A PLANAR DOUBLET-LATTICE CODE FOR TEACHING AND RESEARCH IN AEROELASTICITY

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Abstract. *In the time domain the physical phenomena related to aeroelasticity are easier to understand, but aeroelastic stability analysis for practical purposes are mostly performed, today, in the frequency domain. The present paper discusses an implementation of the doublet-lattice method (DLM) for computing aerodynamic unsteady loads over planar surfaces in the frequency domain. Its results (obtained using the KE and PK solvers of the nonlinear eigenvalue problem associated with the aeroelastic stability) are initially compared with flutter speeds and frequencies observed in experiments. Then, for a wing performing an harmonic motion in plunge, the unsteady aerodynamic loads predicted by this code are compared with those predicted by a CFD code solving (in the time domain) the two dimensional Euler equations. The comparison of the DLM with the CFD suggests a discussion of the differences between the time and the frequency domain formulations of the inviscid unsteady aerodynamic problem. The addional simplifying hypothesis adopted when one goes from the more general potential flow theory (used in the panel methods) to the lifting surface theory on which the DLM is based is also briefly reviewed. The differences in the implementation and between the results of the KE and PK solvers are commented also.*

Keywords*: Aeroelasticity, Unsteady Aerodynamics, Flutter*

1. Introduction

Nowadays there is a large number of numerical methods that can be used for aerodynamics calculation. Computer intensive methods, like finite element, finite volume, lattice-gas and direct Monte Carlo simulation, that could be grouped under the acronym CFD[1], have received a good deal of attention in recent decades, due to the steadily growing availability (at a quite reasonable cost) of powerful computational resources almost everywere in the world. But simpler methods used in preliminary design or in tasks that demand a high number of simulation cases and not too great an accuracy (since the available input data to feed in it are not very accurate), are yet still important. A high number of simulations based on highly simplified structural dynamics models (adjusted to fit ground vibration test results) are frequently demanded in aeroelastic analyses. For this reason, in aeroelasticity, models based on the linearized potential flow theory are those most often used.

The methods based on the linearized potential flow theory can be separated into two categories: (i) panel methods that actually are a sort of boundary element methods and can represent accurately the external geometry of a wing; (ii) lifting surface methods, that can model approximately the plan form, camber and dihedral, but not the thickness of the wing, (the tip and root of the wing must be modeled as being parallel to the flow direction in these methods, due to the wake formed behind the discrete elements used). Lifting surface methods are based on a linear partial differential equation with *linear boundary conditions*, (this boundary conditions type is the most important consequence of the differences between them and the panel methods), that can be easily transformed to the frequency domain. In the

[1] Computational Fluid Dynamics is not really a precisely delimited discipline: it could be understood in a more general sense than that used here, encompassing even the computer assisted experimentation in fluid dynamics.

frequency domain aeroelastic stability analysis (that is the major field of application of computer codes like that discussed in the present work) is greatly simplified and so this is where it is mostly performed.

The doublet-lattice method, of Albano and Rodden (1969) is a lifting surface method developed in the frequency domain, that was incorporated into the NASTRAN and since then become a standard for airplanes aeroelastic stability analyses in the subsonic regime. The implementation of a very simplified version of this method is presented here. In the remainder of the present paper some basic aspects of the doublet-lattice method for a single plane lifting surface and of the PK and KE flutter prediction methods are reviewed and the results of their implementation are compared with experimental results and with the results of a two-dimensional CFD code. The simple implementation discussed here may not be used for a real aircraft analysis, but it is very appropriate, due exactly to its simplicity, to teach how methods like it are implemented and also as a test bed for correction methods (Silva, 2004) used to better the results for the transonic regime in methodologies based on linear potential flow theory.

An interesting aspect of the standard aerodynamic theories used for aeroelastic stability analyses is the absence of boundary layer modeling. There are at least three good reasons for this: (i) lift forces are usually (at least for non-stalled wings) much more important than drag for structural deformation calculations; (ii) over quickly moving boundaries the usual boundary layer models are not as reliable as they are for steady flows and (iii) the transformation of a boundary layer model to the frequency domain will generally produce senseless results.

2. The planar doublet-lattice method

A good starting point for a quick presentation of the doublet-lattice method is the linearized equation modeling the nonsteady potential flow.

$$(1-M_\infty^2)\phi_{xx}+\phi_{yy}+\phi_{zz}-2(M_\infty^2/U_\infty)\phi_{xt}-(M_\infty^2/U_\infty)\phi_{tt}=0 \tag{1}$$

where $\phi(x,y,z,t)$ is the disturbance velocity potential, such that the velocity at any point is given by $\vec{u}=U_\infty\hat{i}+\nabla\phi$, being $\hat{i}$ the unitary vector along the free-stream velocity direction x, U_∞ the modulus of the free stream velocity and M_∞ the free stream Mach number. This equation is not presented in most textbooks dealing with aerodynamics, since transient analytic models for compressible flows are generally too highly elaborated to be of use at an introductory level, but it is derived, from the Euler equations, in textbooks in aeroelasticity like Dowell *et al.* (1994) and in more specialized references, such as the very good monograph written by Blair (1994). For thin wings, approximately represented by an horizontal plane z = 0 (this means that its deviations from the plane geometry may be considered small disturbances), the impenetrability boundary condition is expressed as

$$\phi_z(x,y,0,t)=h_t+U_\infty h_x \tag{2}$$

where $h(x,y,t)$ is the disturbance function describing the deviations in the z position of the wing middle surface from the position of its horizontal plane of reference. The free-stream boundary conditions may be written as $\phi(\pm\infty,\pm\infty,\pm\infty,t)=0$, meaning that the velocity disturbances must disappear at points far apart from the wing.

The transformation of equations (1) and (2) to the frequency domain for subsonic flows may be understood assuming that the disturbance potential function can be written as a superposition of harmonic functions like

$$\phi=\tilde{\phi}\exp(i\omega t) \tag{3}$$

where $i=\sqrt{-1}$ and ω is an angular frequency. Substituting (3) in (1) and (2) one obtains

$$(1-M_\infty^2)\tilde{\phi}_{xx}+\tilde{\phi}_{yy}+\tilde{\phi}_{zz}-2ikM_\infty^2\tilde{\phi}_x+k^2M_\infty^2\tilde{\phi}=0 \tag{4}$$

$$\tilde{w}=ik\tilde{h}/b+\tilde{h}_x \tag{5}$$

where $k = \omega b/U_\infty$ is the reduced frequency, b is a reference length (usually half of the mean aerodynamic chord of the wing under analysis) and $\tilde{w}=\tilde{\phi}_z(x,y,0)/U_\infty$ is the nondimensional upwash amplitude over the wing. To find a solution satisfying Eqs. (4) and (5) (this last along the wing surface), and also the far-field boundary conditions, is not a trivial task, but it can be done by a superposition of appropriate elementary solutions of Eq. (4) having their amplitude constrained to satisfy Eq. (5). The appropriate elementary solutions are those whose amplitude can be put in terms of the distribution of pressure coefficient difference between the lower and the upper sides of the wing, $\Delta\tilde{C}_p(\xi,\eta)$. The form of these elementary solutions, and its derivation from the acoustic potential equation fundamental solution, can be found in references like Dowell *et al.* (1994) and Blair (1994). Here the elementary solutions, (differentiated in relation to the vertical direction *z*), will be represented just by the kernel function $\tilde{K}(x-\xi, y-\eta, k, M_\infty)$ of the integral equation resulting from their superposition (Küssner, 1940)

$$\tilde{w}(x,y) = \iint_{wing} \Delta\tilde{C}_p(\xi,\eta)\,\tilde{K}(x-\xi, y-\eta, k, M_\infty)\,d\xi\,d\eta \tag{6}$$

If the kernel function and the amplitude of the displacements $\tilde{h}(x,y)$ of the wing in relation to the z = 0 plane are known, Eq. (6) (substituting Eq. (5) in its left hand side) may be solved for the pressure coefficients distribution. The doublet-lattice method is a way of discretizing Eq. (6), and some authors, like Bismarck-Nasr (1999) begin their presentation of the method with this equation (that seems to be, for the present authors, a very advanced start).

There is no room here for the details of the method, in some of its alternative versions, as they are presented by Bismarck-Nasr (1999). There would not be room here even for the simpler presentation of it given by Blair (1994). Thus, only general aspects of the method will be mentioned in this paragraph. The wing is subdivided in quadrilaterals, having two of its edges parallel to the flow direction. A unique value of the amplitude of pressure coefficient difference, $\Delta\tilde{C}_p(\xi,\eta)$, is associated with each of these quadrilaterals (that are usually called panels, but it should be remembered that they represent a piece of a wing without thickness, and not a piece of the surface of a thick wing as do the panels in a classical panel method). The kernel function is approximated by a quadratic function along the 1/4 chord line of each quadrilateral and then integrated along this line. The integrals, calculated for each quadrilateral are summed over all quadrilaterals, and so an approximate value of the integral shown in Eq. (6) is obtained. This procedure is used for estimating the nondimensional upwash amplitudes at the 3/4 chord line of each quadrilateral, and the resulting relation between these upwashes and the pressure coefficient differences can be written as

$$\{\tilde{w}\} = [\tilde{B}]\{\Delta\tilde{C}_p\} \tag{7}$$

Equation (7) can be inverted for calculation of the loads due to a sinusoidal oscillating upwash, giving

$$\{\Delta\tilde{C}_p\} = [\tilde{A}]\{\tilde{w}\} \tag{8}$$

where $[\tilde{A}] = [\tilde{B}]^{-1}$ is known as the aerodynamic influence coefficient matrix.

3. Flutter prediction methods

The loads resulting from Eq. (8) can be introduced into the wing structural dynamics modal representation, giving

$$(-\omega^2[\tilde{M}] + i\omega[\tilde{C}] + [\tilde{K}])\{\tilde{X}\} = \frac{1}{2}\rho U_\infty^2 [G]^T [S][\tilde{A}][D_R + ikD_I]\{\tilde{X}\} \tag{9}$$

where, $[\tilde{M}]$ is the diagonal modal mass matrix, $[\tilde{C}]$ is the modal viscous damping matrix (that in general may be nondiagonal, but is absent in the implementation here presented), $[\tilde{K}]$ is the diagonal modal stiffness matrix, $\{\tilde{X}\}$ is a vector of modal participation coefficients, ρ is the specific mass of the fluid, $[G]$ is the spline matrix (Harder and Desmarais, 1972), $[S]$ is the integration matrix (that contains the values of the surface area of each quadrilateral used in the wing discretization, that is multiplied by the half of the quadrilateral mean chord if it corresponds to a panel rotational degree of freedom), $[D_R]$ is a matrix that contains the amplitude of the inclinations $\tilde{h}_x$ associated with each panel for each mode, and $[D_I]$ contains just the dislocations $\tilde{h}$ associated with each panel for each mode. To make the equations shorter, the modal aerodynamic coefficient matrix is defined as $[\tilde{Q}] = [G]^T [S][\tilde{A}][D_R + ikD_I]$.

To evaluate the stability of the system represented by Eq. (9) it is basically necessary to allow that the frequency ω assumes complex values. So, when the imaginary part of the frequency becomes negative, the system becomes unstable. But the fact that $[\tilde{Q}]$ is a complex matrix dependent on the k (and so on the ω) value make the stability analysis a little difficult: it means, in first place, that the eigenvalues problem to be solved is nonlinear, moreover there is no rigorous mathematical support for the calculation of $[\tilde{Q}]$ based on complex k (or ω) values. So, some methods were developed for solving the nonlinear eigenvalues problem associated with aeroelastic stability. Two of them were implemented in the development of the present work, following closely the work of Rodden and Johnson (1994), and are discussed below.

3.1 The KE method

The KE method is the easiest to implement, but, except at the flutter points, its results can hardly be interpreted physically. It relies on the Eq. (9) rewritten as

$$\left[-\left([\tilde{M}]+\frac{1}{2}\rho\frac{b^2}{k^2}[\tilde{Q}(k,M_\infty)]\right)\frac{\omega^2}{1+ih}+[\tilde{K}]\right]\{\tilde{X}\}=0 \tag{10}$$

where the letter h is used for an artificial hysteretic damping coefficient. Equation (10) was obtained neglecting the structural viscous damping matrix $[\tilde{C}]$ and assuming that h is small, and so $1+ih=1$.

In the KE method the generalized complex (but linear) eigenvalues problem represented by Eq. (10) is solved for a given list of reduced frequencies k. The flow velocity associated with each eigenvalue is obtained from the reduced frequency definition $k = \omega b/U_\infty$.

3.2 The PK method

The PK method version implemented in the present work is based on Eq. (9) rewritten as

$$\left[[\tilde{M}]p^2+\left([\tilde{C}]-\frac{1}{2}\rho bU_\infty^2\,\mathrm{Im}([\tilde{Q}(k,M_\infty)])/k\right)p+\left([\tilde{K}]-\frac{1}{2}\rho U_\infty^2\,\mathrm{Re}([\tilde{Q}(k,M_\infty)])\right)\right]\{\tilde{X}\}=0 \tag{11}$$

where $p = \omega(g/2 + i)$ are the complex eigenvalues. If g is small, $p = i\omega$.

Equation (11) is rewritten in the state-space form as a standard eigenvalue problem $([A]-p[I])\{\tilde{X}\}=0$ where the real matrix $[A]$ is given by

$$[A]=\begin{bmatrix} [0] & [I] \\ -[\tilde{M}]^{-1}\left([\tilde{K}]-\frac{1}{2}\rho U_\infty^2\,\mathrm{Re}([\tilde{Q}(k,M_\infty)])\right) & -[\tilde{M}]^{-1}\left([\tilde{C}]-\frac{1}{2}\rho bU_\infty^2\,\mathrm{Im}([\tilde{Q}(k,M_\infty)])/k\right)\end{bmatrix} \tag{12}$$

It is interesting to note that since the modal mass matrix $[\tilde{M}]$ is diagonal, its inversion poses no difficulty. In the PK method, the eigenvalues of the matrix $[A]$ represented in Eq. (12) are calculated for a list of velocities. But this eigenvalue problem is nonlinear, as $[A]$ depends on the k value (that is obtained from a calculated eigenvalue and a given velocity. To deal with this nonlinearity an iterative loop, (where a k value matching the given velocity and the frequency calculated from one of the eigenvalues of $[A]$ is searched for), must be executed for each given velocity and each vibration mode (each mode is associated with two of the eigenvalues of $[A]$).

4. Flutter results

The flutter calculations shown here were performed for the AGARD wing 445.6 weakened # 3, described by Yates Jr. (1988). This is a wing model with a root chord of 0.558 m, an aspect ratio of 3.29, a taper ratio of 0.66, 45° of sweepback at the 1/4 chord and the NACA 65A004 profile. The first 5 natural frequencies were experimentally determined and the normal modes of vibration corresponding to them, given in the Yates Jr. (1988) report, were used in the flutter calculations presented in this section. The experimental results, in terms of flutter velocities are compared with the results of the present doublet-lattice method (DLM) implementation in Table 1.

Table 1. Flutter speeds and frequencies for AGARD wing 445.6 (weakened # 3).

Experimental					DLM	
M_∞	Reynolds	ρ [kg/m^3]	V_F [m/s]	f_F [Hz]	V_F [m/s]	f_F [Hz]
0.678	1.410×10^6	0.2082	231	18.0	238	19.9
0.901	0.911×10^6	0.0995	297	16.1	304	16.8
0.960	0.627×10^6	0.0634	309	13.9	332	14.4

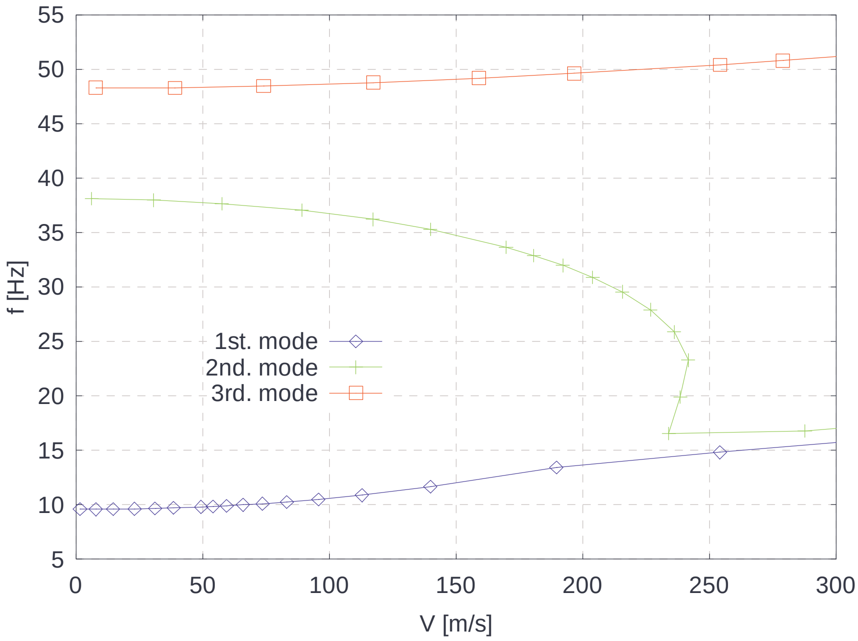

Figure 1. Frequencies f corresponding to the first three modes of vibration, calculated using the KE method, for different flow velocities

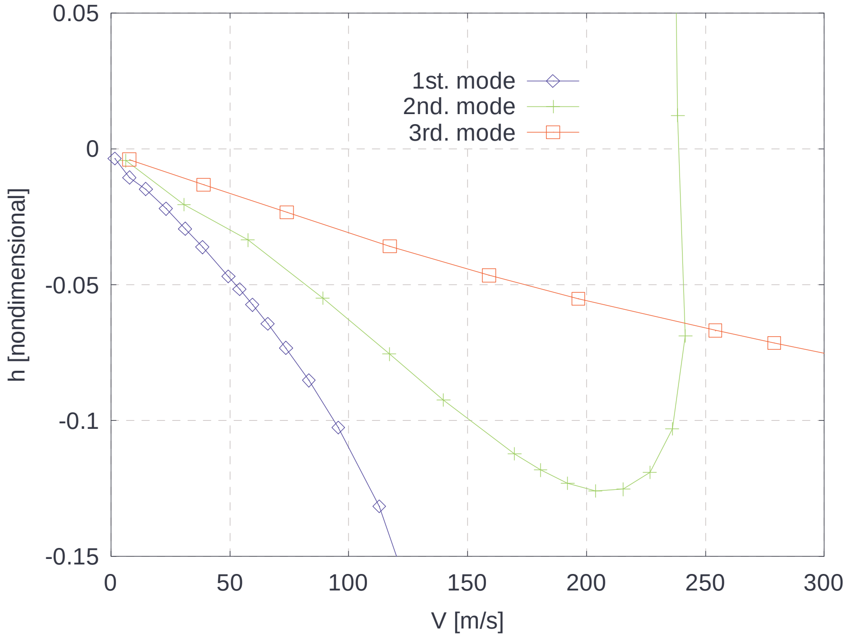

Figure 2. Hysteretic damping coefficients h corresponding to the first three modes of vibration, calculated using the KE method, for different flow velocities

Figures 1 to 4 show the estimates obtained for the frequencies, and for the damping needed for stabilizing each mode of vibration of the aeroelastic system (wing+airflow) provided by the KE and the PK methods for, $M_\infty = 0.678$ and $\rho = 0.2082$ kg/m^3.

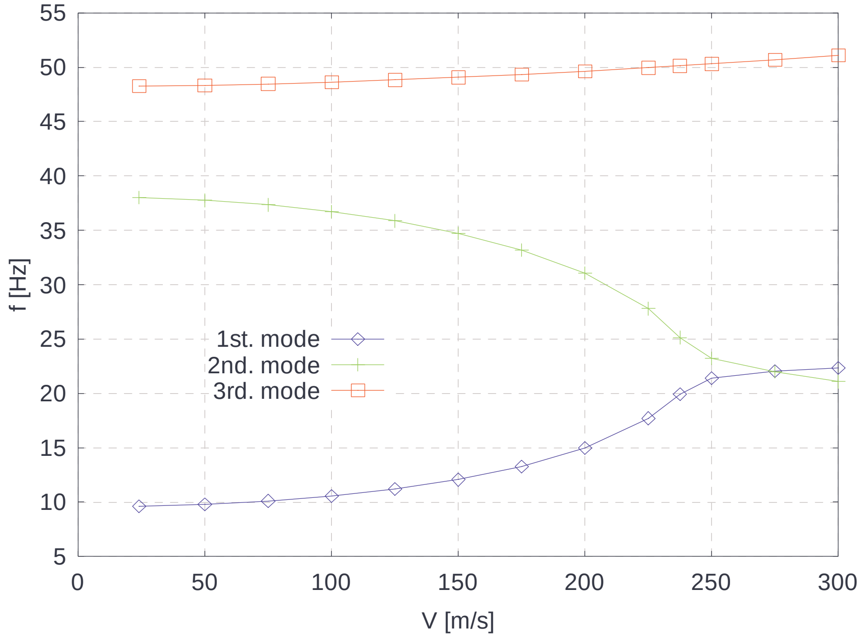

Figure 3. Frequencies *f* corresponding to the first three modes of vibration, calculated using the KE method, for different flow velocities

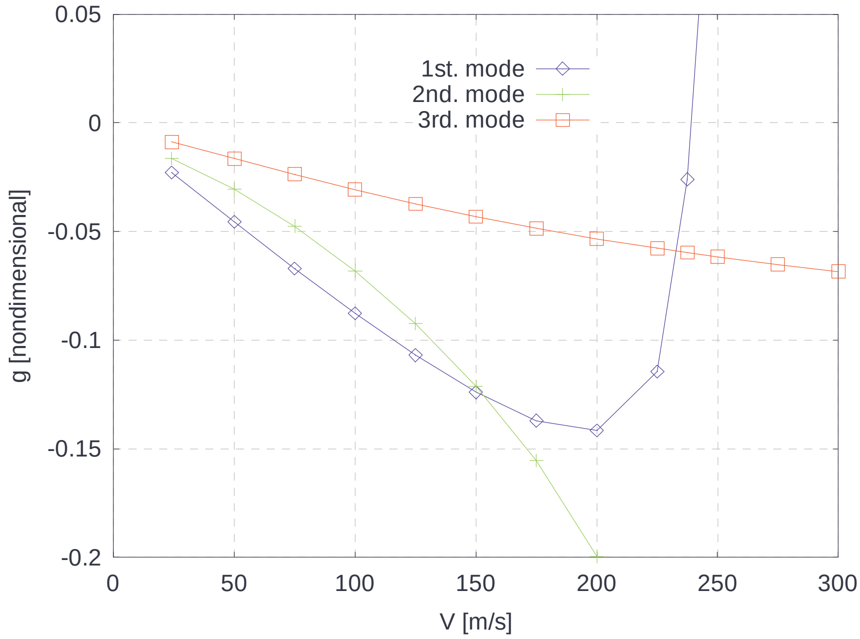

Figure 4. Viscous damping coefficients *g* corresponding to the first three modes of vibration, calculated using the PK method, for different flow velocities

Comparing Fig. 1 with Fig. 3 one can see large differences, being specially impressive the fact that for velocities near the flutter velocity the KE method predicts two different vibration frequencies for the second mode at the same velocity. Comparing Fig. 2 with Fig. 4 the differences are made evident by the fact that the KE method predicts flutter in the second mode, whereas the PK predicts it in the first. This does not means that different flutter modes are being predicted by the two methods. The mode shapes and frequencies of the second and first vibration modes are greatly modified by the aerodynamic effects, so much so that, at the flutter speed, what is called the second mode by the KE method has exactly the same mode shape and frequency of what the PK method calls the first mode at that speed. Notwithstanding the large differences between the frequencies and damping coefficients predicted by the two methods far from the flutter velocities, both predict the same flutter velocities (that corresponding to the point where the damping coefficient needed for stabilizing the system becomes zero) and the same flutter frequency (the frequency of vibration of the mode that became unstable, at the point where the instability begins). The flutter frequency may be difficult to identify in a pair of graphs like that shown in Figs. 1 and 2, because of the appearance of more than one frequency associated with the same velocity. To avoid confusion in such situations one may use reduced frequencies instead of velocities as abscissas in those graphs.

The PK method is more difficult to implement and is generally more expensive computationally than the KE method. But the estimates that it provides for vibrations frequencies and damping, far from the flutter speed, are much more realistic than those given by the KE method. At aeroelastic divergence the reduced frequency becomes zero and so the divergence velocity can be predicted by the KE method only by extrapolation from lower velocities, whereas in the PK method the aeroelastic divergence can be dealt with without special care. Finally, in the PK method, for each velocity the Mach number and the specific mass can be chosen according to the properties given in a standard atmosphere for some altitude. In the KE method, since velocities are not chosen *a priori*, this is not possible.

Some additional stability analyses performed with the code presented here are shown by Santos *et al.* (2005). It is also important to mention that the results of this code were compared with those provided by the aeroelastic module of the MSC/NASTRAN for reduced frequencies to which the aerodynamic matrices were calculated, and the observed differences were small compared to the round-off errors level. When the aerodynamic matrices have to be interpolated or extrapolated, as they are for all velocities in the PK method, and for the frequencies below and above the given frequency list in the KE method, the differences are not so small, because MSC/NASTRAN use splines to interpolate and extrapolate aerodynamic matrices whereas the implementation here describe uses only a two-point linear interpolation and extrapolation formula for this task.

5. Aerodynamic loads over a wing oscillating in plunge

An interesting test realized before the implementation of the flutter prediction methods was the calculation of aerodynamic loads over a straight wing of aspect ratio 20 oscillating in plunge (without rotation) at a reduced frequency k = 1, with 2/5 of its chord as amplitude, immersed in a Mach number 0.3 flow. The results obtained at the central section of the wing were compared with results of a two-dimensional CFD code (Castro, 2001) that solved the Euler equations around a NACA 0014 airfoil oscillating in the same conditions.

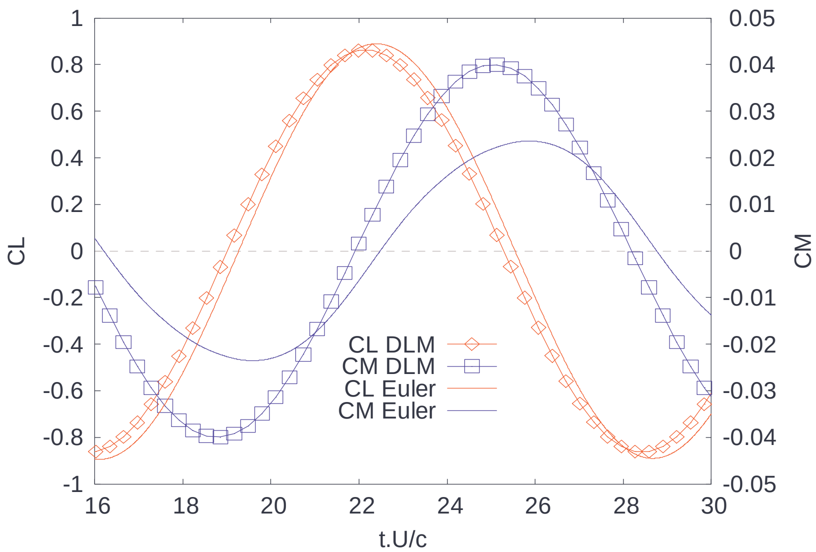

Figure 5. Lift (C_L) and pitch moment (C_M) coefficients for an oscillating wing

Considering that a thickness of 14% of the wing chord is not very small and that the plunge movement with an amplitude corresponding to 2/5 of this same chord is not a small disturbance also, it is not surprising that the differences between the linearized solution represented by the doublet-lattice method and the less simplified CFD solution of the Euler equations are quite visible in Figs. 5 and 6. The fact that the nonlinear effects are more visible in the moment coefficient C_M, than in the lift coefficient could be easily predicted also. More interesting, because not so widely known, is the negative drag (thrust) generated by the oscillating wing (the Knoller-Betz effect). The overestimation of the thrust by the linearized potential theory is in accordance with the literature (Tuncer and Platzer, 1996). These results, besides validating the present doublet-lattice implementation, call attention to the importance of the small disturbances hypothesis that, together with the inviscid flow hypothesis, displaces the doublet-lattice results from those of the complete Navier-Stokes equations.

It is interesting to note also that for calculating these sinusoidal motions (and this is the kind of movement observed in flutter) it is sufficient to determine and invert one matrix of aerodynamic influence coefficients, when a frequency domain method like the doublet-lattice is used. The CFD solution must evolve through thousands of time steps to reach an effectively periodic motion from which results like those shown in Figs. 5 and 6 could be extracted.

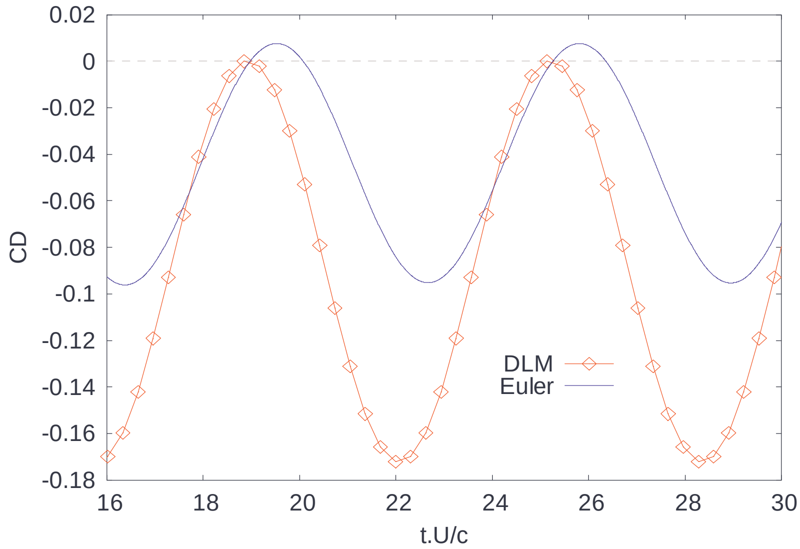

Figure 6. Drag coefficient, C_D for an oscillating wing

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