COMPRESSION-IGNITION ENGINE MODELING

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Abstract. *In this work an algebraic model for the thermodynamic study of the compression-ignition cycle is developed using the concept of efficiencies and a single zone combustion model. The information supplied for the air standard cycle are overestimated because the work fluid is perfect gas, the specific heat are constant, the compression and expansion processes are adiabatic, among others. In this model, the use of efficiency concept in all processes that compose the cycle added to the variation of the specific heat as function of temperature, the heat release based on Wiebe function, and the variation of the initial instant of heat release and heat rejection, makes the air standard cycle to approach to the actual Diesel cycle. This methodology allows to study the engine through a cycle that can be called of non-ideal. The algebraic model serves as didactic tool for the thermodynamic analysis of the compression-ignition engines operation.*

Keywords: *compression-ignition engine, thermodynamic modeling, non-ideal cycle.*

1. Introduction

The operation of an internal combustion engine can be analyzed using an air standard cycle. In the ideal cycle the working fluid is the pure air and it is assumed to be a perfect gas, there is no change in the mass, and the value of specific heat is keeping constant. Heat is assumed to be supplied from a constant high temperature source that is transferred to the air mass, and the excess of heat is rejected to a cold source at ambient temperature (Ganesan, 1995). The air standard cycle provides the parameters that affect the engine performance.

In the early history of the compression ignition engine, the thermodynamic cycle that describes the operation of this engine is the air standard cycle Diesel, also known as the constant-pressure cycle. This cycle was developed by Rudolf Diesel for its equipment. In this cycle, the compression (1-2) and the expansion (3-4) process are isentropic, and the process of heat addition (2-3) occur at constant pressure and heat rejection (4-1) occur at constant volume. Figure 1 show the Diesel cycle on *p-V* and *T-s* diagrams respectively.

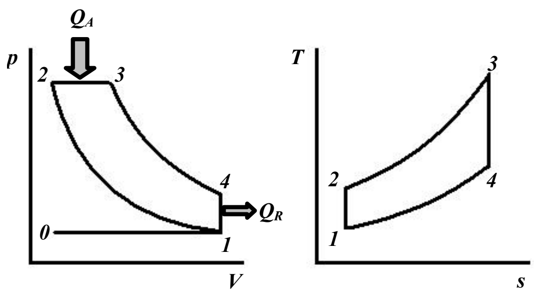

Figure 1. Diesel cycle.

According to Brady (1996), the dual combustion cycle is considered more appropriate to the operating of the actual Diesel engine. This cycle is also called as limited pressure cycle. In a dual cycle a part of the energy is first supplied at constant volume and then the remaining part at constant pressure to the working fluid. Figure 2 show the dual cycle on *p-V* and *T-s* diagrams respectively.

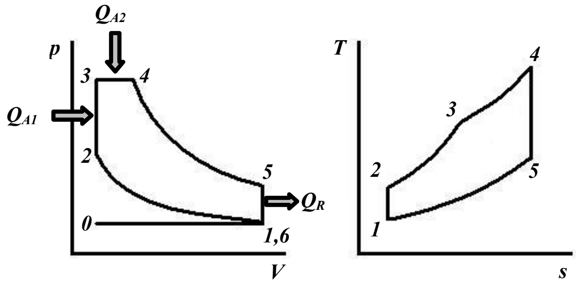

Figure 2. Dual cycle.

The air standard cycle analysis overestimated the performance of actual engines. Recently, Barros (2003) developed an algebraic model for the Otto cycle that uses the concept of efficiency in all the processes that compose the ideal cycle. The efficiency considers the energy losses (irreversibility) in the cylinder in each process. This cycle was called as non-ideal cycle. According to Barros (2003), the advantage of the algebraic model is its low computational load. However this model requires the previous knowledge of efficiencies values. A similar model is presented in this work for the compression ignition engine.

2. Objectives

The objective of this work is to develop an non-ideal cycle algebraic model for the compression ignition cycle based on the classic thermodynamics equations, introducing the efficiency concept and considering other modifications in each process that allow to modify the ideal cycle to approach of actual operating of engine. This model can be used as tool in the teaching of internal combustion engines for analysis of compression ignition cycle.

3. Mathematical models

The equations used to the development of non-ideal model can be divided in two groups. The first group consists in the common equations for all stages of the cycle, that are the following equations: piston stroke $C(q)$, cylinder volume $V(q)$, force exerted for the gas or in the gas $F_g(q)$, tangential force in the crankshaft $F_t(q)$, torque $Tq(q)$, specific heat at constant pressure $c_p(q)$, specific heats ratio $k(q)$, engine power Pt, and specific fuel consumption SFC.

$$C(q) = \frac{C_T}{2}\left(1 - cos(q)\right) + r_{BC}C_T\left[1 - \left(1 - \frac{sen^2(q)}{4r_{BC}{}^2}\right)\right] \quad (1)$$

$$V(q) = V_C + C(q)\frac{pD_p{}^2}{4} \quad (2)$$

$$F_g(q) = \left(p(q) - p_0\right)\frac{pD_p{}^2}{4} \quad (3)$$

$$F_t(q) = F_g(q)\left(sen(q) + \frac{sen(2q)}{4r_{BC}}\right) \quad (4)$$

$$Tq(q) = F_t(q)\frac{C_p}{2} \quad (5)$$

$$c_p(q) = \left(a + bT(q-1) + cT(q-1)^2 + dT(q-1)^3 + eT(q-1)^4\right)R_{ar} \quad (6)$$

$$k(q) = \frac{c_p(q)}{c_p(q) - R_{ar}} \quad (7)$$

$$Pt = \overline{Tq} \times \frac{2p \times N_{rpm}}{60} \tag{8}$$

$$SFC = \frac{(A/C)^{-1}}{Pt} V_d \frac{N_{rpm}}{60} \frac{n_{cil}}{2} \tag{9}$$

The second group of equations of non-ideal algebraic model meets the specific equations of each process (intake, compression, combustion, expansion, blowdown exhaustion, and exhaustion), where the cylinder pressure *p(q)* and temperature *T(q)* are calculated as function of the values of efficiencies. These efficiencies reflect the irreversibility of each process and the values are different of one. The equations for calculate the ignition delay and the heat release during the combustion are part of this group. The working fluid is the pure air and it is assumed to be a perfect gas.

$$p_{admission}(q) = p_0 + (h_V - 1) \times \left[\frac{p_0}{\dfrac{(V_d + V_c)}{V_T(q)}} \right] \tag{10}$$

$$T_{admission}(q) = T_0 \tag{11}$$

$$p_{compression}(q) = p_1 \left(\frac{V_T}{V_T(q)} \right)^{k \times h_C} \tag{12}$$

$$T_{compression}(q) = T_1 \left(\frac{V_T}{V_T(q)} \right)^{\frac{k}{h_C} - 1} \tag{13}$$

$$q_{delay} = q_{inj} + \frac{6 N_{rpm}}{1000} \frac{0,8216}{p_m^{0,386}} exp\left(\frac{4644}{T_m} \right) \left(\frac{40}{NC} \right)^{0,69} \tag{14}$$

$$\frac{dQ}{dq} = 6,9 \frac{Q_p}{q_p} (m_p + 1) \left(\frac{q}{q_p} \right)^{m_p} exp\left[-6,9 \left(\frac{q}{q_p} \right)^{m_p + 1} \right] + 6,9 \frac{Q_d}{q_d} (m_d + 1) \left(\frac{q}{q_d} \right)^{m_d} exp\left[-6,9 \left(\frac{q}{q_d} \right)^{m_d + 1} \right] \tag{15}$$

$$p_{combustion}(q) = \left[p(q-1) + \left(\frac{R_{ar} h_{Comb} \dfrac{DQ}{c_p(q)}}{V_T(q)} \right) \right] \times \left\{ 1 - \left[1 - \frac{1}{\left(\dfrac{V_T(q)}{V_T(q-1)} \right)^{\frac{k}{h_E}}} \right] \right\} \tag{16}$$

$$T_{combustion}(q) = \left[T(q-1) + \left(\frac{h_{Comb} DQ}{m_{ar} c_p(q)} \right) \right] \times \left\{ 1 - \left[1 - \frac{1}{\left(\dfrac{V_T(q)}{V_T(q-1)} \right)^{kh_E - 1}} \right] \right\} \tag{17}$$

$$p_{expansion}(q) = \frac{p(q-1)}{\left(\dfrac{V_T(q)}{V_T(q-1)} \right)^{\frac{k}{h_E}}} \tag{18}$$

$$T_{expansion}(q) = \frac{T(q-1)}{\left(\dfrac{V_T(q)}{V_T(q-1)} \right)^{kh_E - 1}} \tag{19}$$

$$p_{exhaustionBD}(q) = p(q-1) - \left[\frac{p(q-1) - \dfrac{p_0}{h_{BD}}}{\dfrac{V_T - V_T(q-1)}{V_T(q) - V_T(q-1)}} \right] \tag{20}$$

$$T_{exhaustionBD}(q) = T(q-1) - \left[\frac{T(q-1) - \dfrac{T(q-1) p_0 h_{RBD}}{h_{BD}\, p(q-1)}}{\dfrac{V_T - V_T(q-1)}{V_T(q) - V_T(q-1)}} \right] \tag{21}$$

$$p_{exhaustion}(q) = p_6 + (h_{Ex} - 1) \times \left[\frac{p_6}{\dfrac{V_T(q)}{V_c}} \right] \tag{22}$$

$$T_{exhaustion}(q) = T_6 \tag{23}$$

4. Methodology

Equations (1) to (23) constitute a system of algebraic equations that are used to calculate the non-ideal compression ignition cycle. The information required for the model is described in Tab. 1. The coefficients a, b, c, d, e, and f, not presented in Tab. 1, are used to the calculation of specific heat. The results to be obtained by model are described in Tab. 2. These results will be analyzed through comparative tables and diagrams. The diagrams are: *pV*, pressure, temperature, heat release rate, and torque as function of the crank angle, power, torque and specific fuel consumption as function of the rotation, among others.

Table 1. Data required for non-ideal algebraic model.

p_0	Environment pressure	kPa
T_0	Environment temperature	K
R_{ar}	Air constant	J/kgK
C_T	Piston stroke	cm
D_p	Piston diameter	cm
B	Connecting rod length	cm
CC	Cubic capacity of the engine	cm³
n_{cil}	Number of cylinders	
r_c	Compression ratio	
A/C	Air-fuel ratio	
PCI	Lower heating value	J/kgK
N_{rpm}	Crankshaft rotational speed	rpm
q_{inj}	Injection angle	°
NC	Fuel cetane number	
Q_p	Heat release in premixed combustion	J
q_p	Duration of the heat release in premixed combustion	°
m_p	Shape factor to premixed combustion	
Q_d	Heat release in diffusive combustion	J
q_d	Duration of the heat release in diffusive combustion	°
m_d	Shape factor to diffusive combustion	
q_{VEx}	Exhaustion valve opening angle	°
n_V	Volumetric efficiency	
n_C	Compression efficiency	
n_{Comb}	Combustion efficiency	
n_E	Expansion efficiency	
n_{BD}	Blowdown efficiency	
n_{RBD}	Blowdown retaining efficiency	
n_{Ex}	Exhaustion efficiency	

Table 2. Results obtained by non-ideal algebraic model.

p(q)	Cylinder pressure	kPa
T(q)	Cylinder temperature	K
V(q)	Cylinder volume	cm^3
Pt	Engine power	kW
Tq	Engine torque	Nm
SFC	Specific fuel consumption	g/kWh
PME	Mean effective pressure	kPa
C_f	Fuel consumption	kg/s
$\dot{m}_{ar}$	Air flow	kg/s
W_E	Specific work	J/kg
n_t	Thermal efficiency	

The piston stroke *C(q)* is calculated by Eq. (1) and it varies as function of crank angle, total piston stroke C_T, connecting rod length *B*, and total piston stroke ratio r_{BC}. The cylinder volume *V(q)* is calculated by Eq. (2) and it varies as function of the crank angle, and depends of piston stroke and piston diameter D_p. The cylinder volume is the sum of the clearance volume V_c and the displacement volume. The force $F_g(q)$ is calculated by the difference between the external and internal pressure of cylinder volume, described in Eq. (3). The tangential force $F_t(q)$ in the crankshaft is then calculated as function of force $F_g(q)$, crank angle, and ratio r_{BC}, described in Eq. (4). The product of the $F_t(q)$ with the arm of crankshaft is the supplied or consumed torque for the engine, described for Eq. (5). Figure 3 shows the diagram of forces to piston-connecting rod-crankshaft system. Equation (6) is used to calculate the specific heat at constant pressure $c_p(q)$ that depends of *T(q)* and of air constant R_{ar}. This property is obtained through of adjusted polynomial to the JANAF table thermodynamic data (Heywood, 1988). The value of the specific heats ratio *k* is obtained through $c_p(q)$ and R_{ar}, described in Eq. (7). The engine power *Pt* is calculated as function of medium torque $\overline{Tq}$, described in Eq. (8), and the specific fuel consumption *SFC* is obtained by Eq. (9).

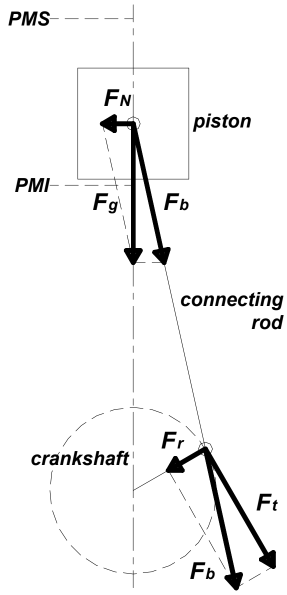

Figure 3. Diagram of forces to piston-connecting rod-crankshaft system.

The equations used in the non-ideal algebraic model for calculate the cylinder pressure and temperature had been obtained of basic equations of air standard dual cycle. However, some assumptions are made that allow to approach the ideal cycle to the actual cycle of the engine. The main one of them is the use of efficiencies in each process that compose the cycle, modifying the pressure *p(q)* and temperature *T(q)* conditions as function of the crank angle. The others assumptions are the following: variation of the specific heat $c_p(q)$, injection advance q_{inj}, ignition delay q_{delay}, angular duration of the combustion (heat supplied) $q_{(p+d)}$, and opening of exhaustion valve q_{VEx}. The stages that compose the motor cycle will be presented following the same numeration on *pV* diagram showed in Fig. 2.

Intake (0-1): the value of the volumetric efficiency is represented by the fall of cylinder pressure during the intake, shows by Eq. (24). The objective of introducing volumetric efficiency is to correct the pressure losses (irreversibility) arising from real flows like the one through the valves. The value of volumetric efficiency for an aspirated engine is always lesser that the unit, but for a turbocharged engine can be equal or bigger that the unit. The pressure of air *p(q)* inside of the cylinder varies as function of atmospheric pressure p_0, the volumetric efficiency (n_V) and the cylinder volume $V(q)$, showed in Eq. (10). The intake process was considered isothermal with $T(q)$ equal to the ambient temperature, described in Eq. (11).

$$h_V = \frac{p_1}{p_0} \tag{24}$$

Compression (1-2): in this process, two modifications are made for calculate the pressure and temperature. The first one considers the irreversibility generated for the viscous friction and the heat transfer, called compression efficiency (n_C), and the second considers the variation of the specific heats ratio $k(q)$. The specific heat c_p and the ratio k are calculated through Eq. (6) and (7) respectively. The compression efficiency corrects the value of k during all the process leading in consideration the irreversibility summarized above. The pressure $p(q)$ is calculated as function of pressure in the end of the compression (p_1) and the volumes ratio, as it shows Eq. (12). In similar way, Eq. (13) describes the calculation of temperature $T(q)$ during the compression.

Combustion (2-3-4): in this model, the energy (or heat) release rate is calculated through the function of Wiebe for compression ignition engine. The energy release corresponds to the period of burning of the fuel inside of the cylinder and can be divided in two stages: premixed burning ($_p$) and diffusive burning ($_d$). The ignition delay q_{delay} was considered and it is defined as the interval between the beginning of the injection (q_{inj}) and the beginning of the burning of the fuel. Equation (14) was used to calculate the ignition delay, described for Ramos (1989), that depends on the pressure and temperature measured during the ignition delay (p_m and T_m), the rotation (N_{rpm}), the fuel cetane number (NC), and the injection angle (q_{inj}). Equation (15) was used to calculate the energy release rates through of two Wiebe functions that describe the process of the premixed and diffusive combustion. This equation depends on the amount of heat release (Q_p and Q_d), the shape factors (m_p and m_d), and the durations of each stage of the combustion process (q_p and q_d). The amount of energy added also depends on combustion efficiency (n_{Comb}) that considers, mainly, the heat transfer for the walls. The calculation of pressure $p(q)$ and temperature $T(q)$, Eq. (16) and (17) respectively, depend on the energy release rate (dQ/dq), the specific heat $c_p(q)$, and the duration of the combustion (q_{Comb}). The expansion of the working fluid during the combustion was considered due to variation of the cylinder volume during the process.

Expansion (4-5): the expansion efficiency (n_E) considers the irreversibility generated within the cylinder, mainly the heat transfer. Similar the compression, it considers the variation of the specific heats ratio, and the value of efficiency corrects the value of k during the expansion process. According to Barros (2003), the value of n_E varies of 0.85 to 0.94 for Otto cycle engines. According to Heywood (1988), the value of n_E is superior in the Diesel engines due to bigger relation air-fuel (lean mixture). Both the modifications change the values of pressure $p(q)$ and temperature $T(q)$ described in Eq. (18) and (19) respectively. The expansion process continues until the opening of exhaustion valve (q_{VEx}).

Blowdown exhaustion (5-6): in the opening of exhaustion valve, part of mass of the combustion gases contained in the cylinder escapes quickly for the exhaustion manifold. The fall of pressure inside of the cylinder in the exhaust valve opening is called of blowdown exhaustion. The blowdown efficiency (n_{BD}) informs which the cylinder pressure in the end of blowdown (p_6) in relation to the atmospheric pressure (p_0), as shows Eq. (25). This fall of pressure depends on the amount of mass that is eliminated of the cylinder. Then, the blowdown retention efficiency (n_{RBD}) relates the amount of gas inside the cylinder in the end of blowdown (m_6) to the amount of gas at exhaust valve opening (m_{VEx}), as shown in Eq. (26). In summary, the blowdown efficiencies are influenced by irreversibilities occurring in the exhaust system. The pressure $p(q)$ and the temperature $T(q)$ are calculated by Eq. (20) and (21) respectively.

$$h_{BD} = \frac{p_0}{p_6} \tag{25}$$

$$h_{RBD} = \frac{m_6}{m_{VEx}} \tag{26}$$

Exhaustion (6-0): in this process, the remaining combustion gases in the cylinder are eliminated for the exhaustion manifold. The exhaustion efficiency (n_{Ex}) considers the energy losses during the process as the gases flow resistance through the exhaustion valves. The pressure $p(q)$ varies with the pressure in the end of blowdown (p_6), the efficiency n_{Ex} and the cylinder volume $V(q)$, described in Eq. (22). The temperature during the process remained constant and equal to the temperature in the end of the blowdown exhaustion (T_6), described in Eq. (23).

If all the efficiencies had been equaled the unit and if the ignition delay and energy release equations will be substituted by the heat addition at constant volume and constant pressure equations, the results obtained by algebraic model will be equals of dual cycle.

5. Results and discussion

As mentioned in item 2, the objective of this model is to serve of tool for the thermodynamic study of non-ideal compression ignition cycle. The results presented are only comparative, showing the capacity of the model to modify the conditions of pressure and temperature of ideal cycle to approach it of the actual operating of the Diesel engine. The results presented in figures 4 to 7 and Tab. 3 and 4 are for the same engine, with the same operating conditions.

Figure 4 shows a comparison of *pV* diagram of dual ideal cycle and non-ideal cycle obtained by model, indicating the reduction in the specific work of engine generated for the irreversibility of each stage of cycle. Figures 5 and 6 show a comparison of cylinder pressure and temperature diagrams, as function of the crank angle, for ideal cycle and non-ideal cycle. The variations of pressure and temperature as function of each efficiency can be studied through these diagrams. Figure 7 shows the heat release rate diagram as function of the crank angle for the ideal cycle and the non-ideal cycle obtained by algebraic model. The amount of the heat release was the same for the two cycles, however in the ideal cycle the amount of the heat release was divided in the constant volume process and the constant pressure process. The process characteristics of Diesel engine combustion can be analyzed through this diagram.

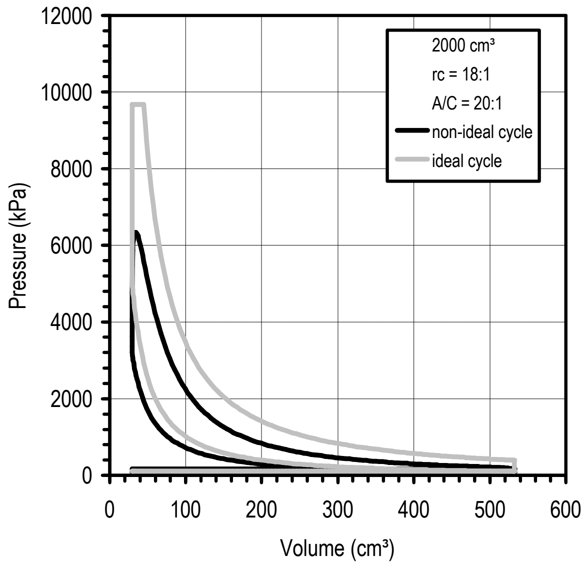

Figure 4. *pV* Diagram showing comparison of dual cycle and non-ideal cycle.

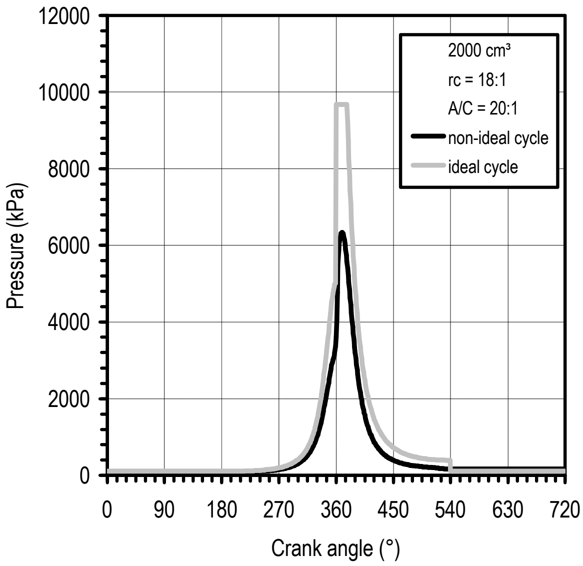

Figure 5. Cylinder pressure diagram of dual cycle and non-ideal cycle.

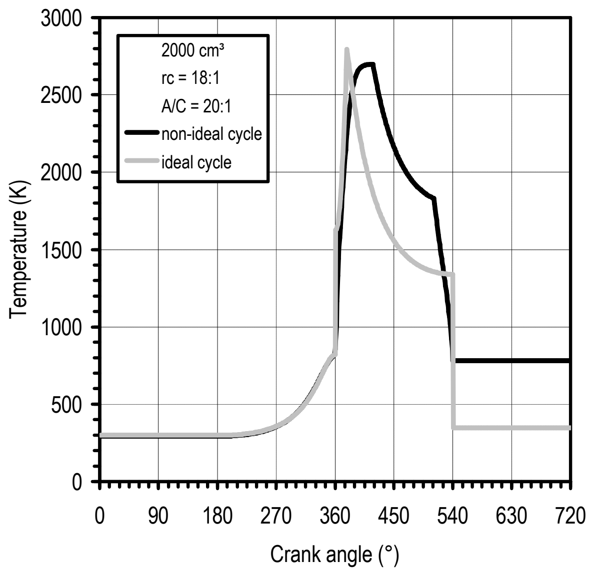

Figure 6. Cylinder temperature diagram of dual cycle and non-ideal cycle.

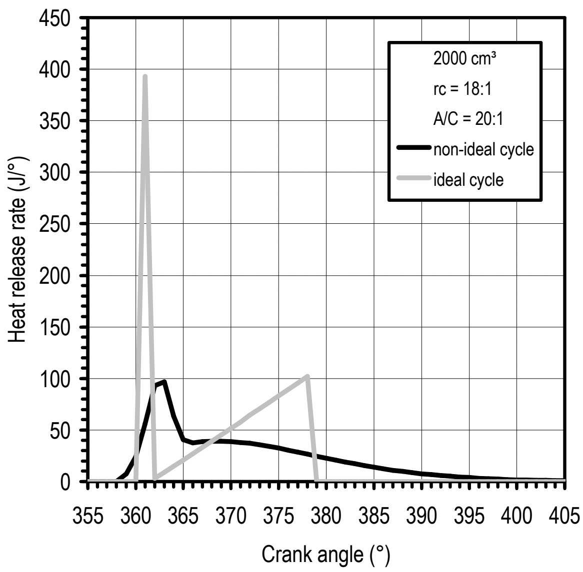

Figure 7. Comparison of the heat release rate of dual cycle and non-ideal cycle.

Table 3 and 4 brings a summary of the influence of the efficiencies in the results of power and specific fuel consumption in relation to the ideal process obtained for non-ideal model. The use of efficiencies in each process allows quantifying the main energy losses of the cycle. The negative and positive values mean, respectively, reduction and increase in the power or specific fuel consumption in relation to the ideal process.

Table 3. Influence of efficiencies in engine power.

Efficiency	Change in the engine power (%)								
	0,80	**0,85**	**0,90**	**0,95**	**1,00**	**1,05**	**1,10**	**1,15**	**1,20**
n_V	-21,35	--	-10,58	--	0,00	--	10,49	--	20,98
n_C	-11,05	-11,09	-10,39	-2,83	0,00	--	--	--	--
n_{Comb}	-20,06	-16,64	-13,22	-3,57	0,00	--	--	--	--
n_E		-34,93	-25,62	-16,07	0,00	--	--	--	--
n_{BD}	-2,19	-1,54	-0,97	-0,46	0,00	--	--	--	--
n_{Ex}	--	--	--	--	0,00	-0,07	-0,14	-0,21	-0,28

Table 4. Influence of efficiencies in specific fuel consumption.

Efficiency	Change in the specific fuel consumption (%)								
	0,80	**0,85**	**0,90**	**0,95**	**1,00**	**1,05**	**1,10**	**1,15**	**1,20**
n_V	1,72	--	0,65	--	0,00	--	-0,44	--	-0,81
n_C	12,42	12,47	11,60	2,91	0,00	--	--	--	--
n_{Comb}	25,10	19,96	15,23	3,70	0,00	--	--	--	--
n_E	--	53,68	34,44	19,15	0,00	--	--	--	--
n_{BD}	2,24	1,57	0,98	0,46	0,00	--	--	--	--
n_{Ex}	--	--	--	--	0,00	0,07	0,14	0,21	0,28

6. Conclusions

The non-ideal cycle model is a useful tool for the thermodynamic study of the compression ignition engines. This model allows to investigate the variations of gas pressure and temperature inside the cylinder on each process of cycle and to verify its influence in the engine performance parameters.

The use of efficiencies diminishes the number of equations of the model in comparison with zero or quasi dimensional models, reducing its computational load and operation time. The use of expressions for the calculation of the ignition delay and energy release rates makes possible the detailed analysis of the combustion process in Diesel engines.

The results presented demonstrate the capacity of model to modify the ideal cycle, transforming it into a cycle that approaches the actual operating of engine. An analysis of influence of efficiencies on results of power and specific fuel consumption was done, and some observations were obtained:

(i) the volumetric efficiency has a strong influence in the values of power and specific fuel consumption due the amount of fuel is dependent of admitted air mass;
(ii) the compression efficiency modifies the values of pressure and temperature in the end of compression, modifying the results of power and specific fuel consumption;
(iii) the combustion efficiency controls the amount of energy release for the work done. It determines the conditions of pressure and temperature in the combustion chamber, and influencing in the power and specific fuel consumption results;
(iv) the expansion efficiency modifies the values of pressure and temperature in the end of expansion, and controlling the of power and specific fuel consumption results;
(v) the blowdown exhaustion and exhaustion efficiencies little affect the results of power and specific fuel consumption, but they modify the values of pressure and temperature during the process that are important in the calculation of the pumping work and the use of turbo-compressors.

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8. Responsibility notice

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