{C}\hat{\$}_{ry}$, ${}^{C}\hat{\$}_{v6} = {}^{C}\hat{\$}_{rz}$, where the normalized screws $\hat{\$}_{px}$, $\hat{\$}_{py}$, $\hat{\$}_{pz}$, $\hat{\$}_{rx}$, $\hat{\$}_{ry}$ and $\hat{\$}_{rz}$, are given in Eq.(3).

The C-frame is chosen in order to make its origin coincident with the V-frame origin. So, the position vector of one origin with respect to the other is null as well as the matrix *W* given in Eq.(5). The matrix of screws transformation ${}^{V}T_C$ is obtained calculating the rotation matrix between the C and V frames (${}^{V}R_C$). To this end it should be observed that the C-frame was chosen parallel to the inertial frame (I-frame) and so, ${}^{C}R_V = {}^{I}R_V$, where ${}^{I}R_V$ is the rotation matrix that gives the orientation of the vehicle in the I-frame commonly measured in the RPY (Roll-Pitch-Yaw) angles (Antonelli, 2003). By transposing ${}^{C}R_{V}$ we obtain ${}^{V}R_{C}$ that is substituted in Eq.(4) to calculate ${}^{V}T_{C}$ and, using Eq.(6) results

$$ {}^{V}\$_{V} = {}^{V}T_{C} \sum_{i=1}^{6} {}^{C}\hat{\$}_{vi} \Psi_{vi} \tag{7} $$

2.3 The UVMS kinematics

In an UVSM the manipulator has a mobile base. So, the manipulator end effector motion with respect to an inertial frame is obtained by adding the manipulator end effector motion with respect to its base to the base motion (Fig.3), i.e.,

$$ {}^{I}\$_{E} = {}^{I}\$_{V} + {}^{I}T_{B} \, {}^{B}\$_{E} \tag{8} $$

where ${}^{I}T_{B}$ is the matrix of screws transformation between the base manipulator frame and the considered inertial frame.

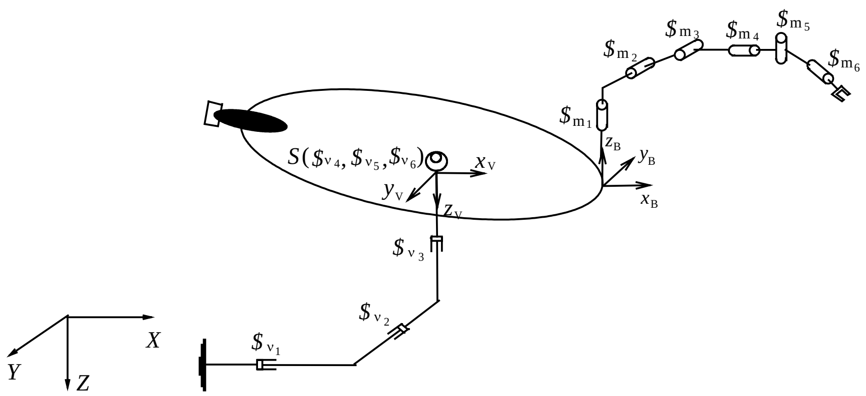


Figure 2 – An Underwater Vehicle-Manipulator System.

Consider an inertial frame instantaneously coincident the vehicle frame (V-frame) defined above. In this case Eq.(8) results

$$ {}^{V}\$_{E} = \sum_{i=1}^{6} {}^{V}\hat{\$}_{vi} \Psi_{vi} + \sum_{i=1}^{6} {}^{V}\hat{\$}_{mi} \Psi_{mi} \tag{9} $$

where ${}^{V}\hat{\$}_{vi} = {}^{V}T_{C} \, {}^{C}\hat{\$}_{vi}$, i=1,6, and ${}^{V}\hat{\$}_{mi} = {}^{V}T_{B} \, {}^{B}T_{R} \, {}^{R}\hat{\$}_{mi}$.

Eq.(9) may be rewritten as

$$ {}^{V}\$_{E} = J\Psi \tag{10} $$

where

$$ J = \left[{}^{V}\hat{\$}_{v} \quad {}^{V}\hat{\$}_{m} \right] \tag{11} $$

$$ \Psi = \left[\Psi_{v} \quad \Psi_{m} \right]^{T} \tag{12} $$

Eq. (10) expresses the UVMS kinematics as an open-loop chain constituted by the manipulator chain attached to the vehicle chain. It should be observed that the matrix J in Eq.(11) has six rows and twelve columns and outlines the system redundancy. In this paper, we present a new methodology to invert the UVMS kinematics (Eq.(10)) derived from the Davies Method and the virtual kinematic chain concept presented in the sequence.

3. Davies Method

Davies method is a systematic way to relate the joint velocities in closed kinematic chains. It is based on the so-called Kirchhoff-Davies circulation law (Davies, 1981, 2000). Davies solves the differential kinematics of closed chain mechanisms from the well-known Kirchhoff law for electrical circuits. This Kirchhoff-Davies circulation law states that ``The algebraic sum of relative velocities of kinematic pairs along any closed kinematic chain is zero'' (Davies,1981).

Using this law, the relationship between the velocities of a closed kinematic chain may be obtained in order to solve its differential kinematics, as is presented in (Guenther et al. 2005).

The kinematic pairs connecting link i to itself form a closed kinematic chain and, for this closed chain, the Kirchhoff-Davies circulation law, is given by

$$\sum_{i=1}^{n=1} \$ = \mathbf{0} \tag{13}$$

where **0** is a zero vector which dimension corresponds to the dimension of twists $.

According with normalized screw definition (Hunt, 2000) this equation may be rewritten as

$$\sum_{i=1}^{n=1} \hat{\$}_i \Psi_i = 0 \tag{14}$$

where $\hat{\$}_i$ represents the normalized screw of twist $\$_i$ and Ψ_i represents the velocity (angular in this case) magnitude of the twist i.

In general, the constraint equation of a mechanism with movements in a *d* order screw system is given by

$$N_{(d\times F_b)} \Psi_{(F_b \times 1)} = 0_{(d\times 1)} \tag{15}$$

where *N* is the network matrix containing the normalized screws which signs depend on the circuit orientation, Ψ is the magnitude vector and F_b is the gross degree of freedom, *i.e.* the sum of the degrees of freedom of all mechanism joints ($F_b = \sum f_i$), with f_i being the degree of freedom of the *ith* joint.

Closed kinematic chains, unlike open kinematic chains, contain passive kinematic pairs, in addition to active kinematic pairs. An external actuator, e.g. a servomotor, gives the velocity of an active kinematic pair. The velocities of the passive kinematic pairs are functions of the velocities of the active kinematic pairs due to the closure of the kinematic chain.

The use of the constraint equation, Eq. (15), allows calculating the passive joint velocities as functions of the active joint velocities. To achieve this solution, the constraint equation needs to be rearranged highlighting the actuated and the passive pair velocities. The magnitude vector Ψ is rearranged in *d* secondary or unknown magnitudes Ψ_s and F_N primary or known magnitudes Ψ_p, *i.e.* $\Psi = \left[\Psi_s \;\vdots\; \Psi_p\right]^T$. Rearranging the network matrix $\left[N\right]_{(d\times F_b)}$ coherently with the magnitude division, we get $\left[N\right]_{(d\times F_b)} = \left[\left[N_s\right]_{(d\times d)} \;\vdots\; \left[N_p\right]_{(d\times F_N)}\right]$, where the secondary network sub-matrix N_s corresponds to the secondary joints and the primary network sub-matrix N_p corresponds to the primary joints. This results in

$$\left[\left[N_s\right]_{(d\times d)} \vdots \left[N_p\right]_{(d\times F_N)}\right] \begin{bmatrix} \left[\Psi_s\right]_{(d\times 1)} \\ \cdots \\ \left[\Psi_p\right]_{(F_N\times 1)} \end{bmatrix} = \left[0\right]_{(d\times 1)} \tag{16}$$

This equation may be rewritten as $N_s \Psi_s = -N_p \Psi_p$ and the joint space kinematics solution is given by

$$\Psi_s = -N_s^{-1} N_p \Psi_p \tag{17}$$

This outlines that the Davies method constitutes a systematic way to express the joint rates of passive joints as funtions of the joint rates of the actuated joints in closed kinematic chains.

In the next section, we introduce the virtual kinematic chain concept, which allows closing open kinematic chains in order to apply the Davies method.

4. The virtual kinematic chain concept

The virtual kinematic chain, virtual chain for short, is essentially a tool to obtain information about the movement of a kinematic chain or to impose movements on a kinematic chain.

In this paper, we use the virtual kinematic chain concept introduced by Campos (2004), who defines a virtual chain as a kinematic chain composed by links (virtual links) and joints (virtual joints) satisfying the following three properties: a) the virtual chain is open; b) it has joints whose normalized screws are linearly independent; and c) it does not change the mobility of the real kinematic chain.

In the next section, the PPPS chain is used to close the UVMS open-loop chain in order to obtain information about the end effector motion in a Cartesian frame and to impose movements to the UVMS chain.

5. UVMS inverse kinematics

The objective of inverse kinematics is to find suitable vehicle and joint trajectories that correspond to a desired end effector trajectory.

The end effector motion description in a Cartesian frame may be obtained closing the UVMS chain by introducing a PPPS virtual chain between the base and the end effector as in Fig.4.

In this case, regarding the circuit orientation indicated in the figure, the closed-loop chain network matrix is given by

$$N = \left[\hat{\$}_{v1} \quad \cdots \quad \hat{\$}_{v6} \quad \hat{\$}_{m1} \quad \cdots \quad \hat{\$}_{m6} \quad -\hat{\$}_{rz} \quad \cdots \quad -\hat{\$}_{px}\right] \tag{18}$$

where the subindex V indicating that all the normalized screws are represented in the vehicle frame is omitted for simplicity. The negative terms in Eq.(18) are in accordance with the definitions of the screws that represent the virtual joint motions given in (Guenther et al. 2005). The corresponding magnitude vector is

$$\Psi = \left[\Psi_{v1} \quad \cdots \quad \Psi_{v6} \quad \Psi_{m1} \quad \cdots \quad \Psi_{m6} \quad \Psi_{rx} \quad \cdots \quad \Psi_{px}\right]^T \tag{19}$$

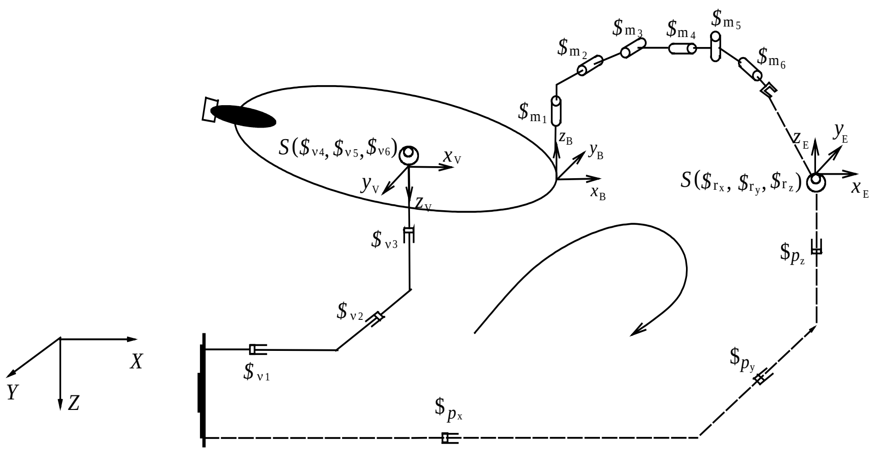


Figure 3 – The Underwater Vehicle-Manipulator System with the PPPS virtual chain.

To obtain the inverse kinematics, we select the velocity UVMS kinematic pairs magnitudes as the components of the primary magnitude vector Ψ_p and velocity magnitudes corresponding to the operational space (virtual kinematic pairs) as the components of the primary magnitude vector Ψ_s. In this case, the primary matrix results $N_p = \left[-\hat{\$}_{rz} \quad \cdots \quad -\hat{\$}_{px}\right]$ and the secondary matrix results $N_s = \left[\hat{\$}_{v1} \quad \cdots \quad \hat{\$}_{v6} \quad \hat{\$}_{m1} \quad \cdots \quad \hat{\$}_{m6}\right]$.

To calculate the secondary pairs magnitudes, we need to invert the secondary matrix. By observing the matrix N_s, it becomes clear that this matrix cannot be inverted because it has twelve columns and only six rows. This drawback was overcome in (Guenther et al. 2005) when six supplementary velocity magnitudes was specified considering energy savings and included it in the primary magnitude vector, in order to obtain a secondary matrix that could be inverted.

Therefore, the primary matrix results $N_p = \left[-\hat{\$}_{rz} \quad \cdots \quad -\hat{\$}_{px} \quad \hat{\$}_{v1} \quad \cdots \quad \hat{\$}_{v6}\right]$ and the secondary matrix is $N_s = \left[\hat{\$}_{m1} \quad \cdots \quad \hat{\$}_{m6}\right]$. This secondary matrix could be inverted even it is full rank, i.e. even the kinematic chain is not at a singularity.

This outlines a systematic way to obtain the inverse kinematics allowed by the approach proposed in this paper only by choosing adequately the primary and secondary velocity magnitudes. This makes the resulting motion coordination algorithm computationally efficient, as it may be verified in the simulations results presented in the section seven. The next section shows a brief description of some methods based in the Pseudoinverse that will be compared with our approach.

6. Some Redundancy Resolutions applied to the UVMS

The kinematically redundant feature of the UVMS are receiving growing attention as the demand increases for more flexible and versatile robotic systems capable of solving sophisticated tasks at the underwater environment. So far, a comparative analysis of different methods to solve the redundancy of these robotic systems is suitable. Thus, with the objective to develop this analysis, some redundancy resolution methods are briefly described as follows.

6. 1 Damped least square

The application of the damped least-squares method to redundant systems is related mainly to the singularity avoidance problem. It is well known that if a nonredundant system is at a configuration close to a singular point, the robot will yield undesirable high joint velocities. The singularity problem still exists to redundant manipulators according Gottlieb (1986). To overcome this problem, Wampler (1986) suggest the damped least squares formulation defined as

$$J^{\#} = J^T \left(J\, J^T + \lambda^2\, I\right)^{-1} \tag{20}$$

where $\lambda \in \Re$ is a damping factor and I is the identity matrix. Nakamura and Hanafusa (1986) extend this formulation using a weighted version of the damped least square

$$J^{\#} = W_1\, J^T \left(J\, W_1\, J^T + W_2\right)^{-1}, \tag{21}$$

where W_1 and W_2 are positive defined weighting matrices , with the damping factor λ incorporated in W_2. The Eq. (21) was named the *singularity robust inverse* and will be simulate in this work.

6. 2 Weighted generalized inverses

An alternative method for solving the redundancy of robotic systems is to use a weighted generalized inverse:

$$J_W^{*} = W^{-1} J^T \left(J\, W^{-1}\, J^T\right)^{-1}, \tag{22}$$

which yields the generalize expression of the Pseudoinverse solution. By means of the weighting matrix W $(6+n)\times(6+n)$, where n is the redundant degrees of freedom, various performance criterion functions can be satisfied. However, according Nenchev (1989), it is difficult to obtain weighting matrices for multicriterial performance functions. This work is limited to the manipulability criterion (see Sciavicco (1996) for more details). The advantage of this method in relation to the Pseudoinverse method is the possibility to coordinate the movement between the vehicle and the manipulator,

$$W^{-1} = \begin{bmatrix} (1-\beta) I_6 & 0_{6\times n} \\ 0_{6\times n} & \beta I_n \end{bmatrix}, \tag{23}$$

where I is the identity matrix and β is the manipulability factor belonging to the interval [0,1] such that $\beta = 0$ corresponds to sole vehicle movement and $\beta = 1$ to sole manipulator movement. Therefore, this method allows an efficient exploration of the degrees of freedom from the UVMS.

6. 3 Singular robust task-priority

According to Liégeois, (1977) the kinematics from a mechanism can be solved in terms of a minimization problem of the quadratic cost function. In the case of the UVMS this function is composed from the vector of joint velocities of the manipulator and vehicle $\varsigma^T \varsigma$ what give the follow general solution :

$$\varsigma = J_p^* \dot{x}_{p,d} + \left(J_s\left(I_N - J_p^* J_p\right)\right)^* \left(\dot{x}_{s,d} - J_s J_p^* \dot{x}_{p,d}\right) \tag{24}$$

where, N is the $(6+n)$ dimensional vehicle/joint velocities into the m-dimensional end-effector task velocities, and $\dot{x}_{p,d}$, $\dot{x}_{s,d}$, J_s, J_p, J_p^* are the primary velocity, the secondary velocity, the secondary Jacobian, the primary Jacobian and the Pseudoinverse of the primary Jacobian, respectively. The Eq. (24) represents the Task-Priority redundancy resolution.

However, this solution presents the weakness of algorithmic singularities unsolved. This happens due the possibility of J_s and J_p are full rank but the matrix $J_s\left(I_N - J_p^* J_p\right)$ loses rank. In the sense to overcome this weakness, a robust solution of the algorithmic singularities occurrence is based on the following mapping:

$$\varsigma = J_p^* \dot{x}_{p,d} + \left(I_N - J_p^* J_p\right) J_s^* \dot{x}_{s,d}. \tag{25}$$

This algorithm has a geometrical interpretation: the two tasks are separately inverted by the use of the pseudoinverse of the corresponding Jacobian; the joint velocities associated with the secondary task are further projected in the null space of the primary task J_p. However, simulation results in (Antonelli, 1998) showed that this resolution for an UVMS does not keep the system away from singularities. This drawback is just overcome in Antonelli, (2002), when a fuzzy technique approach was applied to avoid these singularities. The simulations results showed here are about the Singular Robust Task-Priority.

7. Simulation Results

This section presents the simulation results for a task performed using the conventional methods from the section 6 and the kinematic constraint approach presented in this paper.

The UVMS model is based on a real model of a UVMS showed in (Antonelli, G., 2003). The vehicle has a length of 5 meters and each link of the manipulator has a length of 2 meters. In order to simplify the attainment of initial results, the simulations will be restricted to tasks in the manipulator plan, who is mounted horizontally.

Let the initial configuration of the vehicle be: $x = 0$ m, $y = 0$ m, $\psi = 0$ rad and the manipulator joint angles be: $q = [1.47 \quad -1 \quad 0.3]$ rad, corresponding to the end-effector location: $x_E = 5.92$ m, $y_E = 4.29$ m, $\psi_E = 0.77$ rad.

The task consists in exploring the system's redundancy in order to reduce the energy consumption from the vehicle, as presented in the section five, while the end-effector task is developed. Therefore, it is desired to keep the vehicle position constant, if possible, while its orientation will change to minimize the effect of the ocean current (Antonelli, 1998), here considered equall to 0.52 rad. Simultaneously, the end-effector perform a repetitive linear trajectory with a complete length of 12 meters during a period of 50 seconds (see Fig. 4).

This simulation uses a integration step of 0.1 seconds with the Euler method and was developed with the SIMULINK - MATLAB software version 6.1.

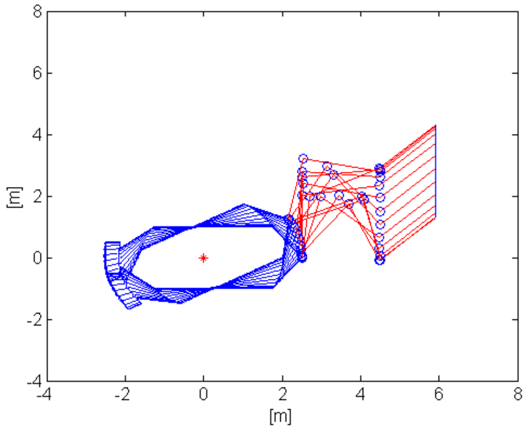
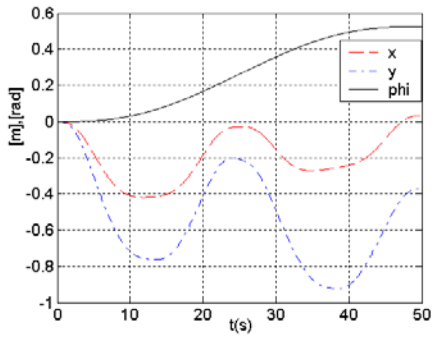
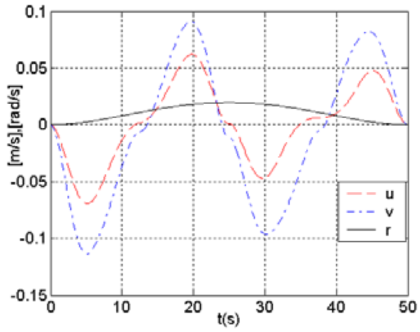


Figure 4 – Alignment with the ocean current and linear repetitive trajectory to the end-effector.

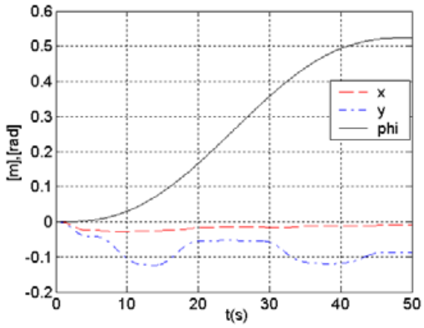


5 (a)

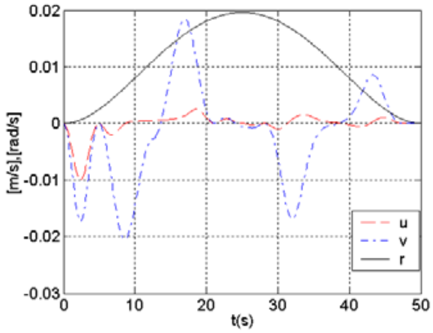


5 (b)

Figure 5– Vehicle positions (a) and velocities (b) with the Damped Least Square.

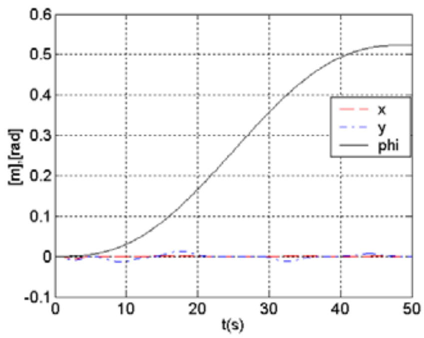


6 (a)

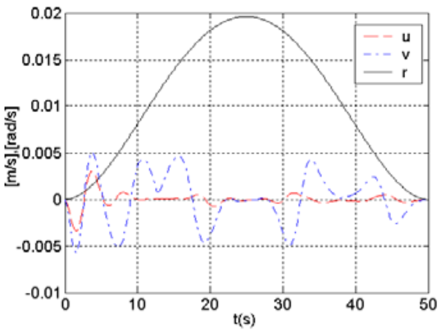


6 (b)

Figure 6 – Vehicle positions (a) and velocities (b) with the Weighted Pseudoinverse.

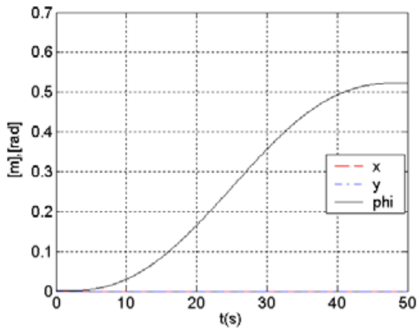


7 (a)

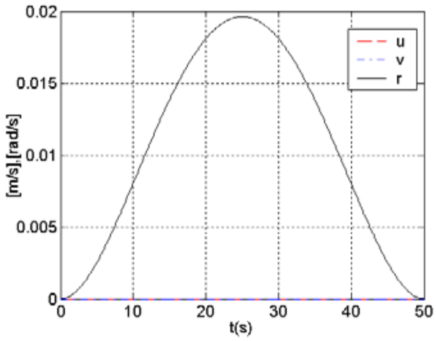


7 (b)

Figure 7 – Vehicle positions (a) and velocities (b) with the Singular Robust Task-Priority.



8 (a)



8 (b)

Figure 8 – Vehicle positions (a) and velocities (b) with the kinematic constraints.

The simulation results show unnecessary and undesirable movements of the vehicle using the Damped Least Square method in Fig. (5) when compared with the others. The Weighted Pseudoinverse shows in the Fig. (6) a notable reduction of these undesirable movements but it depends upon the chosen weighted matrix. Another slight increase in performance is observed in the Singular Robust Task-Priority method as shown in Fig. (7).

Finally, our approach kept the vehicle at the rest, ie without any undesirable movement, by imposing the corresponding kinematic constraint,as shown Fig (8).

These results shows that the algorithm movements resulting from the other conventional methods may be avoided by imposing kinematic constraints, like in this paper.

8. Conclusions

Conventional methods to solve the redundancy of robotic systems were applied to a UVMS and their performances are compared with our approach of kinematic constraints.

The use of movement screws representation results in a computationally efficient coordination algorithm, as it was verified in simulation results.

The approach presented in this paper allows to introduce kinematic constrains avoiding the use of methods that minimize the vehicle movements but don’t keep it at rest completely.

9. Acknowledgements

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