STATIC ANALYSIS OF LAZY-WAVE STEEL RISERS

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Abstract. *Catenary risers have been extensively used in the late years on the offshore oil industry. Its simplicity and relatively low cost are some of the main reasons. However, as submerged oil fields become deeper, lazy-wave configurations become an interesting solution. Unlike catenary, finding the best configuration of a lazy-wave is not an easy task. It requires several simulations due to the great number of parameters involved. Lazy-wave configurations include different segments, being an intermediate one provided with floaters. Not only the lengths of these segments need to be chosen but also the floaters' characteristics as well. To help solving this problem a software of parametric analysis was developed. The parameters taken into account are the lengths of the segments and, in the case of presence of floater, its diameter, length and spacing along the segment span. This work focuses on the static problem in two stages. Firstly, only the cable's own weight is considered. Platform offset and sea current loads are considered after. The selection of the feasible configurations is made based on some criteria such as top angle, tension at the top and at the touchdown point, curvature and cable's wave heights, among others. A case study using this homemade software is performed and shows how helpful is parametric analysis in obtaining the feasible lazy-wave configurations. Also, the results are highly explored to provide a better understanding about lazy-wave configurations and to choose the best among them.*

Keywords*: Parametric Analysis, Lazy-Wave, Steel Riser, Static Analysis*

1. Introduction

Catenary risers have been extensively used in the late years on the offshore oil industry. It is a simple well-known configuration, which can be described by means of one parameter: the cable's total length or the top angle. Its cost is relatively low once it requires little maintenance effort. However, as submerged fields become deeper, catenary configurations present high top tension levels once the suspended length is large. In order to reduce top tension, one possible solution to deep water is the use of floaters along the riser's span. This configuration is known as lazy-wave and although it is more expensive than simple catenary (installation costs are higher and the presence of floaters requires additional maintenance), it has become an interesting possible solution mainly due to top tension reduction. According to Tanaka (2002), top tension reduction may be up to 50% depending on the floaters' characteristics.

However, lazy-wave configurations have not been exhaustive studied and they still need to be better analyzed. Lazy-wave configurations include different segments, being an intermediate one provided with floaters. Not only the lengths of these segments need to be chosen but also the floaters' characteristics as well. The number of design parameters is then greatly increased when compared to simple catenary. To help solving this problem a software of parametric analysis was developed. The parameters taken into account by this software are the lengths of the segments and, in the case of presence of floater, its diameter, length and spacing along the segment span. Fig. (1) illustrates both catenary and lazy-wave configurations.

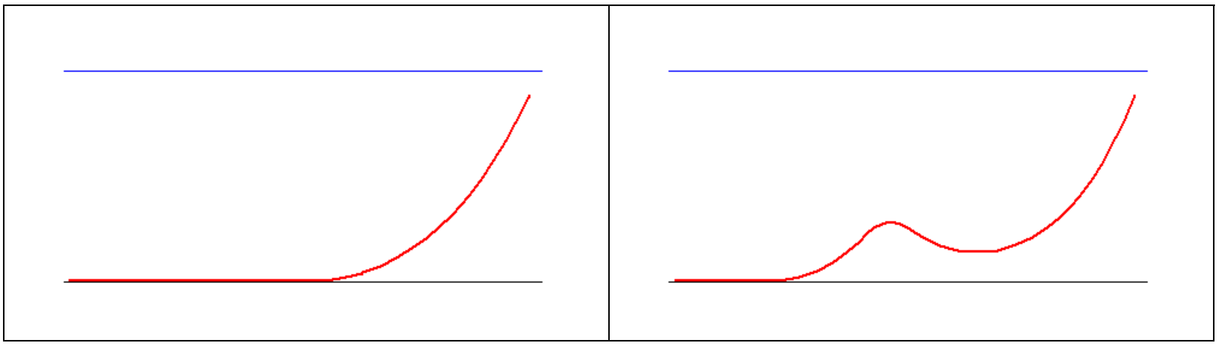

Figure 1. Simple catenary (left) and lazy-wave (right) configurations

This work focuses on the static problem of a lazy-wave configuration composed by three segments, being the second one provided with floaters. The physical properties of the three segments and of the floaters are kept unchanged; only the lengths of the segments act as parameters. The static problem is carried on in two stages. Firstly, only the cable's own weight is taken into account. Geometrically inconsistent configurations, i.e., configurations with total length too small (smaller than the distance from top to anchor) or total length too large (in such a way that the cable does not become totally tensioned) are discarded. The geometrically possible configurations are numerically solved and, based on the comparison of the results to some design criteria (e.g. top angle, top tension, curvature radius), the unfeasible configurations are discarded. On the other hand, the most promising configurations move to the second stage, when platform offset and sea current are considered.

In the following, the static model is presented in section 2. Section 3 shows all data from cables, floaters and other parameters; section 4 explains the analysis methodology and presents design criteria. Section 5 presents the results obtained from the parametric analysis for a case study based on real data and discusses it. The results obtained aim providing a better understanding on lazy-wave configurations and therefore some conclusions are exposed in section 6.

2. Static Model

In this section, the mathematical model for the three-dimensional static problem for a submerged ideal (i.e. infinite axial stiffness and zero bending stiffness) cable hanging from a floating platform from one end and resting on the seabed, free of friction, in the other end and submitted to a steady sea current is presented. Bending stiffness effect is incorporated *a posteriori* by using a boundary-layer technique. Fig. (2) illustrates the static problem, stated as follows. Given the cable's physical properties (diameter $D(s)$, weight per unit length $\gamma(s)$, bending stiffness $EI(s)$), the geometry of the problem (top position X_T, Y_T, Z_T, total length l_T), the environmental conditions (gravity acceleration g, water density ρ, water depth h, current velocity profile: speed along the depth $v_c(z)$ and direction along the depth $\psi_c(z)$) and the drag coefficient $c_d(s)$, one must obtain the static configuration of the cable.

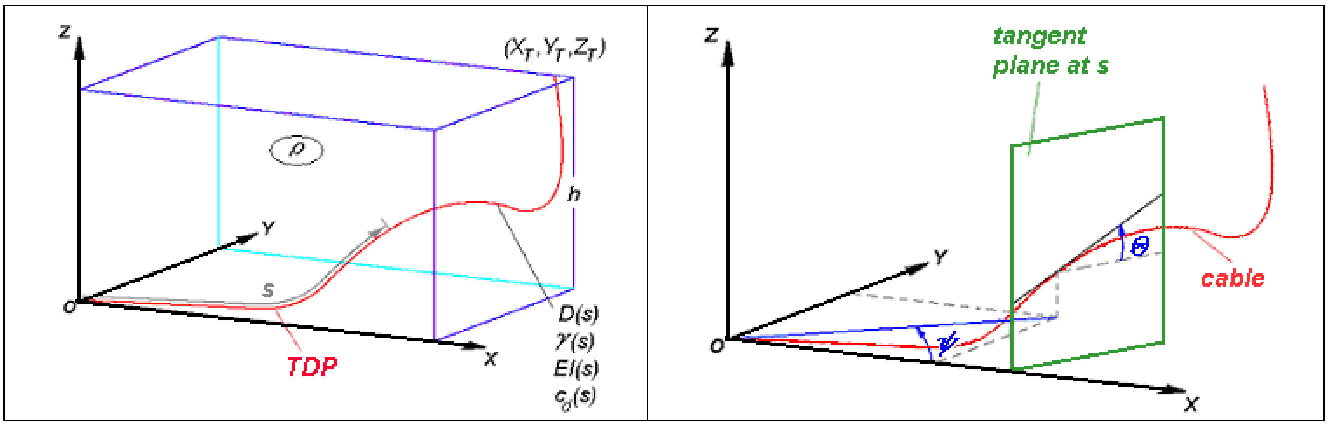

Figure 2. Sketch of the three-dimensional static problem

The three-dimensional static problem model of an ideal cable can be derived by means of the usual equations of geometric compatibility, force equilibrium, moment equilibrium and constitutive relations, assuming three hypotheses: (i) the cable has infinite axial stiffness, (ii) the cable is free for twist and bend, (iii) the seabed is plane, horizontal and perfectly rigid. The first hypothesis can be justified by saying that the cable always works in the elastic regime (small deformations) and the inclusion of axial stiffness reduces the effective tension acting on the cable, see Tanaka (2001). The second one can be justified by saying that twisting and bending are local effects and could be included using a boundary layer technique (the homemade software incorporates bending stiffness). The third hypothesis can be justified by saying that the soil rigidity is important only at the touchdown region and affects weakly the static configuration of the cable, although a perfectly rigid soil implies in a shear force discontinuity at the touchdown point (TDP), see Pesce et. al (1998). By doing so, one obtains quite easily the following system of ordinary differential equations (ODEs):

$$\begin{cases} \dfrac{dx}{ds} = \cos\theta\cos\psi \\ \dfrac{dy}{ds} = \cos\theta\sin\psi \\ \dfrac{dz}{ds} = \sin\theta \\ \dfrac{dF_x}{ds} = -c_x \\ \dfrac{dF_y}{ds} = -c_y \\ \dfrac{dF_z}{ds} = \gamma_{ef} - c_z \\ \dfrac{d\psi}{ds} = \dfrac{c_y\cos\psi - c_x\sin\psi}{F_x\cos\psi + F_y\sin\psi} \\ \dfrac{d\theta}{ds} = \dfrac{c_x\sin\theta - c_z\cos\theta\cos\psi - F_z\cos\theta\sin\psi}{F_x\cos\theta + F_z\sin\theta\cos\psi} \end{cases} \tag{1}$$

In Eq. (1), the curvilinear coordinate s, the Cartesian coordinates (x, y, z) and the orientation angles (θ, ψ) can be seen in Fig. (2); $\vec{\mathbf{F}} = F_x\vec{\mathbf{i}} + F_y\vec{\mathbf{j}} + F_z\vec{\mathbf{k}}$ is the effective force acting upon the cable; $\gamma_{ef}(s) = \gamma(s) - \overline{\rho}g\pi D^2(s)/4$ is the cable's effective weight per unit length and $\overline{\rho} = \rho$ if the cable is submerged or $\overline{\rho} = 0$ if the cable is emerged at s. Also, c_x, c_y, c_z are the projections in the x, y and z directions, respectively, of the current total force per unit length $\vec{\mathbf{c}}(s) = \rho c_d(s)D(s)v_c\vec{\mathbf{n}}_{\mathbf{c}}\left|v_c\vec{\mathbf{n}}_{\mathbf{c}}\right|/2$, where $\vec{\mathbf{n}}_{\mathbf{c}} = \vec{\boldsymbol{\alpha}}_{\mathbf{c}} - \left(\vec{\boldsymbol{\alpha}}_{\mathbf{c}}\cdot\vec{\mathbf{t}}\right)\vec{\mathbf{t}}$ (Note: $\vec{\mathbf{t}} = \cos\theta\cos\psi\vec{\mathbf{i}} + \cos\theta\sin\psi\vec{\mathbf{j}} + \sin\theta\vec{\mathbf{k}}$ is the tangent versor at s) and $\vec{\boldsymbol{\alpha}}_{\mathbf{c}} = \cos\psi_c\vec{\mathbf{i}} + \sin\psi_c\vec{\mathbf{j}}$. These expressions are given by Morison's formula, see Santos (2003).

If $s = \tilde{s}$ is the TDP position (unknown *a priori*), the boundary conditions are then expressed by

$$\begin{cases} x(\tilde{s}) = \tilde{s}\cos\psi_{TDP} \\ y(\tilde{s}) = \tilde{s}\sin\psi_{TDP} \\ z(\tilde{s}) = 0 \\ \theta(\tilde{s}) = 0 \\ \psi(\tilde{s}) = \psi_{TDP} \\ F_z(\tilde{s}) = 0 \\ x(l_T) = X_T \\ y(l_T) = Y_T \\ z(l_T) = Z_T \end{cases} \tag{2}$$

One may notice that there are 9 unknowns ($x(s)$, $y(s)$, $z(s)$, $\theta(s)$, $\psi(s)$, $F_x(s)$, $F_y(s)$, $F_z(s)$ and $\tilde{s}$) and 9 boundary conditions, as presented in Eq. (2). By neglecting bending stiffness, there are no shear forces acting on the cable; also, there is curvature discontinuity at the top, TDP and also points where there are changes in the submerged weight (i.e. where floaters are located), see Silveira & Martins (2004). The inclusion of bending stiffness effect is made by using a boundary-layer technique, in a similar way to the works of Martins (2000) and Balena (2003). According to Aranha et. al (1997), bending stiffness affects weakly the global statics of the problem. However, locally it becomes important, especially when analyzing curvature levels at the TDP, for instance.

The system of ODEs presented in Eq. (1) is numerically solved by a fourth-order Runge-Kutta method. The boundary conditions presented in Eq. (2) imposes a two-point boundary-value problem and can be solved by a shooting method, see Keller (1968).

3. Case study definition

The case study consists of a lazy-wave defined by a single cable (constant physical properties, see Tab. (1)) with floaters (constant physical properties, see Fig. (3) and Tab. (2)) placed along a certain length of the span. In practical terms, it defines a cable formed by three different segments with constant properties but variable lengths, see Fig. (3). These three variable lengths will be the only parameters considered in this work. Tab (3) shows the equivalent properties of each segment based on the values of Tab. (1) and Tab. (2).

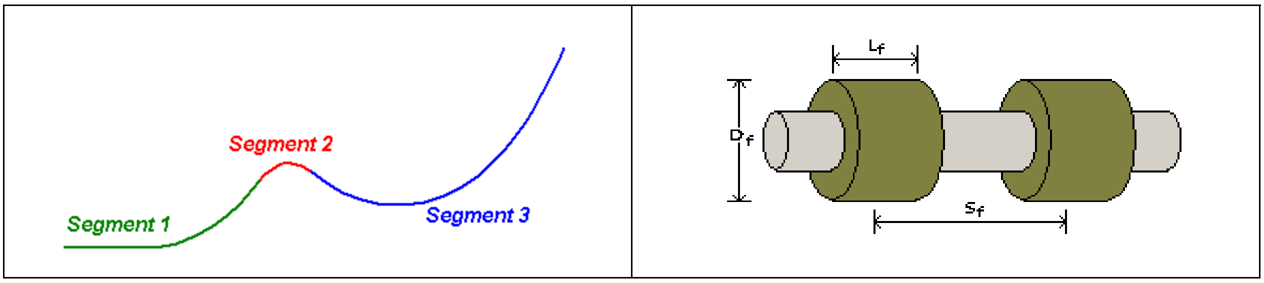

Figure 3. Segments and Floater description

Table 1. Cable's physical properties

Outer diameter	0.4572 m
Inner diameter	0.4064 m
Density	7850 kg/m^3
Contents density	917.0 kg/m^3
Young modulus	207800 MPa
Ultimate Stress	530.7 MPa

Table 2. Floater's physical properties

Diameter (Df)	1.370 m
Length (Lf)	1.300 m
Pitch (Sf)	2.000 m
Density	521.0 kg/m^3

Table 3. Segments' equivalent properties and parameter variation

	Segment 1	Segment 2	Segment 3
Hydrodynamic diameter (m)	0.4572	1.040	0.4572
Effective weight (kN/m)	2.161	-2.090	2.161
Bending stiffness (kNm^2)	167500	167500	167500
Length (m) – Range (Step)	0 – 3500 (100)	0 – 2500 (100)	0 – 2500 (100)
Number of possible lengths	36	26	26

The field marked as "Length (m) – Range (Step)" refers to the range of variation of the corresponding segment length and corresponding incremental length step. For example, *Segment 2* will take lengths of 0m, 100m, 200m, …, 2400m, 2500m. Note: *Segment 1* is connected to the anchor while *Segment 3* is connected to the platform, see Fig. (3).

Notice that for these ranges of segment lengths and for these chosen length steps, one has 36 x 26 x 26 = 24336 different combinations! Remember also that floater parameters are kept unchanged; by varying these parameters, the number of different combinations could become much bigger. This justifies the need of a software to perform parametric analysis. Other important values are: depth 1225m; top position coordinates (2340.0; 0.0; 1248.0) m; water density 1025 kg/m^3; gravity acceleration 9.800 m/s^2; drag coefficient 1.000.

4. Methodology

4.1 Static analysis: loads and flowchart

The static problem is carried on in two stages, see Fig. (4). Firstly, only the cable's own weight is taken into account. The geometrically inconsistent configurations are eliminated and the geometrically possible configurations are numerically solved. Based on the comparison of the results to some design criteria (discussed next), unfeasible configurations are discarded. Only the most promising configurations move to the second stage, when platform offset and sea current are considered. New design criteria may be applied in order to discard other unfeasible configurations.

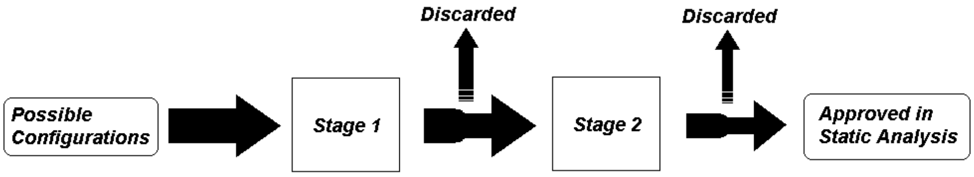

Figure 4. Static analysis flowchart

This procedure tries to eliminate as soon as possible all the unfeasible configurations from the design process. By eliminating a certain configuration in "Stage 1" of the static analysis, one avoids dispending more simulation time on a non-promising configuration. As discussed in three accompanying papers (Tanaka et. al (2005), Balena et. al (2005) and Takafuji et. al (2005)), the developed parametric analysis software also, in the presented order, runs dynamic analysis, calculates internal stresses on key points (e.g. TDP, top) and estimates fatigue life. Only configurations approved in static analysis are qualified to move to the dynamic analysis and so on.

For "Stage 2" of the static analysis, each approved configuration on "Stage 1" is submitted to as many different combinations of platform offset and sea current as desired. In this present work, eight combinations (each combination will be hereafter called a load case) were selected, see Fig. (5). These load cases try to cover all directions (north, north-east, east, …, north-west) and imposes a centenary sea current, i.e., an extreme load. Offsets are 61.25m (this value corresponds to 5% of water depth) in the same direction of the corresponding sea current. The configuration is approved in static analysis only if it fulfills all design criteria for all the load cases.

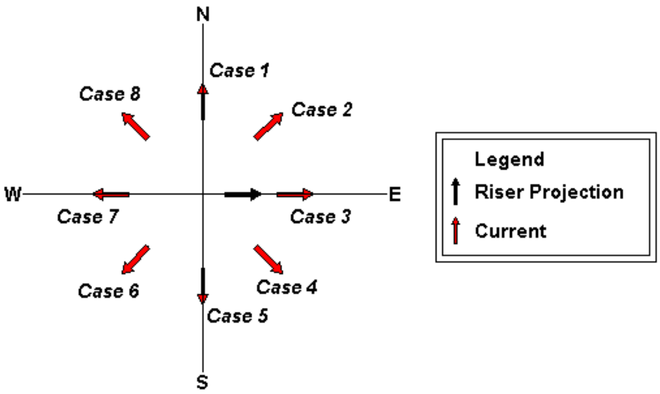

Figure 5. Load cases for "Stage 2" of static analysis

4.2 Design criteria

The design criteria are used to discard some bad, not-promising configurations. In the present work, three criteria will be used: top tension, curvature radius and TDP position.

Top tension. A simple catenary configuration for a riser with data from Tab. (1) shows top tension variation in the range 2700 – 4600 kN depending on the top angle. Suppose there is a constructive restriction in the floating platform, which restricts top angle to 20º. In this case, top tension is around 4000 kN. The main goal in lazy-wave configurations is top tension reduction. Therefore, configurations with top tension above 3000 kN in "Stage 1" will be discarded.

Curvature radius (R). Based on the cable's ultimate strength and based on the approximation for the bending stress $\sigma \approx (1/R)(D/2)E$, the minimum curvature radius is around 80m. According to Pesce & Martins (2004), typical dynamic conditions impose curvature levels of up to 1.4 times the static curvature. Therefore, configurations with curvature radius lower than 150m in "Stage 1" or "Stage 2" will be discarded in order to keep allowable stress levels even after the imposition of dynamic loads in future design stages, see Tanaka (2005).

TDP position. The imposition of platform offset of approximately 60m in "Stage 2" requires a minimum portion of the cable laying on the seabed in order to deny forces at the anchor. In this way, a criterion of minimum TDP position of 100m will be applied on "Stage 1".

5. Results and discussion

From the 24336 initial configurations, only 2870 were geometrically possible with total length varying in the range 2800 – 5800m. However, the 931 configurations effectively approved in "Stage 1" have total length varying in a narrower range: 2800 – 4800m. Some results for the approved configurations are illustrated in Fig (6).

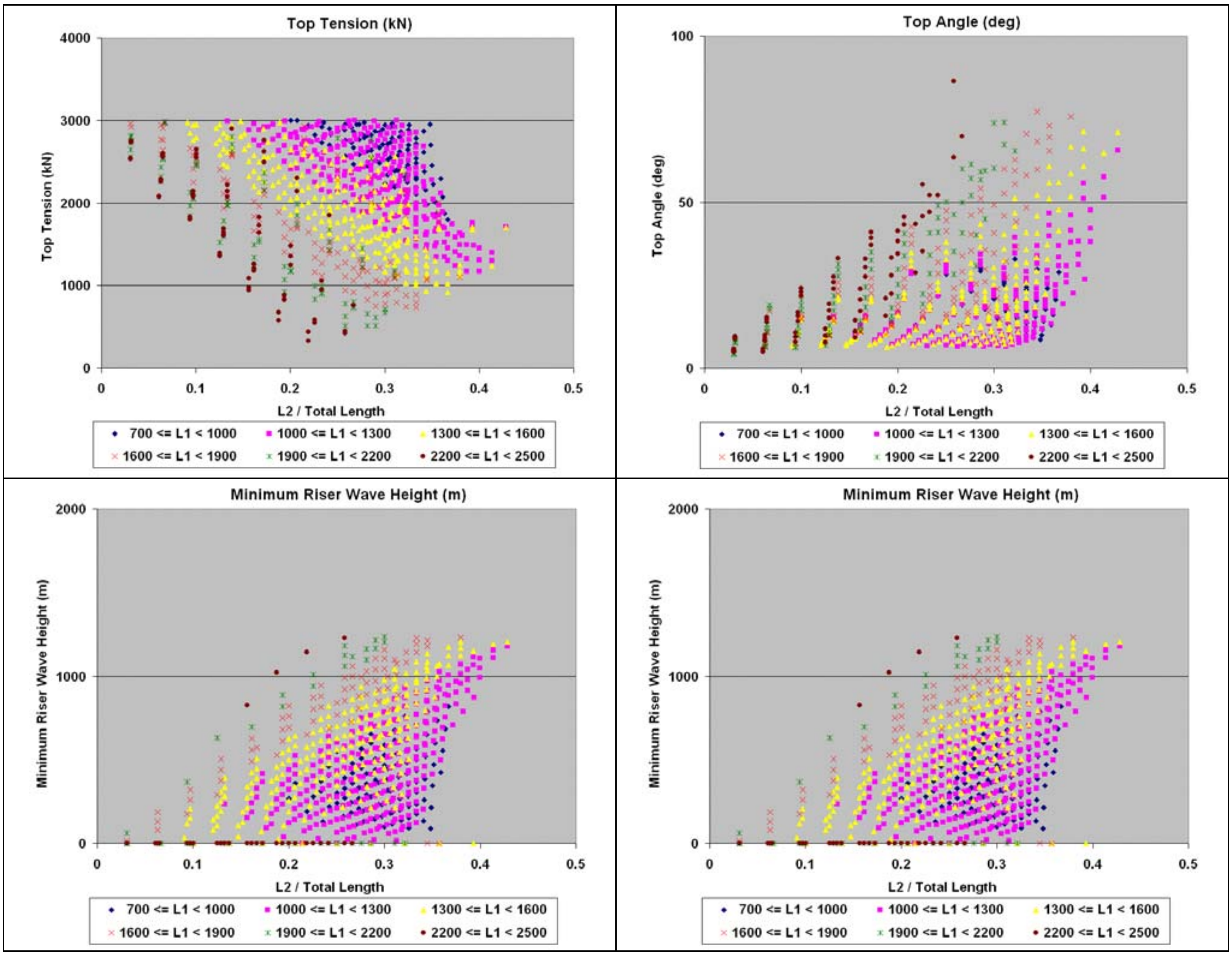

Figure 6. Results for the case study, "Stage 1".

The results show patterns for each of the variables of interest. Some additional simulations were made with constant total length and it is possible then to see the influence of each segment length. For a fixed length (graphs in the columns of Fig. (7)), the length of segment 1 (hereby called L1) plays a key role by defining the starting point of the floaters.

If L1 is small, the minimum riser wave height may get next to zero, i.e., the geometry of the lazy-wave is possible with the riser wave next to the seabed. Also, for a small L1, one may see an abrupt change in the top tension behavior. The reason is that the geometry goes directly from a well defined lazy-wave to a simple catenary. Otherwise, by decreasing L2 (and increasing L3), one would have a portion of the floater touching the seabed.

If L1 is large, the minimum riser wave height is considerably greater in comparison to the small L1 behavior, i.e., the riser wave is not next to the seabed. Also, for a large L1, the riser wave is not always well defined. Many times one has a deformed catenary, without any wave. This reflects on the abrupt changes in the minimum curvature levels but does not in the top tension levels, where the larger L2 range allows a smooth curve (riser waves generally introduces greater curvature levels, catenaries generally presents high curvature in the TDP). Fig. (8) illustrates this discussion.

By changing total length, one observes changes in the ranges of the variables, but the qualitative response is similar.

Finally, 614 from the 931 configurations approved in "Stage 1" were approved also in "Stage 2". This fact shows that the selection of the configurations, i.e., the criteria applied on the 2870 possible configurations ("Stage 1") were adequate. This case study took approximately 2 hours of simulation on a Pentium IV 3.2GHz with 1GB RAM.

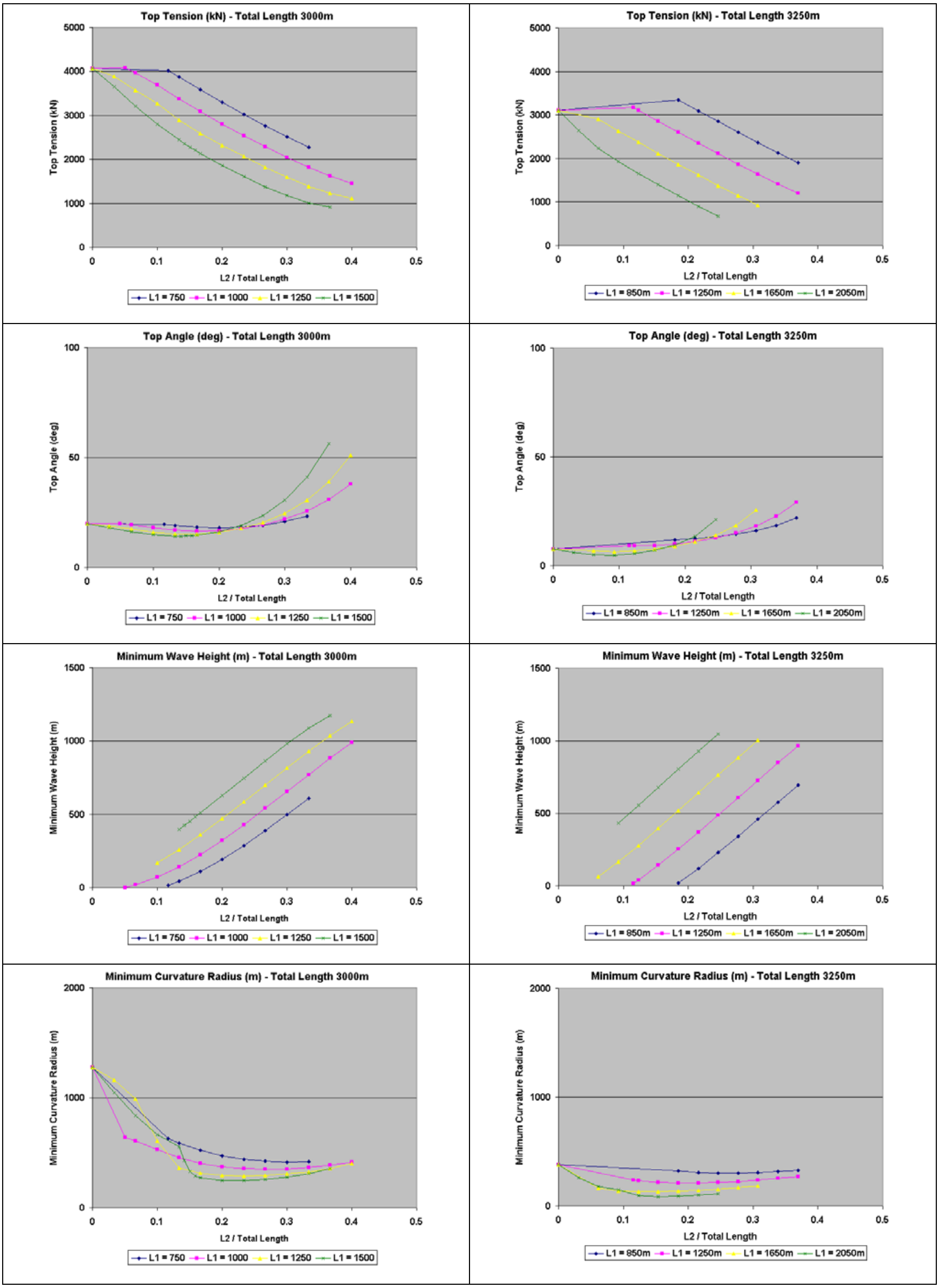

Figure 7. Additional results for fixed total length

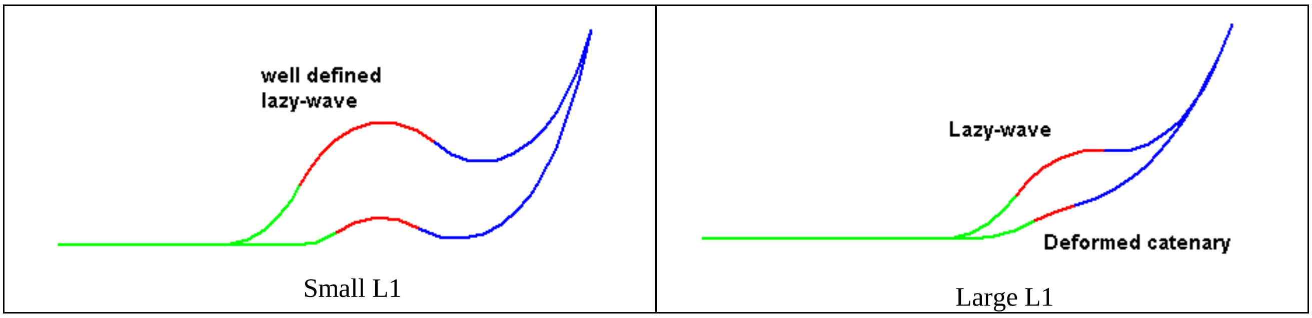

Figure 8. Configurations for small and large L1

6. Conclusions

The paper presented a set of graphs with a non-exhaustive but important study on lazy-wave configurations, especially on top tension, top angle, curvature and geometry. It was shown how the choice of the segments' lengths changes the response of a lazy-wave configuration.

The paper also showed how effective a parametric analysis might be in designing a riser once the number of possible configurations dropped from 24336 to 614 with little computational effort.

7. Acknowledgements

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