A MIXTURE THEORY DESCRIPTION FOR UNSATURATED FLOWS CONTAINING POLLUTANTS

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Abstract. *The flow of a polluted fluid through a wellbore is modeled under a mixture theory assumption in which the mixture consists of n+3 overlapping continuous constituents – namely one solid (porous medium), one liquid (a Newtonian fluid), n distinct pollutants and an inert gas. The simplified mechanical model consists of momentum and mass balance equations for the constituent representing the Newtonian fluid and continuity for those representing the n pollutants. The radial flow of the mixture through a cylindrical shell porous matrix is mathematically described by a nonlinear hyperbolic system of n+2 equations, which may admit discontinuous solutions, being simulated by combining Glimm's scheme with an operator splitting technique to account for the non-homogeneous part of the hyperbolic operator. Some representative results illustrating the numerical methodology are presented.*

Keywords*: continuum theory of mixtures, unsaturated porous medium, pollution, shock waves.*

1. Introduction

Transport phenomena in unsaturated porous media are present in a large number of Engineering applications such as groundwater flows, enhanced oil recovery processes, contamination of soils by hazardous wastes, storage of nuclear waste material and pollution movement. These few examples are able to show an important impact of such phenomena in the energy self-sufficiency and the environmental state. The study of transport in porous media dates from the 1920s, most of the works employing a volume averaging technique to study the momentum transport which allows considering a classical continuum mechanics context, as may be viewed by consulting a comprehensive review, by Alazmi and Vafai (2000), comparing different models for complex problems. This work studies the flow of a mixture of n+1 fluids – a Newtonian fluid and n pollutants – through a rigid porous matrix by employing a mixture theory approach to build the mechanical model. The mixture consists of n+3 overlapping continuous constituents: one solid (porous medium), one liquid (Newtonian fluid), n distinct pollutants (either in solid, liquid or gas phase) and an inert gas (accounting for the compressibility of the mixture as a whole). A cylindrical geometry is considered in order to obtain a preliminary local description for the flow of the polluted fluid through a wellbore.

Supposing an isothermal flow through a rigid porous matrix at rest and since the gas constituent is present for allowing changes in the liquid fraction only – so that no equation is required to describe its behavior, it is not necessary to solve momentum and mass balance equations neither for the gas nor for the porous matrix. The simplified model considers also an important additional hypothesis – a very small amount of pollutants present, at any time instant, compared with the amount of the Newtonian liquid, so that a pseudo-constituent representing a mixture of the n pollutants and the Newtonian fluid may be considered. As a consequence, the motion problem is reduced to solving mass and linear momentum balance equations for this pseudo-constituent combined with the n pollutants mass balance.

The radial flow of the above described mixture through a cylindrical shell porous matrix is mathematically described by a nonlinear hyperbolic system of n+2 equations, which may admit discontinuous solutions – in addition to the classical smooth ones. This hyperbolic nature of the problem ones requires specific numerical tools, such as, for instance, Glimm's scheme or Godunov's one, since the classical numerical tools widely employed to simulate parabolic and elliptic problems are not adequate to their numerical simulation. In the present work the numerical simulation is performed by combining Glimm's scheme with an operator splitting technique accounting for the non-homogeneous part of the hyperbolic operator. Glimm's scheme – a reliable method whose accuracy is mathematically ensured (Glimm, 1965; Chorin, 1976) is based on a theory presenting a solid thermodynamic basis (Smoller, 1983). It approximates hyperbolic nonlinear problems by appropriately gathering the solution of a certain number of associated Riemann problems. Two important features of Glimm's scheme, actually the method that better preserves the shock identity among the numerical procedures adequate to cope with discontinuous problems, deserve a special remark. First, if the width of the steps tends to zero, the approximation obtained by Glimm's method tends to the exact solution of the problem – considering its weak formulation. Another characteristic of this scheme is that it does not dissipate shocks, preserving their magnitude (no diffusion being observed) and position.

2. Mechanical model

The flow of a polluted fluid through a porous medium is described by using a mixture theory viewpoint in which a continuous mixture of a rigid solid constituent at rest (the porous matrix), a polluted liquid constituent and an inert gas playing the role of an additional constituent is considered. The polluted fluid is composed by a Newtonian liquid and a very small amount of *n* distinct pollutants – either very small solid particles or liquids, or gases, dispersed in the Newtonian fluid. Since the quantity of pollutants is small, the mixture of a Newtonian liquid and the pollutants may be viewed as a pseudo-constituent of a mixture, which, from now on will be conveniently denoted as fluid constituent, being considered as a single constituent of a mixture, concerning mass and momentum balance equations. The simplified mechanical model for an isothermal flow of a polluted fluid consists in solving mass and momentum balance equations for the fluid constituent (pseudo-constituent) combined with continuity for the constituents representing the *n* pollutants. The balance equations for the pseudo-constituents are given by (Atkin and Craine, 1976; Rajagopal and Tao, 1995)

$$\begin{aligned} &\frac{\partial \rho_F}{\partial t} + \nabla \cdot (\rho_F \mathbf{v}_F) = 0 \\ &\rho_F \left[\frac{\partial \mathbf{v}_F}{\partial t} + (\nabla \mathbf{v}_F)\mathbf{v}_F \right] = \nabla \cdot \mathbf{T}_F + \mathbf{m}_F + \rho_F \mathbf{b}_F \end{aligned} \tag{1}$$

where ρ_F stands for the fluid constituent (pseudo-constituent) mass density – representing the local ratio between the fluid constituent mass and the corresponding volume of mixture, $\mathbf{v}_F$ is the fluid constituent velocity in the mixture, $\mathbf{T}_F$ represents the partial stress tensor – analogous to Cauchy stress tensor in Continuum Mechanics – associated with the fluid constituent, $\mathbf{b}_F$ stands for the body force (per unit mass) and $\mathbf{m}_F$ for the momentum supply acting on the fluid constituent due to its interaction with the remaining constituents of the mixture.

Assuming the mass transfer as caused by an advection-diffusion process of the pollutant in the n+1 constituents mixture (representing the Newtonian fluid and the n pollutants), the pollutants mass balance is given by

$$\frac{\partial (\rho_F \omega_j)}{\partial t} + \nabla \cdot (\rho_F \omega_j \mathbf{v}_F) = \nabla \cdot (\rho_F D \nabla \omega_j) + r_j \qquad j = 1, n \tag{2}$$

in which ρ_F stands for the pseudo-constituent (Newtonian fluid + pollutants) mass density and $\mathbf{v}_F$ for its velocity. The concentration of the pollutant in the binary mixture (representing the Newtonian fluid and the pollutant), ω_j, is defined as the concentration of each *j*-constituent in the *n*+1 constituents mixture, being expressed as $\omega_j \equiv \rho_{pol_j} / \rho_F$, in which ρ_{pol_j} stands for the *j*-pollutant mass density regarded as a constituent of the mixture – in such a way that the pollutant actual mass is obtained by its integration over the whole mixture. Besides, D represents the diffusion coefficient of the pollutant in the mixture and r_j the rate of production of the *j*-pollutant per unit volume.

The ratio between the fluid fraction (combining the Newtonian fluid and the *n* pollutants) φ and the porous matrix porosity ε is defined as the saturation ψ, so that

$$\psi = \frac{\varphi}{\varepsilon} = \frac{\rho_F}{\varepsilon \rho_f} \qquad 0 < \psi \leq 1 \quad \text{everywhere} \tag{3}$$

in which ρ_f is the actual mass density of the polluted fluid – regarded as a single continuum, in contrast to ρ_F defined as the fluid constituent (the pseudo-constituent) mass density.

Constitutive relations for the momentum source term and the partial stress tensor are required to build the mechanical model for a mixture representing an unsaturated flow of an incompressible Newtonian fluid (representing the fluid and the *n* pollutants) through a porous matrix. Considering the normal fluid stresses dominant over shear stresses and interphase tractions (Allen, 1986), the momentum source and the partial stress are given by (Williams, 1978, Costa-Mattos et al., 1993)

$$\begin{aligned} &\mathbf{m}_F = -\alpha \psi^2 \mathbf{v}_F - \beta \psi \nabla \psi \qquad \text{with} \quad \alpha = \frac{\varepsilon^2 \mu_f}{K} \quad \beta = \frac{\varepsilon^2 \mu_f \mathsf{D}}{K} \\ &\mathbf{T}_F = -\varepsilon^2 \overline{p} \psi^2 \mathbf{I} \end{aligned} \tag{4}$$

where μ_f represents the fluid viscosity (measured considering a Continuum Mechanics viewpoint), K the porous matrix specific permeability, D a diffusion coefficient – analogous to the usual mass diffusion coefficient, $\overline{p}$ is a pressure (assumed constant while the flow is unsaturated) and $\mathbf{I}$ is the identity tensor.

2.1. Mathematical model

Combining Eqs. (1)-(4) a model to describe the radial flow of a polluted fluid through a cylindrical shell porous medium is obtained. Some additional hypotheses are now considered: absence of chemical reactions, which could alter the quantity of the any of the *j* constituents with concentration ω_j, leading to $r_j = 0$, gravitational effects omitted and diffusion neglected, when compared to advection – a reasonable assumption for an isotropic explosion. Also, assuming all the quantities depending only on the time *t* and on the position *r* and that *v* is the only non vanishing component of the fluid constituent (pseudo-constituent) velocity $\mathbf{v}_F$, a one dimensional approach for radial flow of a polluted fluid through a cylindrical shell porous matrix is described as

$$\begin{aligned}
&\frac{\partial \psi}{\partial t} + \frac{\partial}{\partial r}(\psi v) = -\frac{\psi v}{r} \\
&\rho_f \varepsilon \left[\psi \frac{\partial v}{\partial t} + \psi v \frac{\partial v}{\partial r} \right] = -\frac{\partial}{\partial r}\left(\varepsilon^2 \psi^2 \overline{p} \right) - \frac{\beta \rho_f^2 \varepsilon^2}{2} \frac{\partial \psi^2}{\partial r} - \alpha \psi^2 v \\
&\frac{\partial}{\partial t}(\psi \omega_j) + \frac{\partial}{\partial r}(\psi \omega_j v) = -\frac{\psi v \omega_j}{r} \qquad j = 1, n
\end{aligned} \tag{5}$$

A convenient dimensionless form is employed by considering the following dimensionless quantities

$$u = v\sqrt{\frac{\rho_f}{\varepsilon p_0}} \qquad \tau = \frac{t}{r_*}\sqrt{\frac{\varepsilon p_0}{\rho_f}} \qquad \gamma = \frac{\alpha r_*}{\rho_f \varepsilon}\sqrt{\frac{\rho_f}{\varepsilon p_0}} \qquad \xi = \frac{r - r_i}{r_e - r_i} \tag{6}$$

in which p_0 represents a reference pressure defined as defined as $p_0 = \overline{p} + \frac{\beta \rho_f^2}{2}$, $r_* = r_e - r_i$ with r_e and r_i standing for the external and internal radii of the cylindrical shell matrix. It is worth mentioning that the external radius r_e will be used as a reference in the numerical simulation only, since the numerical results will be presented in a region defined in the interval from r_i to r_e. Boundary conditions will be imposed solely at the cylindrical shell internal surface.

As a consequence, the mathematical representation of the above mentioned flow of a polluted fluid through a porous matrix may be expressed as:

$$\begin{aligned}
&\frac{\partial \psi}{\partial \tau} + \frac{\partial}{\partial \xi}(\psi u) = -\frac{\psi u}{\xi} \\
&\frac{\partial}{\partial \tau}(\psi u) + \frac{\partial}{\partial \xi}\left(\psi u^2 + \psi^2 \right) = -\frac{\psi u^2}{\xi} - \gamma \psi^2 u \\
&\frac{\partial}{\partial \tau}(\psi \omega_j) + \frac{\partial}{\partial \xi}(\psi \omega_j u) = -\frac{\psi u \omega_j}{\xi} \qquad j = 1, n
\end{aligned} \tag{7}$$

3. Numerical procedure

The numerical procedure to approximate the nonlinear hyperbolic system (7) consists in combining Glimm's scheme, a numerical methodology specifically developed for simulating non-linear hyperbolic problems, with an operator splitting technique, for treating the non-homogeneous portion of the hyperbolic operator. The simultaneous problem is actually treated as a sequential one. Examples of a wide range of non-linear hyperbolic problems that have already been simulated by combining Glimm's scheme and an operator splitting technique are shock propagation in gas dynamics problems, response of non-linear elastic rods, wave propagation in gas pipelines or in a damageable elasto-viscoplastic pipe and flow through unsaturated porous media (see Martins-Costa and Saldanha da Gama, 2001, and references therein). The numerical methodology employed in the present work to simulate the an unsaturated flow of a Newtonian fluid containing several pollutants through a porous matrix may be regarded as a natural extension of a previous work (Martins-Costa et al., 2004), in which a single pollutant was considered.

Essentially the first step consists in decomposing the operator in such a way that its purely time evolutionary portion is split away from its hyperbolic part, this latter being simulated through Glimm's method by adequately combining the solution of the associated Riemann problem. Each time step – from a time n to a time $n+1$ – is performed by using the solution of the associated Riemann problem for each two consecutive time steps. Glimm's method is based on a theory whose mathematical formulation has a solid thermodynamic basis, which could be expressed by the entropy condition (Smoller, 1983). Concerning Glimm's method simulation of nonlinear hyperbolic systems, a comprehensive approach is presented in Martins-Costa and Saldanha da Gama (2003) where this method is employed to the simulation of a pollutant transport in the atmosphere. Some features, however deserve a comment. Glimm's scheme has been specifically developed to deal with discontinuous problems, preserving the shock waves magnitude and position, within an uncertainty of $\Delta\xi$ (width of each step). These characteristics are not usually found in widely employed numerical methodologies such as finite elements and finite differences. Besides, Glimm's method presents a clear advantage of saving computer storage memory, when compared to other methodologies such as a finite element method associated with a shock capture procedure, however its limitation to one-dimensional problems is an important shortcoming.

The sequential way of simulating the problem consists in building a first approximation for the variables – like a prediction step – from time $\tau = \tau_n$ to time $\tau_{n+1} = \tau_n + \Delta\tau$, by solving the homogeneous problem via Glimm's method. The "correction" step is performed by solving the time evolutionary problem. The first step to implement Glimm's scheme consists in approximating the arbitrary initial condition – given by a function of the position ξ – by step functions, piecewise constant functions, conveniently chosen with equal width steps. In the sequence a Riemann problem – an initial value problem whose initial condition must be a step function, is to be solved for each two consecutive steps. The adequate combination of the solution of as many Riemann problems as required by a previously chosen precision, allows marching from time $\tau = \tau_n$ to time τ_{n+1}. Considering the following convenient redefinition of variables: $F \equiv \psi$, $G \equiv \psi u$ and $H_j \equiv \psi\omega_j$, and using the values of $\left(F, G, H_j\right)$ at time $\tau = \tau_n$ as initial data, a first approximation for the solution at time $\tau = \tau_{n+1}$ is obtained by simulating the corresponding homogeneous problem

$$\left.\begin{aligned} &\frac{\partial F}{\partial \tau} + \frac{\partial G}{\partial \xi} = 0 \\ &\frac{\partial G}{\partial \tau} + \frac{\partial}{\partial \xi}\left(\frac{G^2}{F} + F^2\right) = 0 \\ &\frac{\partial H_j}{\partial \tau} + \frac{\partial}{\partial \xi}\left(\frac{G H_j}{F}\right) = 0, \qquad j = 1, n \end{aligned}\right\} \quad \text{with} \quad \left.\begin{aligned} F &= \hat{F}_n\left(\xi\right) \\ G &= \hat{G}_n\left(\xi\right) \\ H_j &= \hat{H}_{j_n}\left(\xi\right) \end{aligned}\right\} \quad \text{at} \quad \tau = \tau_n \tag{8}$$

via Glimm's method. The solution of (8) evaluated at the time τ_{n+1} is given by the fields $\tilde{F}_{n+1}\left(\xi\right)$, $\tilde{G}_{n+1}\left(\xi\right)$ and $\tilde{H}_{j_{n+1}}\left(\xi\right)$ to be subsequently used as initial data when approximating the time evolutionary problem. In order to employ Glimm's scheme a piecewise constant function is used to approximate the initial data, as follows:

$$\left.\begin{aligned} F = \hat{F}_n\left(\xi\right) &\approx F_{n_i} = \hat{F}_n\left(\xi_i + \theta_n \Delta\xi\right) \\ G = \hat{G}_n\left(\xi\right) &\approx G_{n_i} = \hat{G}_n\left(\xi_i + \theta_n \Delta\xi\right) \\ H_j = \hat{H}_{j_n}\left(\xi\right) &\approx H_{j_{n_i}} = \hat{H}_{j_n}\left(\xi_i + \theta_n \Delta\xi\right) \end{aligned}\right\} \quad \text{at} \quad \xi_i - \frac{\Delta\xi}{2} < \xi < \xi_i + \frac{\Delta\xi}{2} \tag{9}$$

in which θ_n is a number randomly chosen in the open interval $\left(-1/2\ ,\ +1/2\right)$ and $\Delta\xi$ is the width of each step ($\Delta\xi = \xi_{i+1} - \xi_i$). The above approximations for the initial data give rise, for each two consecutive steps, to the following Riemann problem – whose detailed solution is presented in Martins-Costa and Saldanha da Gama (2003):

$$\left.\begin{aligned} F = \hat{F}_n\left(\xi\right) &\approx F_{n_i} = \hat{F}_n\left(\xi_i + \theta_n \Delta\xi\right) \\ G = \hat{G}_n\left(\xi\right) &\approx G_{n_i} = \hat{G}_n\left(\xi_i + \theta_n \Delta\xi\right) \\ H_j = \hat{H}_{j_n}\left(\xi\right) &\approx H_{j_{n_i}} = \hat{H}_{j_n}\left(\xi_i + \theta_n \Delta\xi\right) \end{aligned}\right\} \quad \begin{aligned} &\left(F, G, H_j\right) = \left(F_{n_i}, G_{n_i}, H_{j_{n_i}}\right) \quad \text{for } t = t_n\ , -\infty < \xi < \xi_i + \frac{\Delta\xi}{2} \\ &\left(F, G, H_j\right) = \left(F_{n_{i+1}}, G_{n_{i+1}}, H_{j_{n_{i+1}}}\right) \quad \text{for } t = t_n\ ,\ \xi_{i+1} - \frac{\Delta\xi}{2} < \xi < \infty \end{aligned} \tag{10}$$

Denoting by $F^R_{n_i}$ $G^R_{n_i}$, and $H^R_{j_{n_i}}$ the generalized solution of the Riemann problem (10), the approximation for the solution of (10) at the time τ_{n+1} is given as follows:

$$\left.\begin{aligned} F &= \tilde{F}_{n+1}(\xi) \approx F^R_{n_i}(\xi, \tau_{n+1}) \\ G &= \tilde{G}_{n+1}(\xi) \approx G^R_{n_i}(\xi, \tau_{n+1}) \\ H &= \tilde{H}_{j_{n+1}}(\xi) \approx H^R_{j_{n_i}}(\xi, \tau_{n+1}) \end{aligned}\right\} \qquad \text{for} \quad \xi_i < \xi < \xi_{i+1} \tag{11}$$

In order to prevent interactions among nearby shocks of adjacent Riemann problems, the time step $\Delta\tau$, and consequently, τ_{n+1} must be chosen in such a way that the Courant-Friedrich-Lewy (Smoller, 1977) condition $\tau_{n+1} - \tau_n \le \Delta\xi / 2|\lambda|_{\max}$, with $|\lambda|_{\max}$ representing the maximum (in absolute value) propagation speed, considering all the Riemann problems, is satisfied:

Once the approximation $\tilde{F}_{n+1}(\xi)$, $\tilde{G}_{n+1}(\xi)$ and $\tilde{H}_{j_{n+1}}(\xi)$ has been evaluated, the numerical approximation for the solution (F, G, H) at time $\tau = \tau_{n+1}$ is finally reached by advancing in time to solve the following time evolutionary problem, with the same step $\Delta\tau = \tau_{n+1} - \tau_n$ through equations:

$$\begin{cases} \dfrac{\partial F}{\partial \tau} = -\dfrac{1}{\xi} G \\ \dfrac{\partial G}{\partial \tau} = -\dfrac{1}{\xi}\dfrac{G^2}{F} - \gamma F G \\ \dfrac{\partial H_j}{\partial \tau} = -\dfrac{1}{\xi}\dfrac{G H_j}{F} \quad j = 1, n \end{cases} \quad \text{with} \quad \left.\begin{aligned} F &= \tilde{F}_{n+1}(\xi) \\ G &= \tilde{G}_{n+1}(\xi) \\ H &= \tilde{H}_{j_{n+1}}(\xi) \end{aligned}\right\} \quad \text{at} \quad \tau = \tau_n \tag{12}$$

Equation (12) is approximated by the following first order Euler approximation

$$\begin{aligned} F = \hat{F}_{n+1}(\xi) &\approx \tilde{F}_{n+1}(\xi) - \left\{\frac{1}{\xi}\tilde{G}_{n+1}(\xi)\right\}\Delta\tau \\ G = \hat{G}_{n+1}(\xi) &\approx \tilde{G}_{n+1}(\xi) - \left\{\frac{1}{\xi}\frac{\left[\tilde{G}_{n+1}(\xi)\right]^2}{\tilde{F}_{n+1}(\xi)} - \gamma\tilde{F}_{n+1}(\xi)\tilde{G}_{n+1}(\xi)\right\}\Delta\tau \\ H_j = \hat{H}_{j_{n+1}}(\xi) &\approx \tilde{H}_{j_{n+1}}(\xi) - \left\{\frac{1}{\xi}\frac{\tilde{G}_{n+1}(\xi)\tilde{H}_{j_{n+1}}(\xi)}{\tilde{F}_{n+1}(\xi)}\right\}\Delta\tau \qquad j = 1, n \end{aligned} \tag{13}$$

4. Numerical results

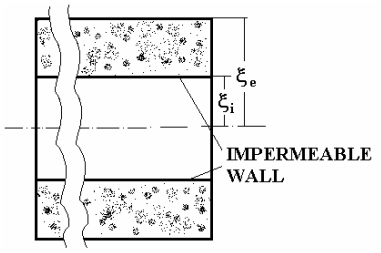

Figure 1. Problem scheme.

Some results are now presented in order to illustrate the numerical methodology described in the previous section. Results have been obtained for the radial propagation of three distinct pollutants, aiming at building a preliminary model for the pollutants propagation in soils, starting from a cylindrical contaminated well, as depicted in Fig. 1. These results have been obtained by approximating the problem stated in equation (7), with distinct initial data, subjected to a boundary condition characterizing an impermeable wall at the inner surface of the cylindrical shell porous matrix ($u = 0$ at ξ_i or $\xi = 0$). All numerical results have been obtained by employing Glimm's difference scheme combined with an operator splitting technique as described in the previous section employing 10^4 evolutions in time while the

spatial domain encompassed 900 steps. In order to exclude edge effects or, in other words, to assure that the external edge is not reached by a shock or a rarefaction wave – allowing simulating transport phenomena in an infinite environment – the results are shown for a domain including only the first 450 steps. So, in all depicted graphs, the cylindrical well internal radius ξ_i ($\xi = 0$) is depicted at the left-hand side, the initial data being given by zero velocity, an increasing step function for ω_A, and decreasing step functions for ψ, ω_B and ω_C.

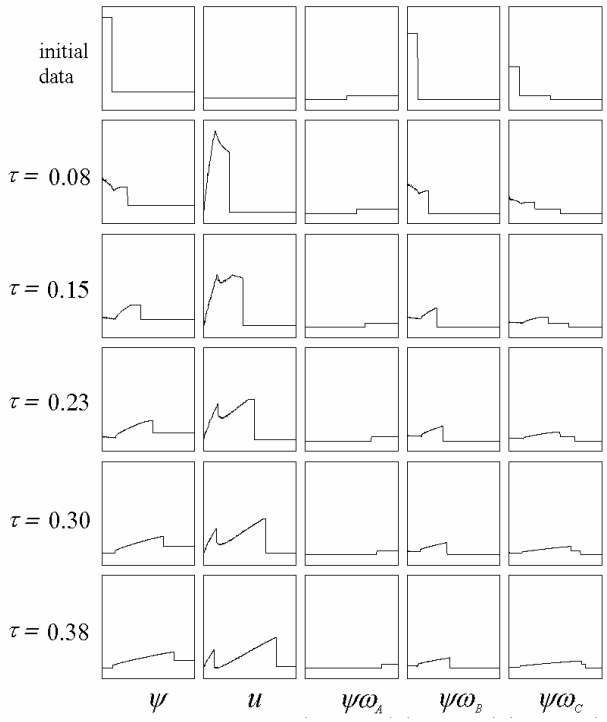

Figure 2. Saturation ψ, pseudo-constituent velocity u and pollutants concentration $\psi\omega_{j(j=A,B,C)}$ variation with radial position ξ for $\gamma = 10$ and $r_i = 0.05$. Initial data: step functions for ψ, ω_A, ω_B, ω_C and zero u.

The qualitative results presented in Figs. 2 to 4, consist of a set of six lines – representing the initial condition and five other selected time instants τ – and five columns of graphs, showing the evolution of saturation ψ, fluid constituent (pseudo-constituent) velocity u and pollutant concentration $\psi\omega_A$, $\psi\omega_B$ and $\psi\omega_C$ – representing the three pollutants *A*, *B* and *C* – along with radial position, displayed from left to right. The vertical axes correspond to these variables numerical values while the horizontal axes show the spatial coordinate ξ. A convenient normalization, accounting for maximum and minimum values of ψ, u and ω_j was performed in such a way that the minimum and maximum displayed values correspond to zero and unit values for ψ, u and ω_j.

Figure 2 shows the pollutants propagation in the radial direction from the internal radius ξ_i to $(\xi_e + \xi_i)/2$, the pollutant being initially concentrated in the internal half of the cylindrical shell, considering a Darcian coefficient $\gamma = 10$ and internal radius $r_i = 0.05$. The mixture velocity, initially zero, presents an increment caused by the jump in the saturation. The velocity variation is caused by two distinct factors: the presence of the Darcian term, as well as a geometrical factor, since the flow area increases linearly with the radius of the cylindrical shell. Also, it may be observed that the entire domain suffers the influence of the perturbation caused by the saturation and pollutants concentration.

The influence of the curvature may be noticed by comparing figures 2 ($r_i = 0.05$) and 3 ($r_i = 1.05$), while the parameter $r_* = r_e - r_i$ – the difference between for the external and internal radii of the cylindrical shell matrix remains constant and equal to unity. As expected, the curvature effect tends to vanish when the problem tends to a plane geometry.

In Fig. 4 the Darcian term coefficient is the only different parameter ($\gamma = 1$), when compared to Fig. 3. Observing these two figures allows concluding that the Darcian term coefficient provokes a small variation on the behavior of the saturation, the pseudo-constituent velocity and the pollutants concentration, when it is made ten times smaller. Actually,

even when the Darcian term coefficient is made equal to zero – in other words, the Darcian term is not considered, no appreciable variation on the plotted parameters is observed.

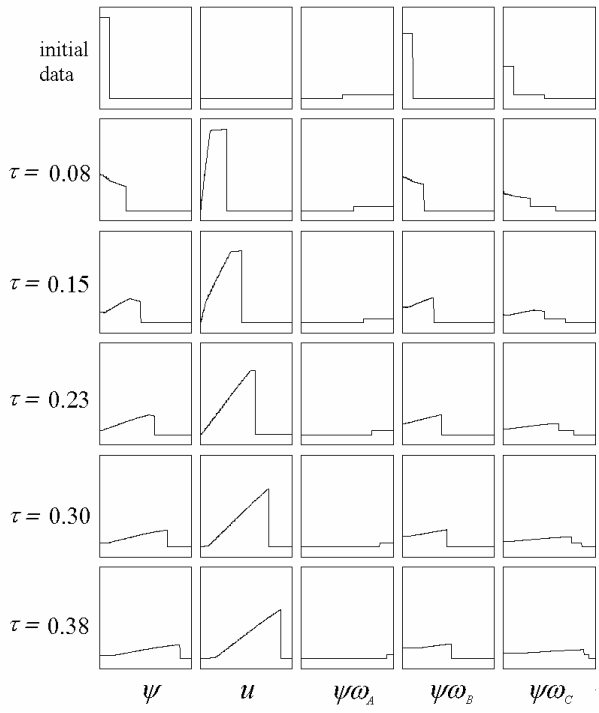

Figure 3. Saturation ψ, pseudo-constituent velocity u and pollutants concentration $\psi\omega_{j(j=A,B,C)}$ variation with radial position ξ for $\gamma = 10$ and $r_i = 1.05$. Initial data: step functions for ψ, ω_A, ω_B, ω_C and zero u.

An important feature, present in all depicted results, is that the discontinuities for the variables ψ and u are in the same spatial position. The numerical method accuracy (see Martins-Costa and Saldanha da Gama (2005) and references therein) is mathematically ensured, all quantities being globally preserved in such a way that the maximum error concerning the shock position is of the order of magnitude of the step width while the shock amplitude is preserved – no shock dissipation being present.

5. Final remarks

Glimm's method is a convenient tool for solving one-dimensional nonlinear problems, exhibiting features such as low storage costs and low computational effort, when compared to other numerical procedures to approximate nonlinear problems. Regarding its accuracy, it is free from numerical dissipation – preserving shock waves magnitude and presenting a deviation from the correct position smaller than the width of each step – preserving shock waves position. Besides, if the width of the steps tends to zero, Glimm's approximation tends to the exact solution of the problem. However, two important shortcomings of this methodology should be considered – first its applicability being restricted to one-dimensional problems. Also, its implementation requires previous knowledge of the solution of the associated Riemann problem.

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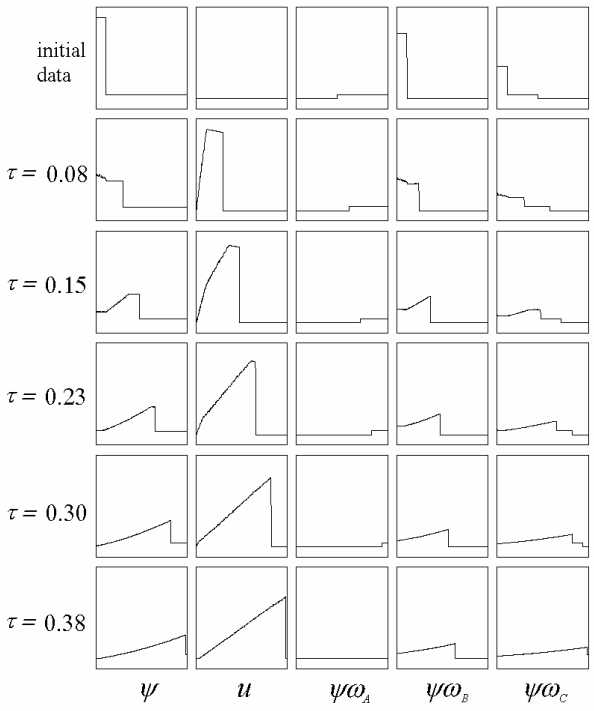

Figure 4. Saturation ψ, pseudo-constituent velocity u and pollutants concentration $\psi\omega_{j(j=A,B,C)}$ variation with radial position ξ for $\gamma = 1$ and $r_i = 1.05$. Initial data: step functions for ψ, ω_A, ω_B, ω_C and zero u.

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8. Responsibility notice

The authors acknowledge full responsibility for all printed material included in this paper.