CONTINUOUS AND INTERMITTENT DRYING (TEMPERING) OF OBLATE SPHEROIDAL BODIES: MODELING AND SIMULATION

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Abstract. *In this work, modeling of continuous and intermittent drying of oblate spheroidal bodies is presented. The model considers the liquid diffusion as the only process of mass transfer inside the body, constant diffusion coefficient and equilibrium conditions at the surface of the solid. The diffusion equation is described in the oblate spheroidal coordinates system, for the two-dimensional case discretized using the finite volume method and fully implicit formulation. Based on the literature, experimental data of the properties and physical dimensions of lentil grains were used in the all simulation. Several processes of intermittent drying for the lentils were simulated: using a same time of tempering (time of rest), beginning in different drying time and for two or more intervals (time steps) along the process. For a same useful time of use of the dryer, it was observed that the intermittent drying always increases the efficiency of the process. Another outstanding result is the improvement of the final quality of the product due to the absence of great moisture content gradients inside of the product during the drying process.*

Keywords: *drying, simulation, mass, oblate spheroidal*

1. **Introduction**

High temperatures facilitate the drying of the lentils seeds; it may however also reduce seed germination. The effect of drying temperature and initial moisture content on lentil viability was investigated by Tang and Sokhansanj (1993). They observed no significant germination loss of lentils for air temperature up to 60 ^{o}C.

Several drying techniques have been proposed in order to rationalize the use of energy, as well as to reduce other problems during the drying process, as loss of germination power and vigor of the seeds. One of these techniques is the intermittent drying. Intermittent drying consists in a process that includes an active drying period, followed by a relaxation period and so on. In the relaxation period (tempering time), equalizing of the moisture content inside the solid takes place. The effect of the tempering time on the drying process is always to increase the moisture migration inside the solid. Knowledge of ideal tempering time is very important for to save of energy during the drying process and for preservation of the product quality.

Several researchers have studied intermittent drying. Lima and Nebra (2001) have reported a mathematical model to predict intermittent drying process in prolate spheroidal bodies. From the numerical results it was found that during the tempering process, the drying rate and the final mean moisture content are affected by the tempering Fourier number, multipass drying and geometrical dimensions of the body. Cinhan and Ece (2001) analyzed the effect of the tempering time on the drying kinetics of the rice through experimental data; the authors observed that the larger the tempering time the larger diffusion coefficient is obtained. Cnossen *et al.* (2003) have reported the effect of the tempering time in the appearance of fissures in the grain of rice. Prochayawarakorn *et al.* (2003) reports a drying process of corn in two stage with a tempering time among them. This alternative drying resulted in a faster process with less damaged grains. In this sense, the objective of this paper is to simulate and to discuss the intermittent drying process in lentils.

2. Mathematical modeling

2.1 Analytical procedure

In this work, the following assumptions were considered:

- The solid is homogeneous and isotropic;
- The drying process occurs under falling rate;
- The heat conduction inside the oblate spheroid in neglected;
- The field of moisture content is symmetric around z-axis and constant and uniform in the beginning of the process;
- The thermo-physical properties are constants;
- The process occurs under equilibrium condition at the surface of the solid.

The Fick's second law is given by:

$$\frac{\partial M}{\partial t} = \nabla \cdot \left(D \nabla M \right) \tag{1}$$

where M is the moisture content (d.b) and D is the mass diffusivity of the material.

To predict the diffusion phenomenon in oblate spheroids, it is necessary to write this equation in an appropriate coordinate system, e.g., in oblate spheroidal coordinates. The Figure 1 shows the body with oblate spheroidal geometry.

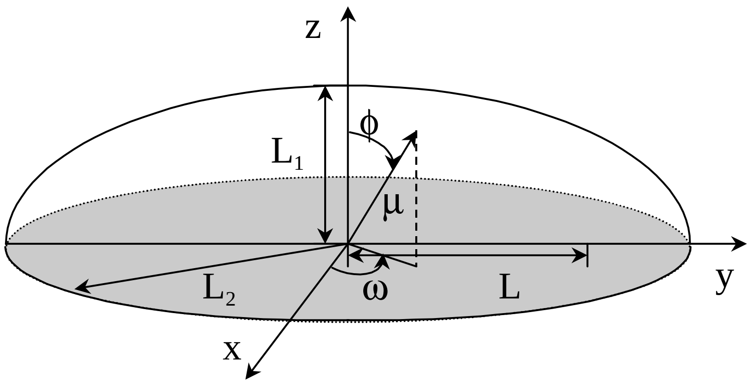

Figure 1. Characteristics of an oblate spheroid

The relationships between the Cartesian (x, y, z) and oblate spheroidal coordinate systems (μ, φ ,ω) are given by (Stratton *et al.*, 1941; Flammer, 1957; Abramowitz and Stegun, 1972):

$$x = L\sqrt{\left(1+\xi^2\right)\left(1-\eta^2\right)} \quad \zeta \; ; \qquad y = L\sqrt{\left(1+\xi^2\right)\left(1-\eta^2\right)} \quad \sqrt{1-\zeta^2} \; ; \; z = L\,\xi\,\eta \tag{2}$$

where L_1 and L_2 are the solids dimensions, $L = \left(L_2^2 - L_1^2\right)^{1/2}$, $\xi = sinh\mu$, $\eta = cos\phi$ *and* $\zeta = cos\omega$. The intervals of the variables ξ, η e ζ (in terms of ω) are: $0 \le \xi \le L_1/L$; $0 \le \eta \le 1$; $0 \le \omega \le 2\pi$.

By calculating the metric coefficients and the laplacian in the new coordinate system, and using the symmetry of the body around the z-axis, equation (1) can be written as follows:

$$\frac{\partial M}{\partial t} = \left[\frac{1}{L^2\left(\xi^2 + \eta^2\right)} \frac{\partial}{\partial \xi} \left(\left(\xi^2 + 1\right) D \frac{\partial M}{\partial \xi} \right) + \frac{1}{L^2\left(\xi^2 + \eta^2\right)} \frac{\partial}{\partial \eta} \left(\left(1 - \eta^2\right) D \frac{\partial M}{\partial \eta} \right) \right] \tag{3}$$

The following boundary conditions during continuous drying periods are used:

- Free surface: the moisture content at the surface is constant during of the process

$$M(\xi = L_1/L, \eta, t) = M_e \tag{4}$$

- Planes of symmetry: the angular and radial gradients of moisture content are equal to zero at the planes of symmetry.

$$\frac{\partial M(\xi, \eta = 1, t)}{\partial \eta} = 0 \qquad \frac{\partial M(\xi, \eta = 0, t)}{\partial \eta} = 0 \qquad \frac{\partial M(\xi = 0, \eta, t)}{\partial \xi} = 0 \tag{5}$$

- Initial conditions in the interior of the solid

$$M(\xi; \eta; 0) = M_o \tag{6}$$

The average moisture content of the body was calculated as follows (Whitaker, 1980):

$$\overline{M} = \frac{1}{V}\int_{v} M dV \tag{7}$$

During tempering, the moisture content profile is calculated along the time, assuming the surface of the solid to be impermeable. This last assumption requires the development of a new mathematical model to describe liquid diffusion inside the solid. Under these conditions, the change in the average moisture content of the product is negligible. Thus, the mathematical modeling that describes the tempering process is given by Equation (3) and the following boundary conditions:

$$\frac{\partial M\left(L_1 / L;\eta;t\right)}{\partial \xi} = 0 \qquad M(\xi;\ \eta;\ 0) = f(\xi;\ \eta) \tag{8}$$

The function $f(\xi;\ \eta)$ in the initial period of tempering corresponds to the moisture content profile in the solid at the end of the continuous drying period. The function $f(\xi;\eta)$ depends on of the radial and angular coordinates. Because of the impermeability condition, the drying rate in the solid must be equal to zero. In this case, the rate of moisture storage in the solid must be equal to the change in moisture content inside it, throughout tempering.

2.2. Numerical procedure

Due to the symmetry in the body, the computational domain showed in Figure 2 was adopted, where the nodal points (P, N, S, W, E) and lines of constant ξ and η are also presented.

It is possible to verify the existence of the symmetry in the plane that contains the points (x=0, y=0, z=0) and (x=0, y=0, z = L), in particular y≥0, z ≥0 and x=0.

The numerical solution of the problem utilizing the finite-volume method is obtained by integrating equation (3) over volume and time. For a fully implicit formulation and practice B, the discretized equation is given by (Maliska, 2004; Patankar, 1980):

$$A_P M_P = A_E M_E + A_W M_W + A_N M_N + A_S M_S + A_P^0 M_P^0 \tag{9}$$

The calculation started with the given initial condition and stopped when the following convergence criterions was satisfied at each point of the computational domain:

$$\left| M^{k-1} - M^{k} \right| \leq 10^{-8} \tag{10}$$

More details about numerical procedure can be found in Carmo and Lima (2001) and Carmo (2004).

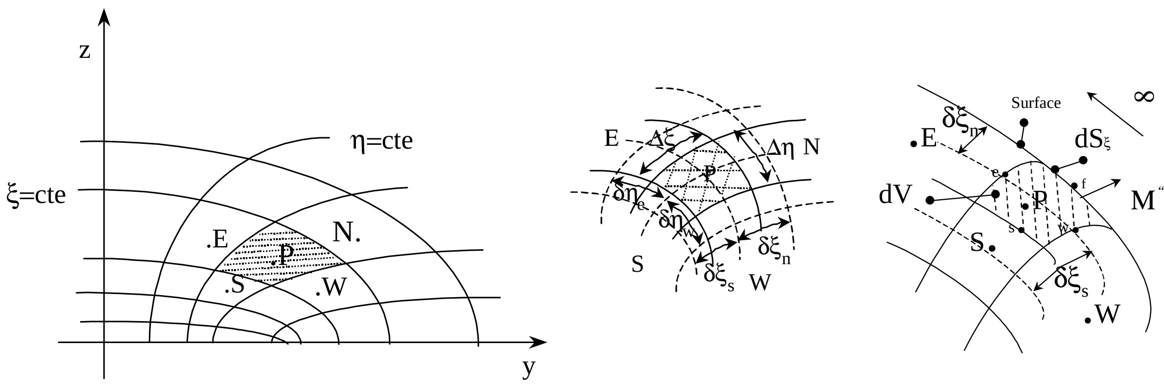

Figure 2. Schema of the control volume used in this work

As an application, this method was used to describe drying of the lentil grains. The following data were used (Carmo, 2004):

L_1 = 0.14 cm, L_2 = 0.34 cm (see in the Figure 1) and D = 6.62 × 10^{-12} m^2/s.

3. Results and discussions

Numerical solutions of the mass diffusion equation for an oblate spheroid solid has been determined using an uniform grid size of 20x20 points and Δt=20s. To validate the numerical methodology used in the present work, the numerical results obtained by the authors were compared with analytical results given by Haji-Sheikh (1986). This comparison can be encountered in Carmo and Lima (2001) and Carmo (2004).

Figure 3 shows the drying kinetics of lentil. The parameters of the drying process and the physical properties of the lentil are given in Carmo (2004). The figure 3 illustrates the comparison between continuous drying and intermittent drying process for 24 hours of drying time. The rest period is 30000 seconds and it begins when 25000 seconds of continuous drying are completed. After this rest period, the drying process always results in an increment in the drying rate. After 24 hours of drying process, the average moisture content of the solid is smaller when the process of intermittent drying is used. This is in agreement with Fioreze (1986) and Lima (1999).

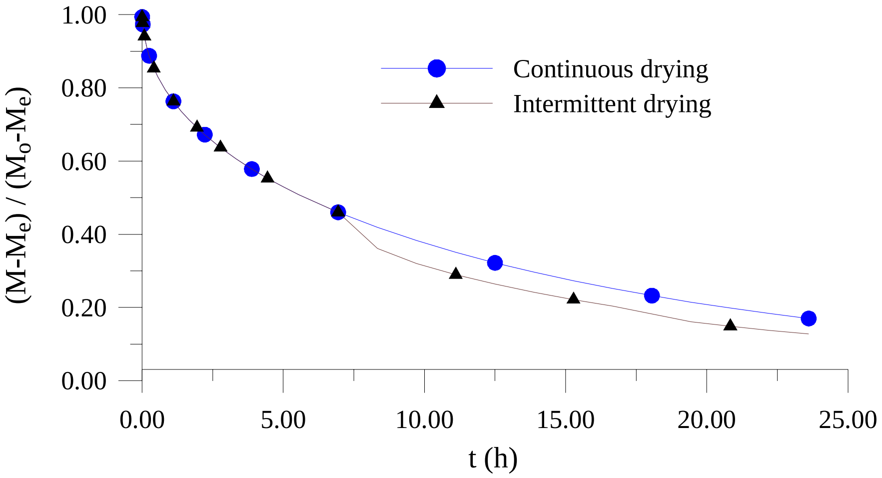

Figure 3. Comparison between the continuous and intermittent drying kinetics of lentil for a tempering time of 8.33h. The tempering period begin at 6.94 h.

Figure 4 displays the drying process of lentils in four different situations. The total drying time is 23.3h in all the cases. The continuous drying presents the smallest moisture content in the end of the process, however a larger useful time of operation of the dryer is used. In the other cases, a total tempering time of 30000s is considered, and the drying process is accomplished in 1, 2 or 3 steps. The drying accomplished only one step it presents the largest final moisture content. With two or three steps the curves present practically the same final moisture content, however, a smaller moisture content is gotten in the end of the process, in relation to that gotten with only one drying steps. The use of two passes of drying presented better results in energy efficiency of the dryer and in reducing the generation of internal stress in the dried material.

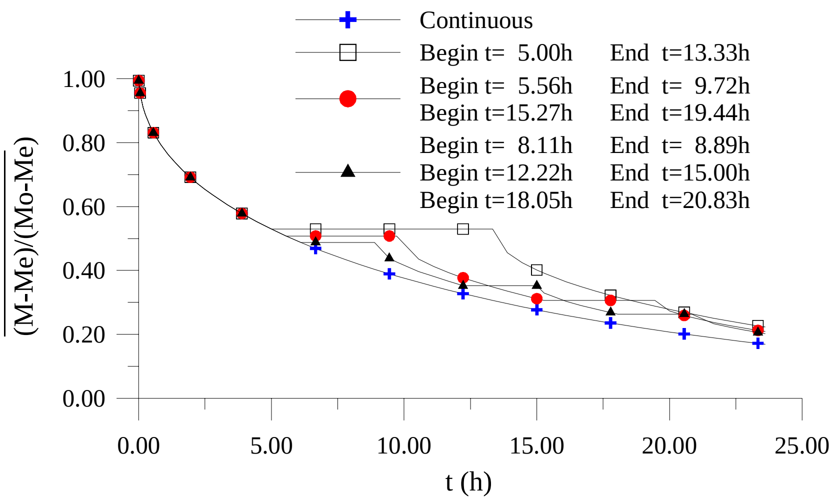

Figure 4. Drying kinetics of lentils (showing the tempering time)

Figure 5 illustrates the same cases presented in the Figure 4, where the drying kinetics is shown without the tempering intervals. It is noticed now that always after a rest interval in the drying process, a larger drying rate is obtained.

Figures 6, 7 and 8 show the moisture content distribution in the initial, intermediate and final times of the tempering process. We can see that with progress of time of rest, the moisture content inside the product becomes more uniform; to long time (total tempering time) the moisture inside the body will be totally uniform (Figure 9).

The solids, Figures 6, 7 and 8 present different moisture profiles but the same average moisture content. In this case, this moisture is that obtained in the beginning of the tempering process (t = 25000s) and its value is 0.4599. Figure 9 illustrates the moisture profile inside the solid after the time denominated "total tempering time". In this instant, the profile is practically uniform, with a difference of 2.9% between the largest and smaller value of local moisture content

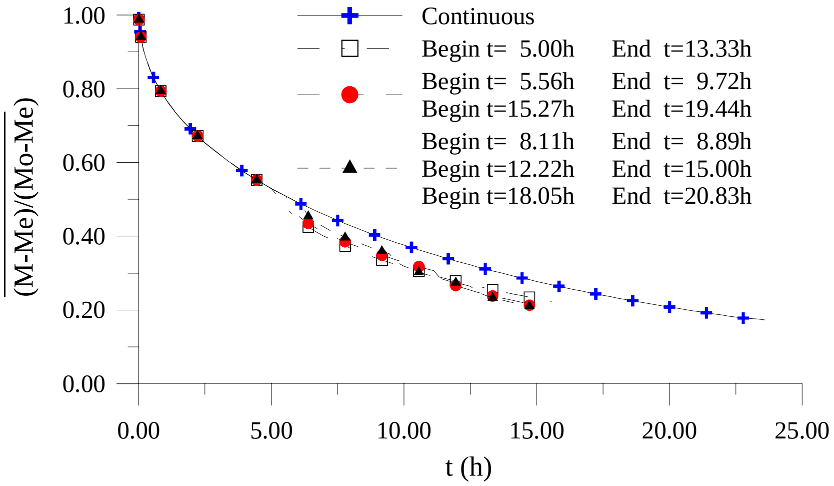

Figure 5 – Drying kinetics of lentils (omitting the tempering time)

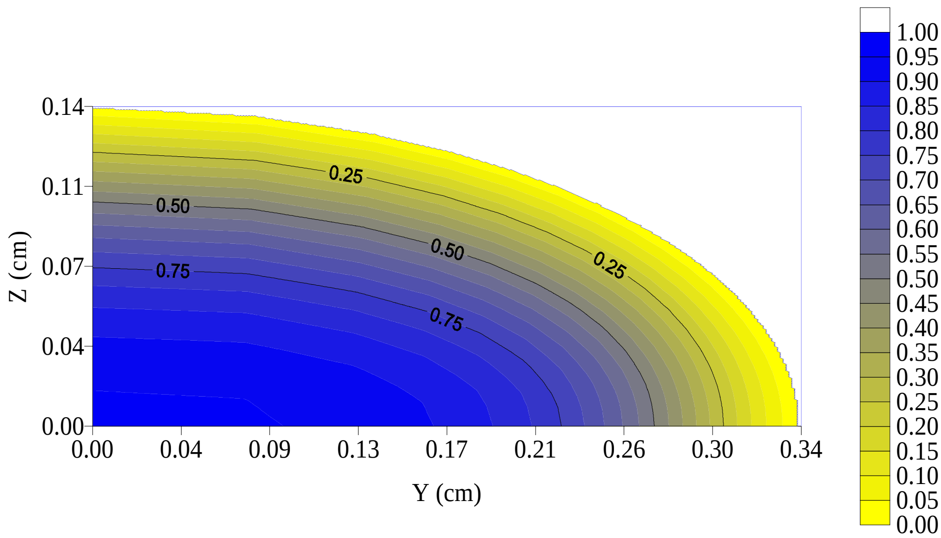

Figure 6. Dimensionless moisture content distribution inside the lentil at t = 6.94h (equivalent time to the beginning of the tempering process). One pass drying (Fig. 3)

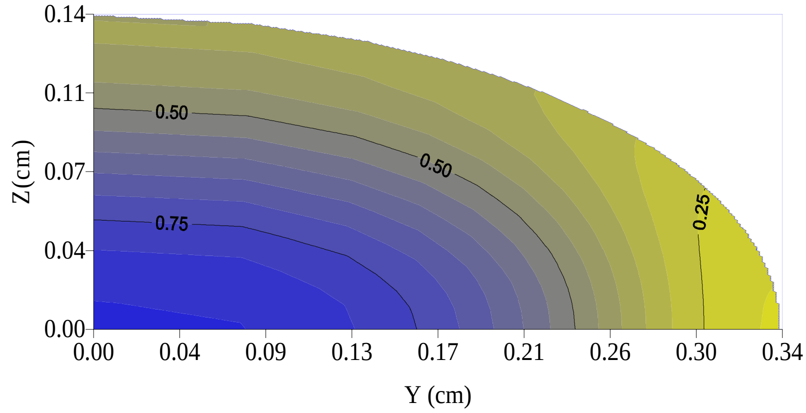

Figure 7 – Dimensionless moisture content distribution inside the lentil at t = 11.11h (equivalent time to the half tempering process). One pass drying (Fig. 3).

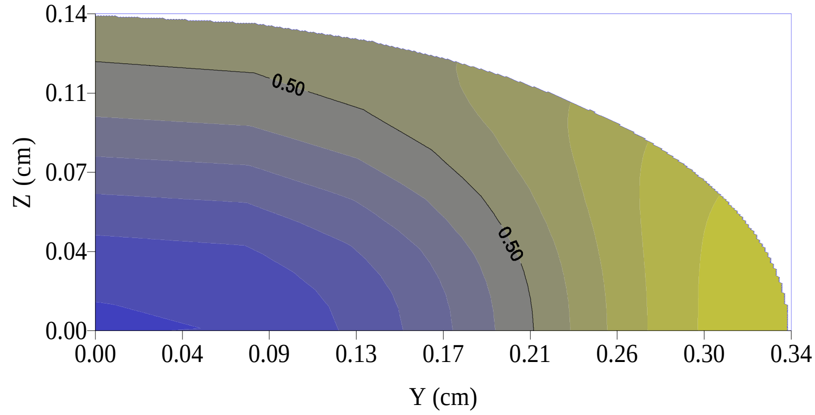

Figure 8 – Dimensionless moisture content distribution inside the lentil at t=15.28h (equivalent time to the end tempering process). One pass drying (Fig. 3).

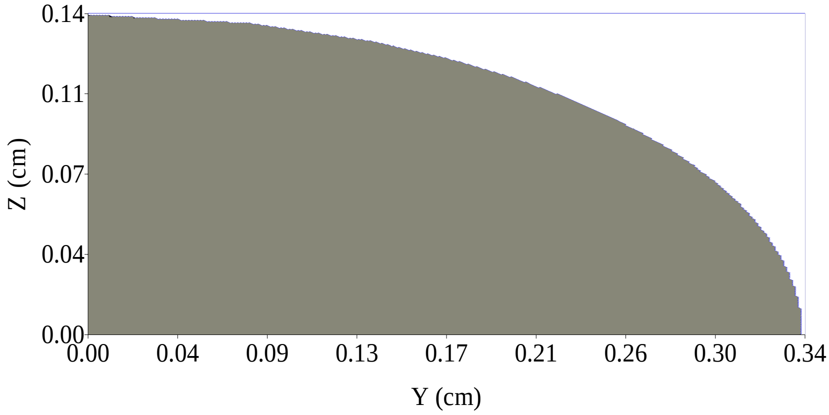

Figure 9 – Dimensionless moisture content distribution inside the lentil at t=111.11h (total tempering time). One pass drying (Fig. 3).

4. Conclusions

The analysis of the results permits us to conclude that the intermittent drying process is an important method to minimize energy in the drying process. In all cases studied, the drying rate after tempering was higher than its value in a continuous process. Numerical analysis using the finite-volume method can be used to simulate the tempering process: to determine a convenient tempering time and the moment which this process should be initiated. Among all the studied cases, the drying process where two tempering times it is used it presents smaller average moisture content in the end. Finally, the beginning and the period of tempering affect the drying rate of the product after tempering.

5. Acknowledgements

The authors would like to express theirs thanks to CAPES (Coordenação de Aperfeiçoamento de Pessoal de Nível Superior, Brazil) and CNPq (Conselho Nacional de Desenvolvimento Científico e Tecnológico, Brazil), for their financial support of this work

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