SIMULATION OF RADIATION CONDITION IN ROTATING MACHINERY USING SIMPLIFIED MODELS

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The article presents a comparison of two techniques to simulate radiation condition in rotating machinery: the linear cone models and infinite elements with exponential decay. The majority of numerical methods employed to simulate dynamic soil-structure interaction, namely the Boundary Element Method and the Infinite Element Method are very precise but are also very time-consuming since no conventional modal base is available in these problems. Also, these methods tend to depend too much on geometric conditions which normally are unavailable at the initial design stages of the installation design. On the other hand, Sommerfeld's or radiation condition is rarely simulated in rotor dynamics because designers tend to rely on massive foundations to isolate soil vibration. Also, soil viscoelastic parameters are frequency-dependent and do permit the usage of classic pseudo-modal techniques. The method of cone models presents a simple and fast way to simulate radiation condition in rotor dynamics even if some of their results are also frequency-dependent. Since today there is a clear tendency to use lighter and stronger foundation structures to enlarge the velocity operation range of these machines, a fast evaluation of the soil dynamic properties are well needed. Results obtained with simple, physical models, indicate that the hypothesis of massive foundations are very difficult to obtain in practice even in the case of very massive foundation. Besides, massive foundations tend to amplify vibration amplitudes in critical velocities and resonance frequencies since radiation condition plays an important role in rotating machinery simulation.

Keywords: *rotor dynamics, finite elements, infinite elements, cone models*

1. Introduction

The numerical analysis of problems of foundation vibration and its dynamic influence on the machine behaviour has presented numerous challenges in the past thirty years. From a historical perspective, the initial models that dealt with the problem of dynamic soil-structure interaction (DSSI) tended at first to ignore some basic phenomena that are well understood today, the main of which is the so-called Sommerfeld´s radiation condition (Wolf, 1985; Barros, 1996). Such boundary condition is generally imposed to displacement fields within the soil to make sure that mechanical waves generated in the soil-foundation interface do not bounce back to the foundation-structure subsystem. Today, sophisticated numerical procedures are available to take into account this radiation phenomenon but there is no definite answer about how much damping is introduced in rotating machinery by its contact with the surrounding soil. The reasons for this is that there is still a tendency to treat the rotor, the frame (pedestals) and the block foundation as if they were all independent and neglecting the mutual interaction between these systems. The function of the foundation is not only to support the weight of the expensive machinery but also to contribute in reducing vibration amplitudes particularly in the vicinities of resonance frequencies and critical velocities. Gaul (1986) was amongst the first who tried a combined approach in which the modelling of soil-structure interaction was obtained trough boundary element methods and combined with the equations of motion of rotor dynamics. But, due to computational limitations, only partial results were presented.

Rotating machines have many applications in today's industrial plants and are viewed as complex machines made primarily of a rotating set where one can find shafts, discs and bearings and of a heavy steel or iron supporting structure which is fixed upon a concrete or steel foundation resting on the surface of a supporting soil. Cases of rotating machinery where the foundation is partially buried in the surrounding soil are not uncommon due to the stiffer dynamic isolation of such alternatives (Romanini, 1995).

The shape, materials and construction techniques involved in making such foundations are complex and demand sophisticated numerical methods for a detailed dynamic analysis (Bonello, 2001). These foundations are connected to and supported by soils that have also a complex dynamic behaviour mainly due to the dissipative effects of these almost infinite media. The modelling procedures involved in the description of the dynamic behaviour of such media can be very time-consuming, depending on the level of precision required for the analysis. Also, methods based on integral formulations such as Boundary Element Methods (BEM) or Green's Functions are unfamiliar to the vast majority of structural engineers and researchers of rotor dynamics. A review of the main aspects involved in modelling the dynamic soil-structure interaction shows that until very recently engineers did not recognise the role of the geometric damping of the soil in the description of the dynamic behaviour of structures connected to it (Gazetas, 1983). Such damping or dissipative mechanism arises in the rotating machinery modelling due to the propagation of the rotor energy throughout the soil via mechanical waves. Due to the presence of these attenuation phenomenon, vibration levels of structures connected to the soil are quite distinct than those isolated by trenches or rigid piles (Barros, 1996).

The purpose of the present paper is, therefore, to create an approximate method to include soil-foundation dissipative effects directly into equations of rotor dynamics and to analyse the influence of the soil modelling technique by two finite element models: the simplified cone models (Wolf, 1985) and the infinite element method (Barros, 1996).

2. Equation of motion for a rotor-foudation-soil model in frequency domain

The mathematical model of the coupled machine-foundation-soil presented above is split in two distinct parts:

- machine or rotor system, represented by mass [M], damping [C] and stiffness [K] matrices of the shaft, disc and bearings elements. In the rotor system one can also include possible stiffness and damping effects of the bearings. One can also include stiffening effects on the shaft due to the presence of axial or shear forces (Lalanne, 1990).
- auxiliary or supporting system, represented by the soil-foundation-structure impedance matrix [S(ω)] also called complex stiffness matrix which, in the case of infinite elements, is obtained directly in frequency domain.

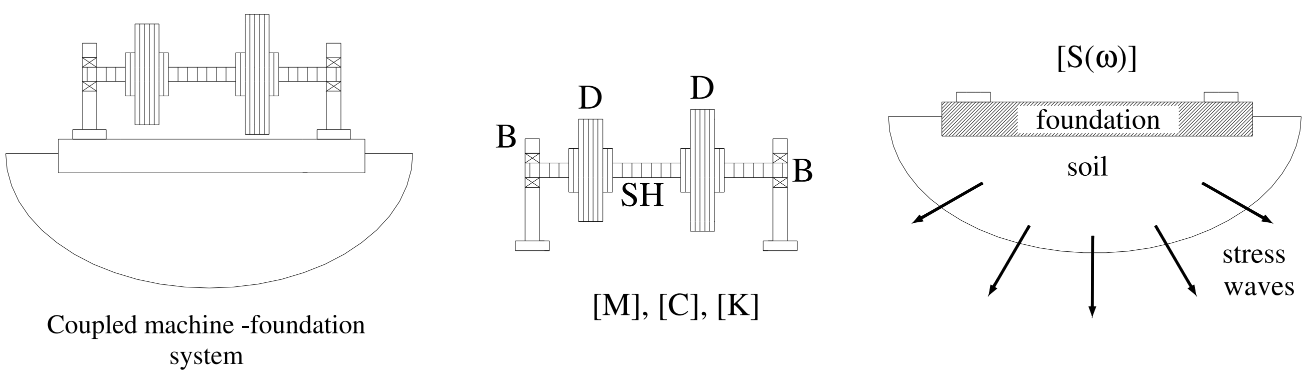


Figure 1: decomposition of the original coupled system into machine and support sub-systems
(B = bearings, D = discs, SH = shaft elements)

One can define the displacement vector {x(t)} containing the degrees of freedom of the rotor system. The vector $\{x_B(t)\}$contains a partion of {x(t)} with all the DOF´s of the connecting nodes lying on the machine-foundation interface and $\{x_S(t)\}$ contains the remaining nodes of {x(t)}. Therefore the rotor equation of motion can be represented as (Lalanne, 1990):

$$[M]\{\ddot{x}(t)\}+[C]\{\dot{x}(t)\}+[K]\{x(t)\}=\{F(t)\} \tag{1}$$

where:

$$\{x(t)\}=\begin{Bmatrix}\{x_S(t)\}\\ \{x_B(t)\}\end{Bmatrix} \quad \text{and} \quad \{F(t)\}=\begin{Bmatrix}F_S(t)\\ F_B(t)\end{Bmatrix}$$

The term {F(t)} represents the vector of external generalised forces and the partition $\{F_S(t)\}$ contains all the forces acting on the rotor system. The partition $\{F_B(t)\}$ represents the interaction forces between the machine structure and the supporting system. In the case of the rotor system, [M], [C] and [K] are obtained via summation of each finite element matrix corresponding to the shaft, disk and bearing elements of the rotor plus any DOF representing the machine structure following the procedures of structural analysis. Partitioning of mass, damping and stiffness matrices and applying the Fourier transform to the rotor system results:

$$\begin{bmatrix} [S]_{SS} & [S]_{SB} \\ [S]_{BS} & [S]_{BB} \end{bmatrix} \begin{Bmatrix} \{X\}_S \\ \{X\}_B \end{Bmatrix} = \begin{Bmatrix} \{F\}_S \\ \{F\}_B \end{Bmatrix} \tag{2}$$

where the B-index indicates those nodes of the structure lying on the machine-foundation interface and the S-index indicates all the remaining nodes in the rotor system. The subscripts in $[S]_{ab}$ for a,b = S,B represents a partition of the system impedance matrix, given by:

$$[S]_{ab} = -\omega^2 [M]_{ab} + i\omega [C]_{ab} + [K]_{ab} \tag{3}$$

and $\{X_S\}$ and $\{X_B\}$ represent complex displacement amplitudes in the rotor system. The interaction forces arising between the machine and the supporting systems are calculated as follows:

$$\{F\}_B = [S]_{BB}^G \left(\{X\}_B^G - \{X\}_B \right) \tag{4}$$

where $\{X\}_B^G$ represents the displacement amplitudes of the interface nodes which are calculated without the machine influence and $[S]_{BB}^G$ is the impedance matrix of the supporting system. In cases where external forces exist only on the rotor, the term $\{X\}_B^G$ vanishes and the equation of motion of the coupled system is simplified as:

$$\begin{bmatrix} [S]_{SS} & [S]_{SB} \\ [S]_{BS} & [S]_{BB} + [S]_{BB}^G \end{bmatrix} \begin{Bmatrix} \{X\}_S \\ \{X\}_B \end{Bmatrix} = \begin{Bmatrix} \{F\}_S \\ \{0\} \end{Bmatrix} \tag{5}$$

The soil-structure impedance matrix $[S]_{BB}^G$ is obtained using the concept of flexibility matrix as given as follows.

2.1 Impedance matrix of the supporting system

Since the terms in $[S]_{BB}^G$ are frequency-dependent and cannot be expressed in terms of a modal base, the method generally used to obtain the equation of motion of the supporting system is through direct inversion of the flexibility matrix for a soil-foundation subsystem excited by an harmonic force for each DOF in $\{X\}_B^G$ for each frequency step. The foundation itself can be treated as flexible or rigid since there are no differences in the formulation. In this case a previous modal analysis of the pedestals and foundation blocks are required to investigate possible flexible modes within the machine operation range. The equation of motion in frequency domain for the supporting system is:

$$[N(\omega)]_{BB} \{F_B\} = \{X_B\} \tag{6}$$

where $[N(\omega)]_{BB}$ represents the flexibility matrix for each structural DOF on the interface between the machine and the supporting system. The soil-foundation impedance matrix is, therefore:

$$[S]_{BB}^G = [N]_{BB}^{-1} . \tag{7}$$

Although one of the conclusions drawn here is that massive foundation tend to increase vibration in the machine only the case of a rigid foundation will be considered in this paper. Ignoring torsional vibrations in z and x axis and considering only in-plane interaction between machine and foundation, the supporting system has only three DOF´s which are related to the foundation rigid body modes in the plane of figure 2

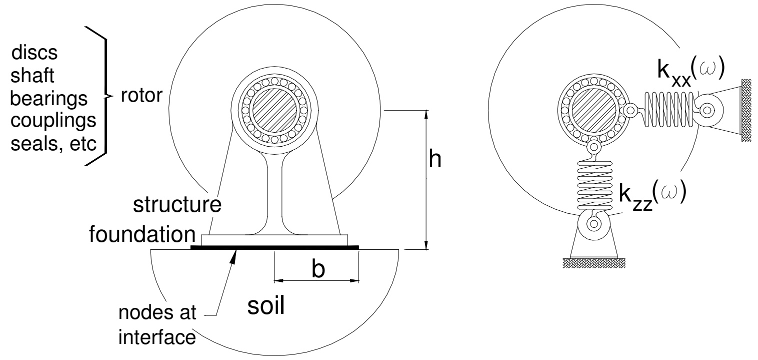


Figure 2: supporting system of a flexible foundation and its approximation by discrete elements.

Neglecting axial forces on the rotor system and considering that the moment acting on the foundation is given only by horizontal reaction forces in the rotor bearing, the equation of motion (7) is given by (Barros, 2005):

$$\left(\begin{bmatrix} N_{wz} & 0 & 0 \\ 0 & N_{ux} & N_{um} \\ 0 & N_{\varphi x} & N_{\varphi m} \end{bmatrix}^{-1} - \omega^2 \begin{bmatrix} m_z & 0 & 0 \\ 0 & m_x & 0 \\ 0 & 0 & I_z \end{bmatrix}\right)\begin{Bmatrix} F_Z \\ F_X \\ F_X \end{Bmatrix} = \begin{Bmatrix} w \\ u \\ \varphi.\dfrac{b^2}{h} \end{Bmatrix}, \quad \text{therefore} \quad \begin{bmatrix} S_{wz} & 0 & 0 \\ 0 & S_{ux} & S_{um} \\ 0 & S_{\varphi u} & S_{\varphi m} \end{bmatrix}\begin{Bmatrix} w \\ u \\ \varphi\frac{b^2}{h} \end{Bmatrix} = \begin{Bmatrix} F_z \\ F_x \\ F_x \end{Bmatrix} \tag{8}$$

where displacements *w, u* and φ are the foundation DOF´s and $N_{ij}(\omega)$ are massless foundation flexibility functions. These functions are obtaining simply by applying a unit excitation in ith direction and obtaining the resultant displacements in x,z and φ directions. The terms m_x, m_z and I_z refer to combined foundation-structure mass and inertia, calculated in the center of the foundation Neglecting the machine structural flexibility in the plane of figure 2 and considering the forces F_Z and F_X acting directly on the rotor axis, the inverse of equation (8) can be changed as:

$$\begin{bmatrix} k_{zz} & 0 \\ 0 & k_{xx} \end{bmatrix}\begin{Bmatrix} w \\ z \end{Bmatrix} = \begin{Bmatrix} F_z \\ F_x \end{Bmatrix} \tag{9}$$

where b and h are, respectively, foundation half-bandwidth and distance from rotation axis to the foundation-machine interface. One can see from equation (9) that there is no dynamic coupling between displacement in vertical direction and the other two. In equation (9), dynamic stiffness k_{zz} and k_{xx} are given in terms of flexibility functions by:

$$k_{wz} = S_{wz} \quad \text{and} \quad k_{xx} = \frac{S_{ux}S_{\varphi m} - S_{um}S_{\varphi x}}{S_{\varphi m} + S_{ux}h^2 - \left(S_{um} + S_{\varphi x}\right)h} \tag{10}$$

The base rocking displacement φ can be calculated before horizontal displacement u(ω) is calculated:

$$\varphi = \left(\frac{S_{\varphi x} - hS_{ux}}{hS_{um} - S_{\varphi m}}\right)u \tag{11}$$

2.2. Infinite Elements

The infinite element used here to obtain flexibility functions $N_{ij}(\omega)$ were developed by Barros (1996). In this element, which can be used for plane strain or plane stress problems, one defines two separated directions that are chosen concerning the element orientation in the global system of coordinates. Vector ε is the main propagation direction and is set to simulate the main energy flow propagating from the source toward the infinite. Geometry of the element is displayed in figure 3. As in soil mechanics, stress waves propagate in several directions, there is not a unique propagation direction for all infinite elements. However, an adequate mesh can be constructed if one considers that near surface, energy propagation is mainly horizontal due to the presence of Rayleigh waves (Wolf, 1985) and in the interior of the soil, propagation is mainly radial due to body waves (pressure and shear waves). Figure 3 also shows a typical mesh constructed to obtain influence functions for rigid foundation on the surface. In this example, the region near the foundation (called near field) is modeled by quadratic lagrangean elements. Previous analyses (Medina and Taylor, 1982) indicate that accurate results in the frequency range $0 < a_0 < 6$ can be obtained extending the near field to a distance of 3 times the foundation half-bandwidth.

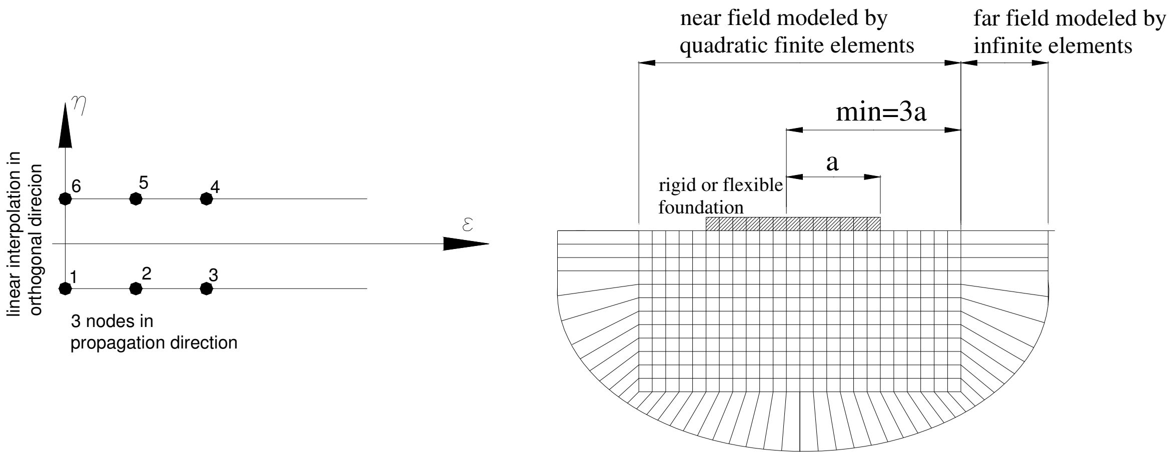


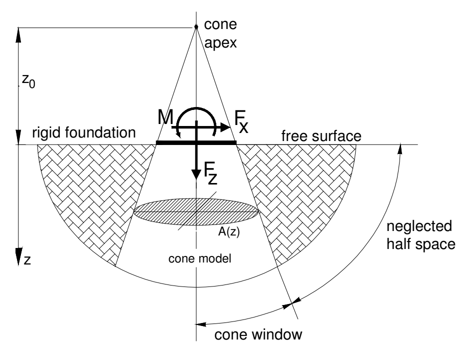
Figure 3: geometry of the infinite element and FEM mesh including these elements in the far filed region

2.3. Cone Models

Cone models, as described by Wolf (1985), present a simple method for obtaining some half- or layered-space flexibility functions. However, since they ignore large portion of the half space very close to the machine, such models does not permit simulation of Rayleigh (or surface) wave propagation which carries more than half of the vibration energy in some frequency ranges. Therefore, such models underestimate the soil damping capacity, particularly in the case of buried foundation or piles. For the case of vertical flexibility function $N_{wz}(\omega)$, the linear cone model permits some results that indeed allow modal techniques to be applied in the coupled soil-foundation-machine system. For a cone with linearly increasing transversal area (linear cone), the vertical flexibility function for rigid foundation is:

$$N_{wz}(\omega) = \frac{1}{A_0 c_p^2 \rho \left(\dfrac{1}{z_0} + \dfrac{i\omega}{c_p} \right)} \qquad \text{where } c_p \text{ is the pressure wave velocity.} \qquad (12)$$

In equation (13), A_0 is the soil-foundation contact area and z_0 is the cone apex. In practice, parameters A and z_0 for linear cone are chosen in such a way that N_{wz} calculated using equation (13) has the same static stiffness of an equivalent foundation of the same contact area Such values can be found in GAZETAS (1983).



$$A(z) = A_0(z - z_0)$$

Figure 4: Linear cone model for $N_{wz}(\omega)$

2.4 Comparison of the methods

The methods described above for obtaining flexibility functions will be applied for the analysis of a soil-foundation-structure system subjected to a vertical loading varying from $a_0 = 0.1$ to $a_0 = 2.5$. This corresponds to a range in rotor velocities varying from 100 to 20000 rpm. The geometry of the problem is described below:

a) soil: the underlying soil was considered isotropic, viscoelastic with η=0.10 , c_p = 259.2 m/s and c_S = 158.9 m/s . This corresponds to G = 48 MPa, ν = 0.25 and ρ = 1900 kg/m^3. Soil modelling was conducted according to the two alternatives described earlier. Welded contact soil-foundation was also admitted.

b) structure: the pedestals were modelled in steel with h = 0.6m. Structural damping was admitted as $\eta = 0.01$ and frequency independent. A 3D full finite element model of the pedestal was constructed in order to verify if rigid conditions could be applied to the model. The weight of the pedestal was 32.8kg without bearings. Figure 5 shows a displacement FRF of the model for an harmonic unit load acting on the center of the bearing and the corresponding response measured in the same location and direction assuming fixed boundary conditions on the nodes resting on the foundation-structure interface. To construct this function, modal superposition was employed with 30 modes of the pedestal taken into account, in the range from 1 Hz to 10.5 kHz. Results indicate that in the frequency range from 0 to 1 kHz (0 to 60000 rpm in rotor velocity) the response function is very stable and the pedestal can be considered a rigid body.

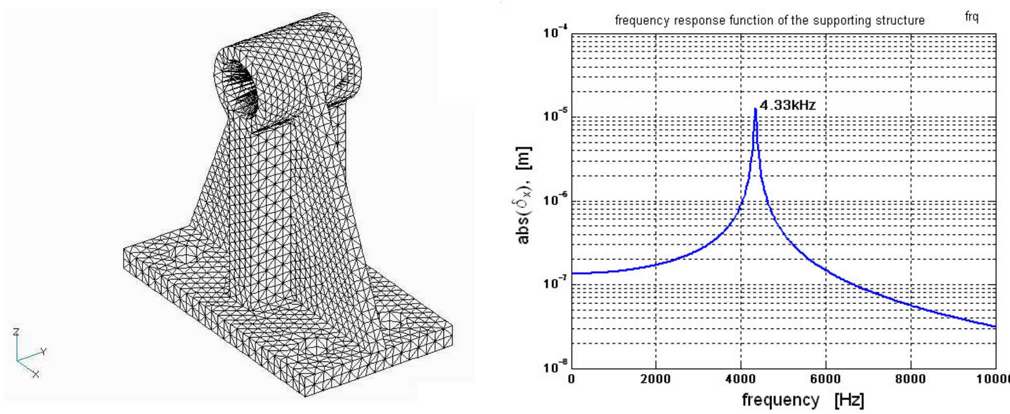


Figure 5: Finite element model and its frequency response function
for an harmonic loading acting in horizontal direction

c) foundation: the surface basemat was admitted as a rigid rectangular block with b = 0.50m and mass varying from 1 to 100 times the mass of the pedestal. Equivalent inertia tensor for this rigid body was also calculated.

Figure 6 shows the frequency behaviour of the combined supporting structure (soil + foundation + soil) as modelled using infinite elements and linear cone model. Results are displayed for several mass ratios mf/ms where mf is the foundation mass and ms is the pedestal mass.

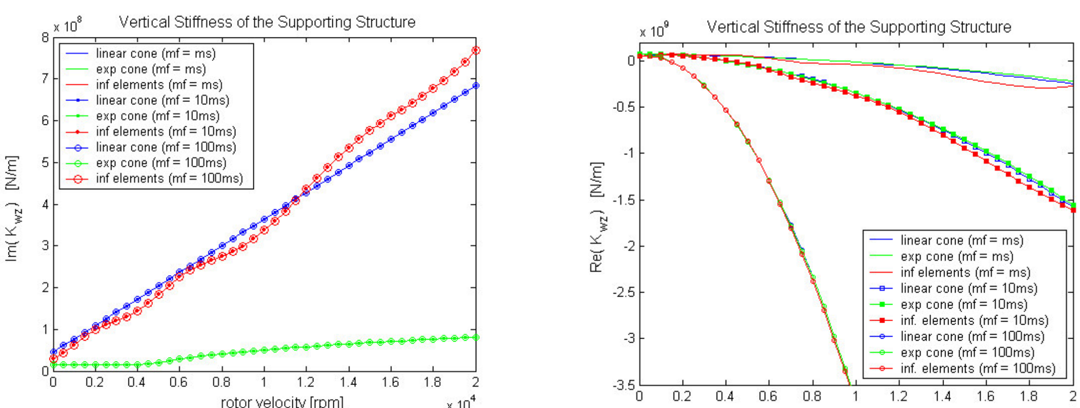


Figure 6: Frequency behaviour of the supporting structure as functions of the mass ratio ms/mf

There is an excellent agreement between the models for the real component of the vertical stiffness and also a good agreement between the linear cone model and the infinite elements for the imaginary component. Also, for massive ($m_f = 10m_s$) and very massive ($m_f = 100m_s$) foundations, the influence of the foundation is more effective as the frequency increases. It is also important to note that the amount of stiffness, particularly for rotor velocities below 10000 rpm and in the range $m_f \leq 10m_s$, are comparable to the bearing stiffness that are usually introduced in the rotor equations. Therefore, soil and foundation flexibility should not be neglected when dealing with rotor dynamics.

2.5 Numerical Implementation: dynamic behaviour of an asymmetric rotor on a viscoelastic half-space

In order to determine the influence of the soil dissipative parameters on the dynamic behaviour of rotating machinery, the flexibility functions described above were used together with the motion equations of rotordynamics to determine some FRF curves of an asymmetric rotor supported by a viscoelastic half-space. A comparison illustrating the influence of the foundation mass in the results was also obtained. Figure 7 illustrates the geometry of the present problem.

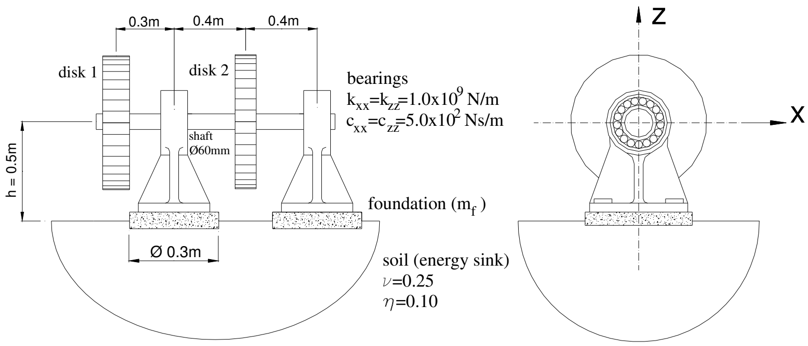


Figure 7: Geometry of the rotating machinery

Table 1: physical properties of the several elements used in the model

bearings (2 elements)	$k_{xx} = k_{zz} = 1.0\text{x}10^9$ N/m , $k_{xz} = k_{zx} = 0$ $C_{xx} = C_{zz} = 5.0\text{x}10^2$ N.s/m , $C_{xz} = C_{zx} = 0$
disc 1	m_d= 0.7 kg, $I_x = I_z = 1\text{x}10^{-3}$ kgm^2 and I_y=2x10^{-3} kgm^2
disc 2	m_d= 1.6 kg, $I_x = I_z = 3.2\text{x}10^{-3}$ kgm^2 and I_y=6.4x10^{-3} kgm^2
shafts (28 elements)	E = 2.09x10^{11} N/m2, $I_x = I_y$ = 7887 mm^4 , ρ = 7920 kg/m^3
pedestals plus bearing (each)	m = 3.5 kg, Ix_A = 0.817 kg.m^2, I_{yA} = 0.251 kg.m^2, I_{zA} = 0.934 kg.m^2
total rotating mass (axis + disks)	m_{total} = 4.2 kg
soil	c_p = 158 m/s , c_s = 198 m/s, η = 0.10, ν= 0.25
foundation + pedestals	m = m_f
pedestals	E = 200 GPa, ν = 0.30, ρ = 7800 kg/m^3 , m = 32.8 kg , η = 0.01 I_z = 0.246 kgm^2 and I_y = 0.939 kgm^2

The system was excited by an unbalancing force corresponding to 0.01 kgm on disc 1. To demonstrate the influence of the foundation mass on the displacements, the soil impedance matrix was calculated via finite/infinite modelling considering four distinct mass relations of m_f /m_{total} = 1.0, m_f /m_{ts} = 5 , m_f /m_{total} = 100 and rigid foundation, were m_{total} is the approximate machine mass including pedestals and bearings. To demonstrate the influence of the modelling technique, figure 8 shows the system response for n = 0rpm considering three alternatives of modelling of the supports: rigid base, infinite method and cone model for a foundation mass of 42kg that corresponds to m_f /m_{total} ≈ 10.

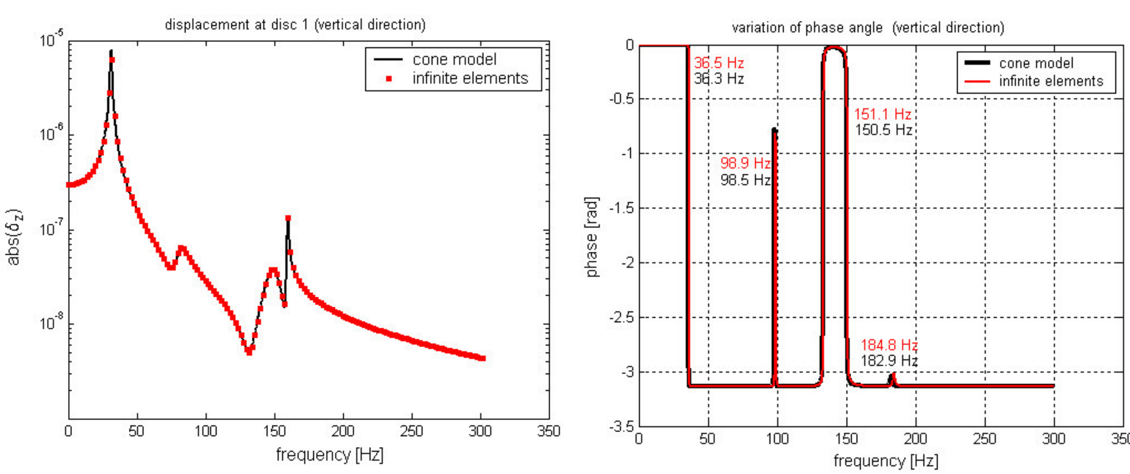


Figure 8: Influence of the modelling technique on frequency response for mf/ms = 10 and n= 0rpm.

Results demonstrate that the modelling technique is also important when considering the response of rotating machinery but there is no appreciable difference between the results obtained with infinite elements or cone models.

This suggests that an economical analysis can be performed simply by considering an equivalent spring-dashpot model for soil attenuation if foundation and supporting structure are rigid enough. The results shown in figure 9 refer to the unbalance response on disc 1 when rotation speed varies from 0 to 150 Hz for several mass relations when the soil response was modelled with cone elements. In this case, the system was excited by an unbalancing mass of 0.01 kg.m on disc 1.

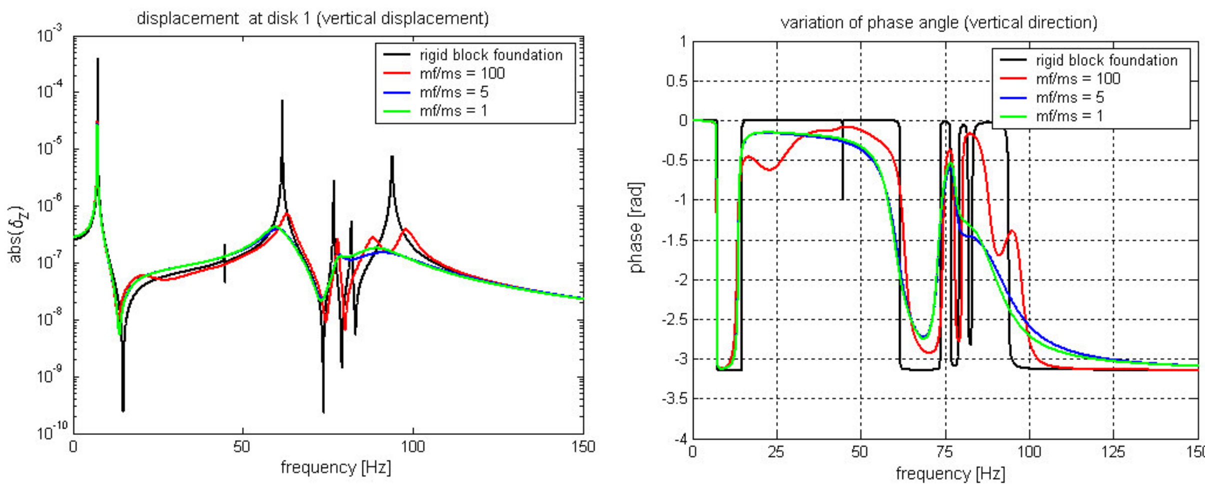


Figure 9: Influence of the modelling technique on the unbalance response for mf/ms = 5 and desb=0.01 kg.m

3. Conclusions

As expected soil flexibility increases the damping in rotating machinery since it introduces an additional energy sink in rotordynamics equations. Also, even for foundation masses of about 100 times greater than the rotor's, there is an appreciable difference between ideal and soil damping models which indicates that the boundary conditions that are normally applied in rotordynamics codes, i.e. rigid block foundation are very difficult to obtain in practice. Also, at initial design stages, when details of foundation geometry and materials are not known, a previous analysis with simplified cone models provides reliable results at low computational costs.

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