AN ERROR SYNTHESIZATION REDUCED MODEL FOR A CMM

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Abstract: *This work presents a mathematical model, called Error Synthesization Reduced Model (ESRM). ESRM consists of a set of algebraic expressions that synthesizes the volumetric error components of a coordinate measuring machine. The expressions for the error components were obtained by means of the combination of the geometric errors at each preferential moving direction of CMM. ESRM, which is characterized by positioning errors Ex, Ey and Ez of probe tip, was developed from the geometric analysis of the machine and error components expressions were determined by errors groups on the analyzed preferential direction. The expressions of the reduced model, geometrically derived, are compacted due to the error grouping that happens in one direction. The use of an indirect calibration method defines the way for grouping the errors. Measurement of positioning errors was accomplished using a bar of holes. The mechanical square was used to measure grouped errors. The developed model was tested and validated by means of the measurement of standard artifacts, such as gauge blocks and ring gauges and leads to conclude that proposed model enables the reduction on the number of calibration tests, decreases the time spent on these procedures and simplifies the propagation of measuring uncertainty analysis.*

Keywords: *Coordinate measuring machines, error grouping, volumetric error*

1. Introduction

Modeling of MM3Cs has been grown in importance because mathematical models allow the determination of errors largeness and behavior. Thus, errors can be compensated (Zhang et al, 1985). Many researchers have been studied and developed models to represent MM3Cs errors and many techniques have been used for this purpose.

Mathematical models used in metrology, for three coordinate measuring, have the function of combine, in an appropriated and weighed manner, individual errors of each preferential direction of the machine moving, which results in the called volumetric error. Such models determine the difference between real way and ideal way that is described by probe tip (Hocken, 1977).

The construction of a mathematical model for errors can be done through these techniques: Geometric Structural Analysis, Vectorial Analysis of Measurement Ways, Matrical Analysis through Homogeneous Transformations and Statistical Analysis (Denavit and Hartenberg, 1955; Di Giacomo et al, 1997; Di Giacomo et al, 1986; Guye, 1978; Paul, 1981).

This work presents a mathematical tool for MM3C modeling, the Error Synthesization Reduced Model (ESRM), which was first developed by Zirondi (2002). This choice was made due to the possibility of observing the influence of individual errors on the positioning of probe tip in distinct positions for machine work volume; there is also the possibility of considering the equipment probing system in calibration process. Besides, after previous comparison with other known models, ESRM seems to have more reduced synthesization equations for Ex, Ey and Ez. It needs less little calibration time, which reduces this activity cost, allows the diagnostic of errors source and guarantees tracking calculated errors.

ESRM allows calculating positioning errors in probe tip, in X, Y and Z directions, from any coordinated point starting from the errors, eventually called undesirable displacements. This sort of displacements was divided in two groups: those which have the same direction of the movement and those which occur perpendicularly to movement direction.

Next sections show: development of ESRM, calibration procedure for coordinate measuring machine starting from the model and, obtained results.

2. Mathematical modeling

A mathematical model, called Error Synthesization Reduced Model, which combines the influence of geometrical errors in each preferential directions of the machine, was used to determine the positioning error of probe tip, in any direction X, Y or Z, of any coordinated point. ESRM was developed from a geometrical analysis of the machine and the expressions of the components in volumetric error were determined by the sum of positioning errors and corrections portions. Such expressions are extremely simple and require a lesser number of calibrations, which reduces practical tests time. One interesting feature of the model is the simplicity of the analysis in uncertainty propagation of coordinated points. The model, arranged with an appropriated calibration method, allows the establishment of a tracking sequence for measures made in the analyzed MM3C.

2.1. Error Synthesization Reduced Model (ESRM)

ESRM allows the determination of positioning errors in any direction X, Y or Z, of any coordinated point, starting from measures in bars of holes and with a mechanical square, in fifteen generators.

The first step in the modeling of a MM3C is defining the position where coordinates reference system must be placed. In this work, the system was positioned on granite straightening, as close as possible of Y axle guide.

After defining the reference system positioning, a geometrical analysis of the machine structure was made in order to define the contribution of each geometric error in their preferential directions.

Table 1 shows obtained results in geometrical analysis of MM3C. Contributions of second order were despised because they are considered irrelevant.

Table 1: Errors in X, Y and Z directions

Volumetric Error Component	Geometric Error	Movement	Offset
X	Position	X	
	Straightness in Y at direction X	Y	
	Straightness in Z at direction X	Z	
	Pitch X	X	Z
	Yaw Y	Y	Y (fixo)
	Yaw Z	Z	Z
	Roll Y	Y	Z
	XY Orthogonallity		Y (fixo)
	XZ Orthogonallity		Z
Y	Position	Y	
	Straightness in X at direction Y	X	
	Straightness in Z at direction Y	Z	
	Yaw Y	Y	X
	Pitch Y	Y	Z
	Pitch Z	Z	Z
	Roll X	X	Z
	XY Orthogonallity		X
	YZ Orthogonallity		Z
Z	Position	Z	
	Straightness in X at direction Z	X	
	Straightness in Y at direction Z	Y	
	Pitch Y	Y	Y (fixo)
	Roll X	X	Y (fixo)
	Roll Y	Y	X

With this information, obtained from geometrical analysis of the machine, synthesization reduced equations for Ex, Ey and Ez were formulated.

2.2. Expression for Y component in volumetric error

Let us consider two generators in XY plan, denoted by G_1 and G_i, which are, both, parallel to Y axle and have distinct X coordinates. G_1 was placed as close as possible of Y scale in order to minimize the effects of Abbé offsets. Thus, one can obtain positioning error of Y axle, properly said. Figure 1 shows, schematically, generators G_1 and G_i.

In Fig. 1 we can see that G_1 and G_i have, respectively, two points P_1 and P_i, which coordinates differ only in X value.

The values of measured errors in P_1 and P_i are different. These differences are due to angular movement which contributions depend on two factors: X position (X axle offsets) and, influences by errors with change position from X_0 to X_i. According to geometrical analysis, Tab. 1, errors which cause differences on probe tip positioning error, in Y direction of XY plan, are: Yaw of Y axle, XY Orthogonallity, Roll of X axle and X Straightness in Y direction. Thus, if positioning error of P_1 in Y direction is supposed known, one can determine error in P_i, through Equation (1):

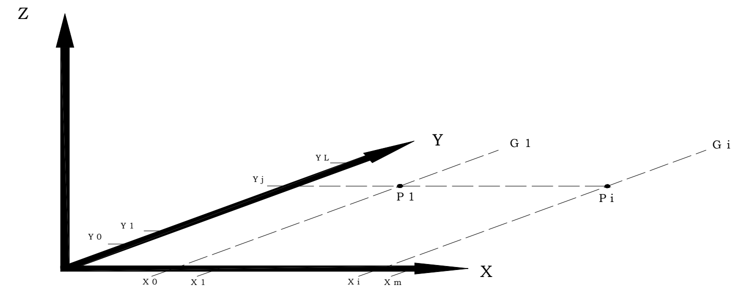


Figure 1: Representation of generators G_1 and G_i in XY plan

$$E_Y\left(X_i, Y_j, Z_0\right) = E_Y\left(X_0, Y_j, Z_0\right) + Straightness\,Y(X) \cdot Yaw(Y) \cdot [X\ arm] + \\ + Roll\,(X) \cdot [Z\ arm] + Orthogonallity\ XY \cdot [X\ arm] \tag{1}$$

where:

- $E_y\left(X_0, Y_j, Z_0\right)$ is the positioning error of Y axle, in some arbitrary point P_1, belonging to G_1;
- $E_y\left(X_i, Y_j, Z_0\right)$ is the positioning error of Y axle, in some arbitrary point P_2, placed in any generator G_i, parallel to G_1 in XY plan;
- X and Z offsets are measures of distance in respective X and Z directions, between probe and Y axle.

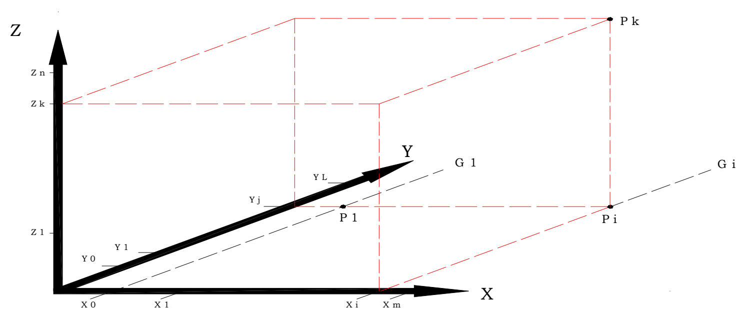


Figure 2: Representation of points belonging to the work volume of MM3C

Proceeding to the analysis in a similar way, one can evaluate E_Y in any position of the work volume of MM3C. Thus, let us consider some point P_k belonging to a generator which is contained in YZ plan and is parallel to Y.

Observing Fig. 2, one can notice that the positioning difference of P_i and P_k in Y direction is due to angular movements which depends, both, on Z offsets and errors caused by movement on Z direction. According to Tab. 1, errors which cause differences in relative positioning of P_2 and P_3 are: Straightness of Z in Y direction, Pitch of Y axle, Pitch of Z axle, Roll of X axle and XY Orthogonallity. As angular Roll of X axle error was already considered in Eq. (1), the value of E_Y calculated on point P_3 can be written in function of the value of E_Y on point P_2, through Equation (2). In this expression, $E_Y\left(X_i, Y_j Z_k\right)$ is the value of Y component of volumetric error, in some point $\left(X_i, Y_j Z_k\right)$ belonging to work volume of the MM3C.

$$E_Y\left(X_i, Y_j, Z_k\right) = E_Y\left(X_i, Y_j, Z_0\right) + Straightness\,Y(Z) + Pitch(Y) \cdot [Z\ arm] + \\ + Pitch(Z) \cdot [Z\ arm] + Orthogonallity\,YZ \cdot [Z\ arm] \tag{2}$$

Replacing Eq. (1) in Eq. (2), one can obtain the synthesization equation of E_y, given by the Eq. (3), which is valid for any point belonging to work volume of the MM3C. In this equation, the term Orthogonallity was simplified by Ort.

$$E_Y\left(X_i, Y_j, Z_k\right) = E_Y\left(X_0, Y_j, Z_0\right) + Straightness\,Y(X) + Straightness\,Y(Z) + \\ + Yaw(Y) \cdot [X\ arm] + Pitch(Y) \cdot [Z\ arm] + Pitch(Z) \cdot [Z\ arm] + \\ + Roll\ X \cdot [Z\ arm] + Ort\ XY \cdot [X\ arm] + Ort\,YZ \cdot [Z\ arm] \tag{3}$$

In order to determine E_Y value for any position in work volume of MM3C it is necessary to quantify the contribution of all geometric errors which appear in expression (3).

According to Eq. (3), the portion which corresponds to the contribution of error Yaw from Y axle, in Y direction of volumetric error, is different and proportional to X coordinates on these points. Figure 3 illustrates this fact.

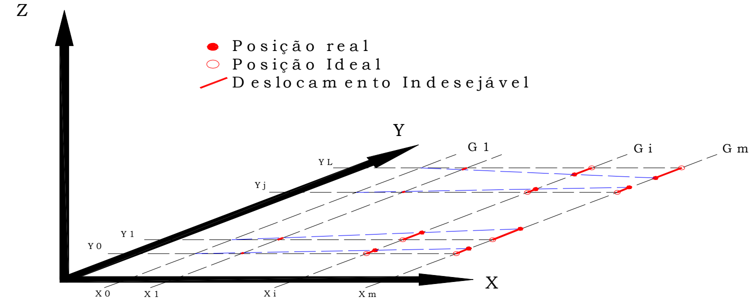


Figure 3: Angular Yaw error of Y axle, due to an offset existence on X direction

The portion of E_y corresponding to Yaw error of Y axle, in any point $\left(X_i, Y_j Z_k\right)$ in work volume of MM3C, can be obtained from measures made in two distinct generators in a given XY plan. As the offsets in Z direction don't interfere in the result, XY plan, which contains the generators, may be in any position in Z. Thus, Yaw error of Y axle in any spatial position can be calculated through Eq. (4), where:

- $\partial\left(X_1, Y_j\right)$ is the positioning of point j, measured in G_1 (in this situation, the offset in X axle is minimum);
- $\partial\left(X_M, Y_j\right)$ is the positioning of point j, measured in G_2 (parallel to G_1, in XY plan), placed in position X_M of X axle (offset in X is maximum);
- $d\_YawY\left(X_i, Y_j\right)$ is the Yaw error of Y axle in any point $\left(X_i, Y_j Z_k\right)$.

$$d\_YawY\left(X_i, Y_j\right) = \frac{\partial\left(X_M, Y_j\right) - \partial\left(X_1, Y_j\right)}{X_M - X_1} \cdot \left(X_i - X_1\right) \tag{4}$$

A similar observation can be done to the Pitch of Y axle, which contribution on Y direction increases proportionally to Z offset.

In Fig. 4, one can visualize the angular Pitch error of Y axle caused by the existence of an offset on Z direction. Equation (5) presents the calculation for Pitch error of Y axle in any spatial position, where:

- $\partial\left(Y_j, Z_0\right)$ is the positioning of point j, measured in G_1 (offset in Z axle is minimum);
- $\partial\left(Y_j, Z_N\right)$ is the positioning of point j, measured in G_3, parallel to G_1 and moved away from that in Z direction (offset in Z is maximum);
- $d\_PitchY\left(Y_j, Z_k\right)$ is the Pitch error of Y axle in point $\left(X_i, Y_j Z_k\right)$.

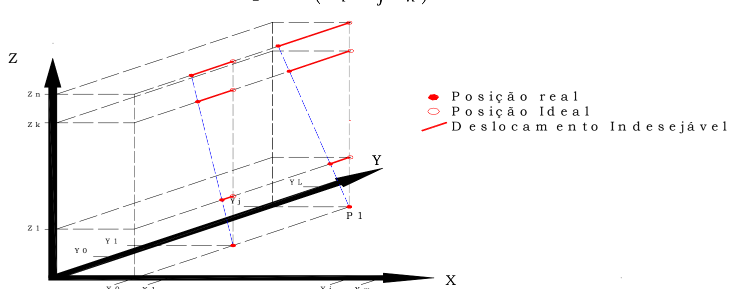


Figure 4: Angular Pitch error of Y axle amplified by the existence of an offset in Z direction

$$d\_PitchY\left(Y_j, Z_k\right) = \frac{\partial\left(Y_j, Z_N\right) - \partial\left(Y_j, Z_0\right)}{Z_N - Z_0} \cdot \left(Z_k - Z_0\right) \tag{5}$$

Analyzing Eq. (3), one can also notice that there are errors that change the positioning in Y direction and depend on X coordinate or on offsets in this direction: X Straightness in Y direction, Roll of X axle and XY Orthogonallity. Let

$d\_XY(X_i)$ be the portion of E_Y that groups the influences of these errors in position X_i. However, it must be observed that the value of $d\_XY(X_i)$ don't remain unchanged with the variation on Z coordinate, due to the influence of Roll error in X axle. In other words, if the measurement of $d\_XY(X_i)$ will be made for a minimum Z offset and, later, for a maximum Z offset, differences between found displacements, for the same X_i, occur only due to Roll error of X axle. Thus, displacement caused by Roll in X axle, XY Orthogonallity and X Straightness in Y direction can be determined by the sum of two portions: the obtained value for a measure found for a minimum Z offset (when the influence of Roll in X is minimum) and the value of the difference between the two measures. As this difference is linear for a given X coordinate, and it is proportional to the offset in Z direction, the resulting error can be found by Eq. (6), where:

- $d\_XY(X_i, Z_k)$ is the undesirable displacement in Y direction caused by Roll error on X axle, XY Orthogonallity and X Straightness in Y direction;
- $dXY(X_i, Z_N)$ is the undesirable displacement in Y direction, measured in point i of G_4, which is in XY plan and placed near X scale (in this case, offset in Z is minimum);
- $dXY(X_i, Z_0)$ is the undesirable displacement in Y direction, measured in point i of G_5, parallel to G_4, moved away from that in Z direction and far away from X axle scale (offset in Z is maximum).

$$d\_XY(X_i, Z_k) = dXY(X_i, Z_N) + \frac{dXY(X_i, Z_0) - dXY(X_i, Z_N)}{Z_0 - Z_N} \cdot (Z_k - Z_N) \tag{6}$$

Besides these mentioned errors, there are also, in expression (3), other errors which depend on Z coordinate or Z offsets and change positioning on Y direction. Such errors are: Z Straightness in Y direction, Pitch of Z axle and YZ Orthogonallity. Let us consider that $d\_YZ(Z_k)$ is the portion of E_Y which groups the influences of these errors for any Z coordinate. This error is measured in a generator G_6 in an YZ plan.

Replacing the errors in Eq. (3), an expression of ESRM to synthesize E_Y is found and results in Eq. (7).

$$\begin{aligned} E_Y(X_i, Y_j, Z_k) = E_Y(X_0, Y_j, Z_0) + d\_YawY(X_i, Y_j) + d\_PitchY(Y_j, Z_k) + \\ + d\_YZ(Z_k) + d\_XY(X_i, Z_k) \end{aligned} \tag{7}$$

Equations for X and Z components of volumetric error were developed in the same way and resulted in Eq. (8) and (9).

$$E_X(X_i, Y_j, Z_k) = E_X(X_i, Y_0, Z_0) + d\_PitchX(X_i, Z_k) + d\_YX(Y_j, Z_k) + d\_XZ(Z_k) \tag{8}$$

$$E_Z(X_i, Y_j, Z_k) = E_Z(X_0, Y_0, Z_k) + d\_ZY(X_{i,} Y_j) + d\_ZX(X_i) \tag{9}$$

3. MM3C calibration

According to proposed model and after machine modeling, calibration strategy could be traced, that is, all geometric errors present in error synthesization equations must be measured.

Bars of holes and a mechanical square were used for calibrating the MM3C. Such artifacts were used due to their low costs, in relation to other calibration systems, and their reasonable easy application (Piratelli Filho, 1997, Poole, 1983).

Measures of the bar of holes were made with probe tip positioned on Z offset, as unique tip qualification, with diameter of 4 mm. This diameter was chosen in order to minimize the influence of superficial roughness on internal wall of the holes in measuring results.

The use of a mechanical artifact for errors calibrating is an interesting proposal since it allows considering, in calibration process, an obligatory using part: the equipment probing system (Zhang, 2000).

The use of a bar of holes as an artifact for picking data to the ESRM showed itself also favorable because it is a simple object, quite easy to be manufactured and handled.

Before calibrating MM3C, the bars were pre-calibrated in a Universal Measuring Machine, SIP, which has a resolution twenty times smaller than the machine used for the MM3C, that is, 0,1µm. All distances between the centre of the first hole and the centre of all the other holes were measured.

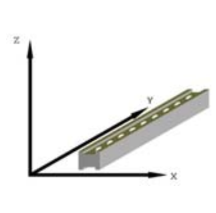
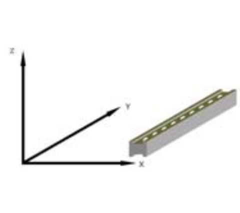
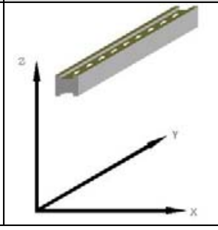
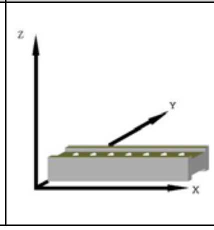
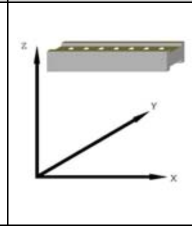
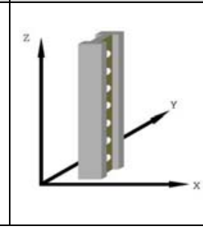
Nine coordinated points were taken, in each hole, with the measurement probe; the measurement occurred always in the first hole and each one of the other holes and so, the distance between their centers were calculated.

Scale errors were calculated using data obtained from machine calibration with bar of holes and the equation Ei = Value found with ProgCalibra - SIP measure.

The bar was positioned parallel to the axle, the closest possible to the axle that had to be evaluated, that is, in 1, 4 and 6 positions and for X, Y and Z, respectively.

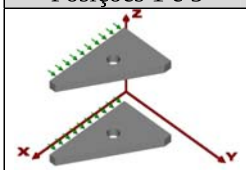
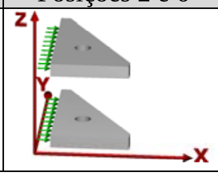
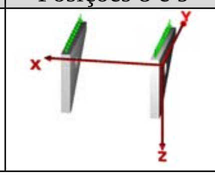
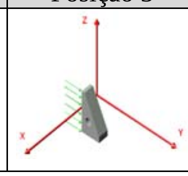
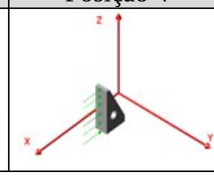
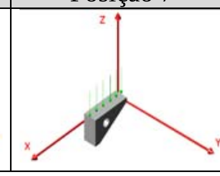
Other errors which occur in the same direction of the movement were also calculated using data obtained from machine calibration with bar of holes, such as Yaw(Y), Pitch(Y) and Pitch(X).

Table 2: Bar positions on measuring machine

Posições					
1	2	3	4	5	6
					

Measures of errors with mechanical square were made with a granite square manufactured by Mitutoyo, which orthogonallity error is 2,5 arcsec and with a digital linear probe manufactured by Tesa, G21 model, which total displacement is 4,3 mm. The square was positioned parallel to one of the axle while the probe touches the other face. The electronic probe was adapted in the Z offset of the MM3C.

Table 3: Placement positions of the square in machine

Posições 1 e 5	Posições 2 e 6	Posições 8 e 9	Posição 3	Posição 4	Posição 7
					

Measurement was made in the following way: linear probe was positioned on the square and zeroed in the first position then the sweeping of the square is done and, in positions showed in Tab. 3, measures indicated by the instrument were taken.

Finishing the calibration and using ESRM equations, one could find the components of the volumetric error in the three evaluated axle, and in all work volume. Graphics showing errors surfaces for application of the ESRM in different measuring plans for three coordinated axes can be seen in Fig. 5 to 10 (all are placed in the next section). Besides, positioning errors of probe tip are presented in volumetric errors graphics, in X, Y and Z directions.

4. Results

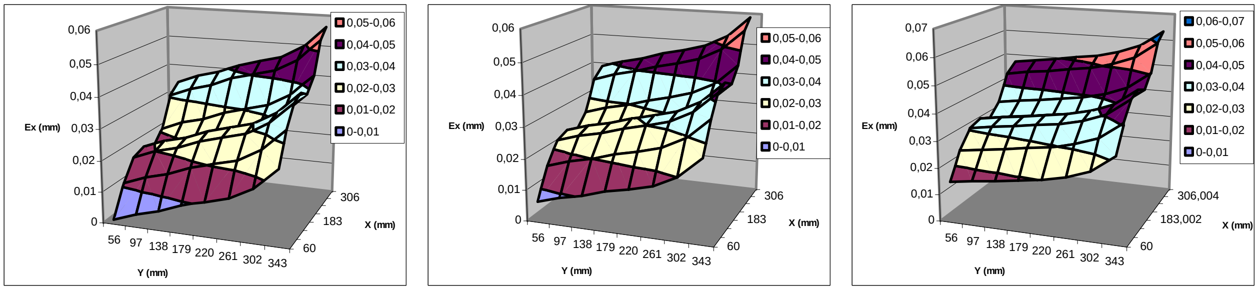
Curves were, for the three components, synthesized in five plans; first three of them are XY plans in different Z positions. Fourth is a XZ plan with Y coordinate positioned in the centre of the axle and the fifth one is a YZ plan with X coordinate also positioned in the centre of the axle.

4.1. X component of volumetric error

Surfaces which show the behavior of MM3C volumetric error component in X direction can be seen in Fig. 5 and 6.

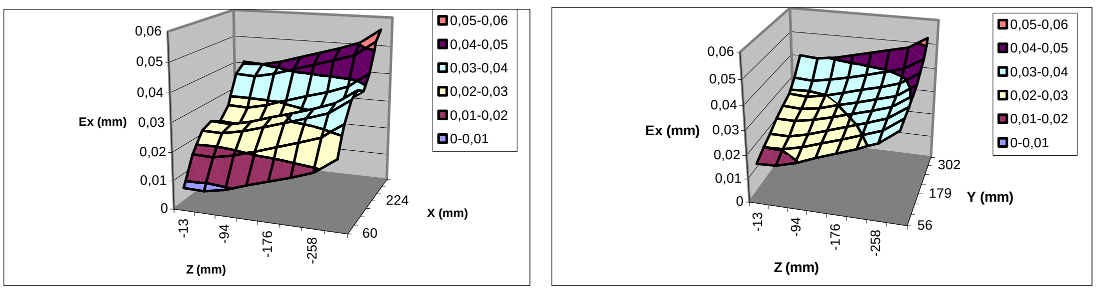
Observing last graphics, one can affirm that Ex component values vary between 1,5 µm and 63,3 µm. Error in each measured plan showed itself in a sufficiently similar form, tending to increase when X and Y coordinates increase. There is also a notable increase in error when Z coordinate is changed: in first point of the first graphic, the value of the error is 1,5 µm; in the second graphic is 6,7 µm and, in the third one, 15,3 µm (for all graphics, X and Y coordinates are, respectively, 60 mm and 56 mm). This is due to the increase in the Z offset.

The graphic to the left of Fig. 6 shows error Ex varying from 8 µm to 55,6 µm while, in the graphic to the right, it varies from 17 µm to 52,4 µm. Both graphics show the increase in error when X and Y coordinates increase and Z coordinate decreases.



Ex in XY plan, z = 0 mm   Ex in XY plan, z = 123 mm   Ex in XY plan, z = 246 mm

Figure 5: Ex surfaces drawn in distinct measuring plans

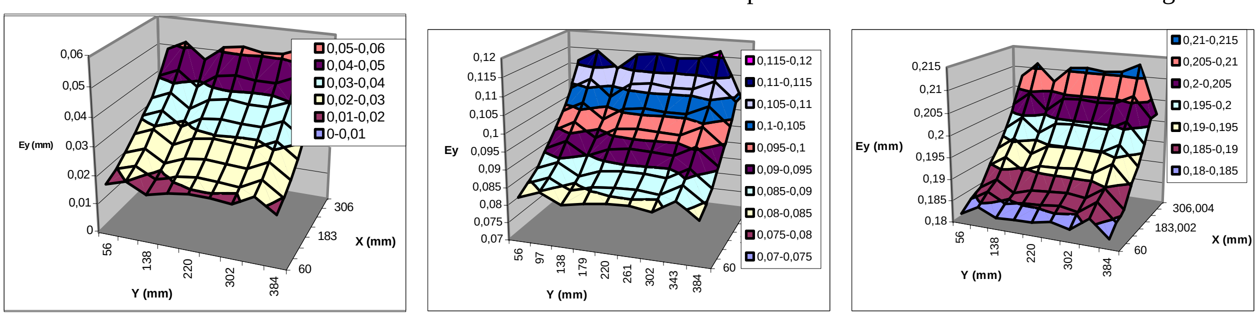


Ex in XZ plan, y = 206 mm   Ex in YZ plan, x = 181 mm

Figure 6: Ex surfaces drawn in XZ and YZ measuring plans

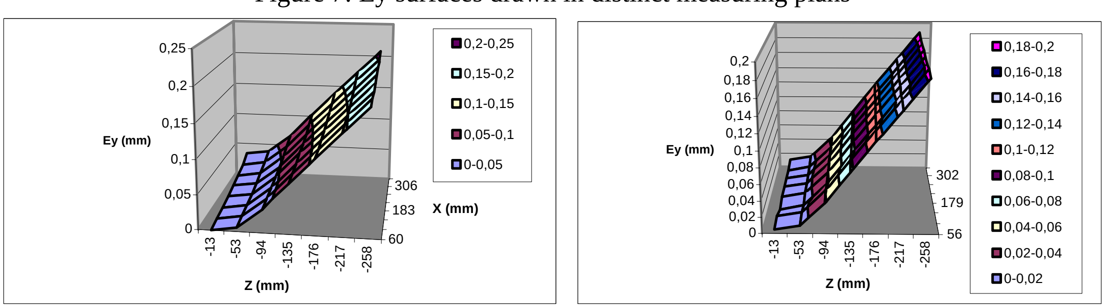
4.2. Y component of volumetric error

Surfaces which show the behavior of MM3C volumetric error component in Y direction can be seen in Fig. 7 and 8.



Ey in XY plan, z = 0 mm   Ey in XY plan, z = 123 mm   Ey in XY plan, z = 246 mm

Figure 7: Ey surfaces drawn in distinct measuring plans



Ey in XZ plan, y = 206 mm   Ey in YZ plan, x = 181 mm

Figure 8: Ey surfaces drawn in XZ and YZ measuring plans

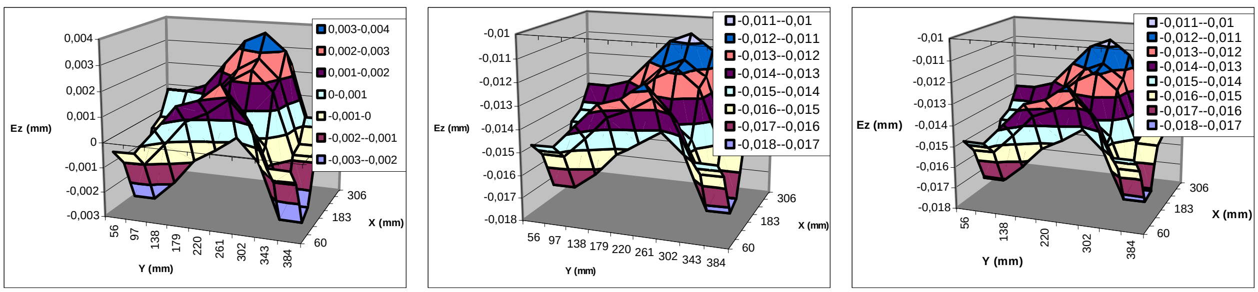
Observing graphic of Fig. 7, one can see that Ey component values vary between 16,6 μm and 212,8 μm. Error in each measured plan showed itself in a sufficiently similar form, tending to increase when X coordinate increases. There isn’t any tendency when Y coordinate increases. There is also a notable increase in error when Z coordinate decreases: in first point of the first graphic, the value of the error is 17,8 µm; in the second graphic is 82,6 µm and, in the third one, 182,3 µm (for all graphics, X and Y coordinates are, respectively, 60 mm and 56 mm). This is due to the increase in the Z offset and to YZ Orthogonallity error, which is high and has great influence in Ey.

The graphic to the left of Fig. 8 shows error Ey varying from 0,7 µm to 245,1 µm while, in the graphic to the right, it varies from 5 µm to 225 µm. Both graphics show the increase in error when Z coordinate decreases and, when X and Y coordinates increase, one can notice a soft tendency of error increase; biggest tendency can be observed in the graphic to the right.

4.3. Z component of volumetric error

Surfaces which show the behavior of MM3C volumetric error component in Z direction can be seen in Fig. 9 and 10.

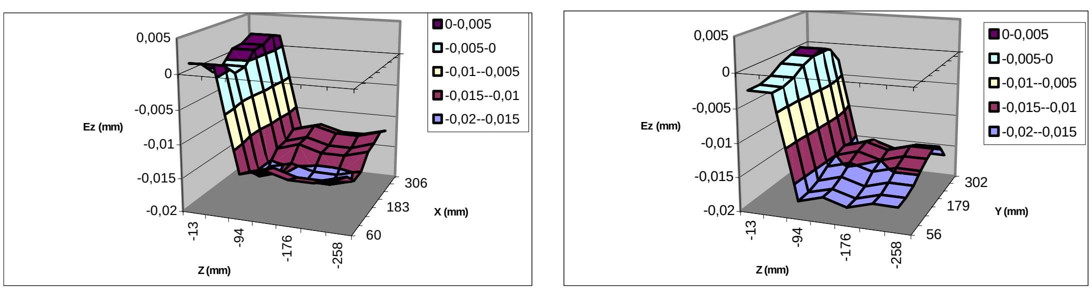
Analyzing graphic of Fig. 9, one can verify that Ez component values vary between -17,6 µm and 3,7 µm. Curves has the same form and initial point of each one were displaced due to the influence caused by Z and Y offsets.



Ez in XY plan, z = 0 mm　　Ez in XY plan, z = 123 mm　　Ez in XY plan, z = 246 mm

Figure 9: Ez surfaces drawn in distinct measuring plans

The graphic to the left of Fig. 10 shows error Ey varying from 2,3 µm to -15,9 µm while, in the graphic to the right, it varies from 0 µm to -17,7 µm. Both graphics show the decrease in error when Z coordinate decreases.



Ez in XZ plan, y = 206 mm　　Ez in YZ plan, x = 181 mm

Figure 10: Ez surfaces drawn in XZ and YZ measuring plans

5. Conclusions

Scientific accomplishments in industrial area are closely linked to the sprouting of new needs. Studies in Metrology follow, or must follow, progress in manufacturing means.

Tri-dimensional measuring technique allows the execution of metrology tasks which first implicated in great efforts. In some applications, this technique maybe represents the only choice for an objective and reproducible measurement.

This work had the objective of presenting a new error synthesization model for coordinates measuring machine. It was verified that the used model reduces significantly the set of data utilized for machine errors compensation.

ERSM also can be applied in other machines but, before the application, a detailed analysis of its structure must be done because of the existence of alterations in errors vectors and in errors grouping formation.

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