INFLUENCE OF THE AXIAL WEDGE EFFECT ON THE STATIC EQUILIBRIUM POSITION OF HYDRODYNAMIC JOURNAL BEARINGS

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Abstract. *The Axial Wedge Effect was originally identified by the author during the derivation of the Reynolds Equation for the study of tilted hydrodynamic journal bearings. The context in which the inclusion of the Axial Wedge Effect in the Reynolds Equation affects the calculation of the static properties of such bearings is defined and exemplified with numerical results.*

Keywords: *hydrodynamic bearings, Reynolds Equation, angular misalignment.*

1. Introduction

Hydrodynamic journal bearings are susceptible to static angular misalignment, resulting from improper assemblage, elastic and thermal distortion of the shaft and bearing housing, and also manufacturing errors. Angular misalignment affects static operational properties such as journal equilibrium position and load capacity, lubricant side leakage, friction and heat generated within the fluid film.

Most previous works on the theme focused on the determination of static properties of angular misaligned bearings. DuBois et al. (1957) provided experimental determination of the hydrodynamic pressure field of tilted journals and corresponding static equilibrium position and couples. Theoretical evaluations of misalignment torques in gas and liquid lubricated bearings were performed by Rice (1965) and Manacorda and Capriz (1965) respectively. Thorough discussion of static misalignment influence on plain bearings was presented by Smalley and McCallion (1966), while grooved bearings were studied in Pinkus and Bupara (1979) and Vijayaraghavan and Keith (1989). Temperature effects were considered in Braun et al. (1983) and Mokhtar et al. (1985), whereas the role of non-Newtonian lubricants was analysed by Buckholz and Lin (1986).

Several authors studied both static and dynamic properties of tilted hydrodynamic bearings, among whom noteworthy contributions were provided by Jakeman (1986, 1989), San Andres (1993) and Qiu and Tieu (1995).

None of the aforementioned authors who conducted theoretical studies noticed that in the context of tilted bearings the traditionally adopted form of the Reynolds Equation could lead to questionable results. The aim of this paper is to present the Axial Wedge Effect, originally derived by the author, and show in which instances it affects significantly the static equilibrium position of the journal and consequently all the other bearing properties.

2. Reynolds Equation and the Axial Wedge Effect

Considering the local coordinates system (ζ,γ,ξ) and the other variables depicted in Fig.1, the Reynolds Equation for laminar, isoviscous flow is (Hamrock, 1995)

$$\frac{1}{\mu}\left[\frac{\partial}{\partial\xi}\left(h^3\frac{\partial p}{\partial\xi}\right)+\frac{\partial}{\partial\zeta}\left(h^3\frac{\partial p}{\partial\zeta}\right)\right]=12\left(v_j-v_b\right)+6h\frac{\partial}{\partial\xi}\left(w_j+w_b\right)+6h\frac{\partial}{\partial\zeta}\left(u_j+u_b\right)\ldots$$
$$-6\left(w_j-w_b\right)\frac{\partial h}{\partial\xi}-6\left(u_j-u_b\right)\frac{\partial h}{\partial\zeta} \quad (1),$$

where p is the pressure and μ is the viscosity of the lubricant.

In order to consider the relative angular misalignment between the journal and the bearing, three other coordinate systems are adopted: (X,Y,Z), which is inertial, (C_b,X_b,Y_b,Z_b) and (C_j,X_j,Y_j,Z_j), that move with the bearing and journal central planes, respectively. The sequence of rotations adopted, sketched in Fig. 2 is: first, rotation $A_{b,j}$ around Z results in $Y_{b,j}$; second, rotation $B_{b,j}$ around $Y_{b,j}$ defines $Z_{b,j}$; the systems of coordinates do not rotate around the bearing and journal longitudinal axes, X_j and X_b respectively..

Since the relative tilt angles $A_r = A_j - A_b$ and $B_r = B_j - B_b$ between the journal and the bearing are very small, one assumes that $\cos A_r = \cos B_r = 1$, $\operatorname{sen} A_r = A_r$, $\operatorname{sen} B_r = B_r$, resulting in the following relation between the unit vectors of the bearing and journal coordinate systems:

$$\begin{Bmatrix} \vec{I}_b \\ \vec{J}_b \\ \vec{K}_b \end{Bmatrix} = \underbrace{\begin{bmatrix} 1 & -A_r & B_r \\ A_r & 1 & 0 \\ -B_r & 0 & 1 \end{bmatrix}}_{T_{bj}} \begin{Bmatrix} \vec{I}_j \\ \vec{J}_j \\ \vec{K}_j \end{Bmatrix}, \tag{2}$$

where $A_r = A_j - A_b$ and $B_r = B_j - B_b$ are the relative tilt angles around axes *Z* and *Y* respectively.

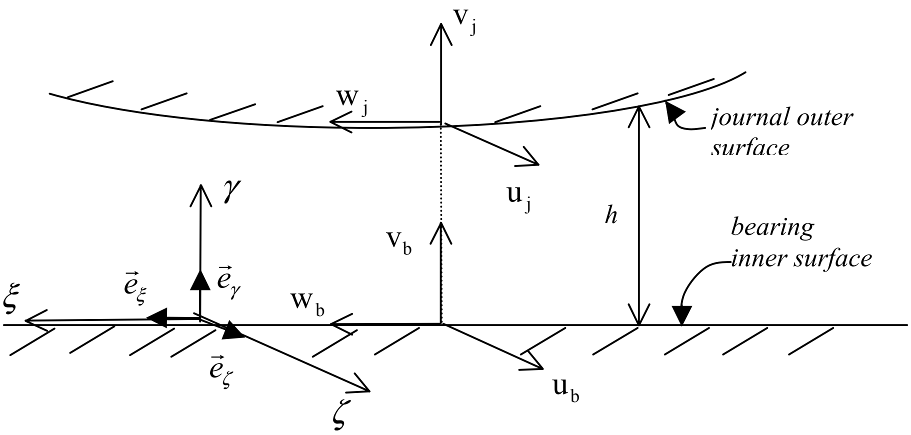

Figure 1. Local coordinates system (ζ,γ,ξ) and respective unit vectors $(\vec{e}_\zeta,\vec{e}_\gamma,\vec{e}_\xi)$, with $-L/2 \le \zeta \le L/2$ and $0 \le \xi \le \pi D$, where *L* is the length of the bearing and *D* its diameter.

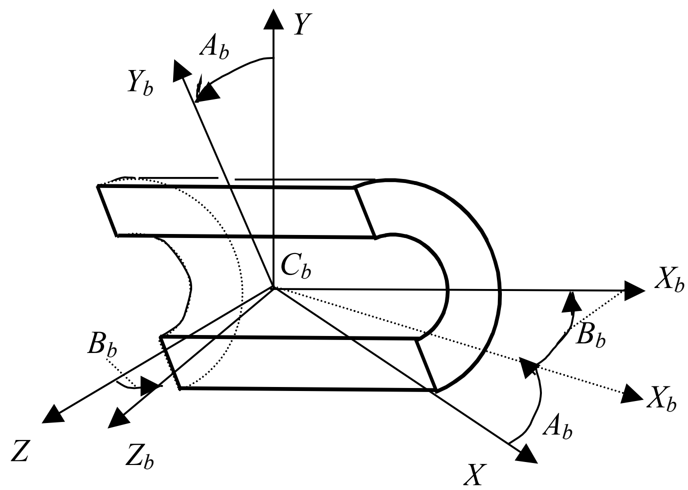

Figure 2 Coordinate systems (X,Y,Z) and (C_b, X_b, Y_b, Z_b).

Accordingly, the angular velocity vectors of the journal and the bearing are,

$$\begin{cases} \vec{W}_b = \left(\omega_b - B_b \dot{A}_b\right)\vec{I}_b + \dot{B}_b \vec{J}_b + \dot{A}_b \vec{K}_b \\ \vec{W}_j = \left(\omega_j - B_j \dot{A}_j\right)\vec{I}_j + \dot{B}_j \vec{J}_j + \dot{A}_j \vec{K}_j \end{cases}, \tag{3}$$

where ω_b and ω_j are the bearing and journal angular velocities around their respective longitudinal axes, $\dot{A}_{b,j} = dA_{b,j}/dt$ and $\dot{B}_{b,j} = dB_{b,j}/dt$. Therefore, the velocities of the points P_j located on the outer surface of the journal and of the points P_b located on the inner surface of the bearing, shown in Fig. 3, are given by

$$\begin{cases} \vec{V}_b = \vec{V}_{C_b} + \vec{W}_b \wedge \left(P_b - C_b\right) \\ \vec{V}_j = \vec{V}_{C_j} + \vec{W}_j \wedge \left(P_j - C_j\right) \end{cases}, \tag{4}$$

with

$$\begin{cases} \left(P_j - C_j\right) = \left(\dfrac{\zeta_b}{\cos B_r \cos A_r} + \dfrac{r\cos\alpha\tan A_r}{\cos B_r} - r\sin\alpha\tan B_r\right)\vec{I}_j + r\left(\cos\alpha\vec{J}_j + \sin\alpha\vec{K}_j\right) \\ \left(P_b - C_b\right) = \zeta_b\,\vec{I}_b + R\left(\cos\theta'\vec{J}_b + \sin\theta'\vec{K}_b\right) \end{cases}, \tag{5}$$

where R is the radius of the bearing, $r = R - c_R$ and c_R is the radial clearance.

All the variables in Eq. (5) are defined in Fig. 3, which shows the bearing cross section s' that contains P_b; points P_j belong to the cross section of the journal, which, due to the relative angular misalignment, is elliptical; the equation of the ellipse is

$$\left(P_j - C_j'\left(\zeta_j'\right)\right) = r\left[\frac{\cos\alpha'}{\cos A_r}\vec{J}_b + \left(\frac{\sin\alpha'}{\cos B_r} - \cos\alpha'\tan A_r\tan B_r\right)\vec{K}_b\right]. \tag{6}$$

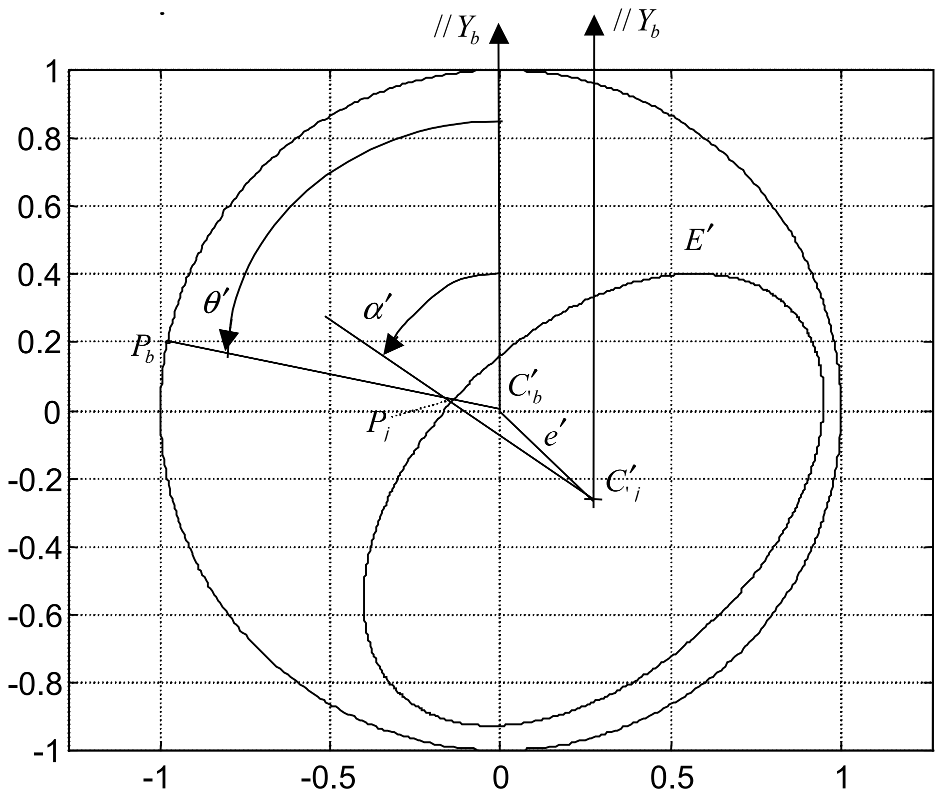

Figure 3. Cross section of a cylindrical tilted bearing, with $A_r < 0$, $B_r > 0$ and $\zeta > 0$.

Within the context of small angles, the lubricant film thickness is

$$h\left(\theta',\zeta\right) = c_R - \left(y_r + \zeta A_r\right)\cos\theta' - \left(z_r - \zeta B_r\right)\sin\theta', \tag{7}$$

and Eqs. (5) and (6) become,

$$\begin{cases} \left(P_j - C_j'\left(\zeta_j'\right)\right) = r\left(\cos a'\vec{J}_b + \sin a'\vec{K}_b\right) \\ \left(P_j - C_j\right) = \left(\zeta_b + A_r r\cos a' - B_r r\sin a'\right)\vec{I}_j + r\left(\cos a'\vec{J}_j + \sin a'\vec{K}_j\right) \end{cases}, \tag{8}$$

showing that the ellipticity of the journal cross section is negligible, as assumed, but not justified, by Smalley and McCallion (1967). Similarly, all other geometric imperfections are considered irrelevant in this context. Therefore, the calculation of $\vec{V}_b$ results in

$$\begin{aligned}\vec{V}_b = & \left[A_b \dot{y}_{C_b} - B_b \dot{z}_{C_b} + R\left(\dot{B}_b \sin\theta' - \dot{A}_b \cos\theta' \right) \right] \vec{I}_b \ \ldots \\ & + \left[\dot{y}_{C_b} + \zeta_b \dot{A}_b - R\left(\omega_b - B_b \dot{A}_b \right) \sin\theta' \right] \vec{J}_b + \left[\dot{z}_{C_b} - \zeta_b \dot{B}_b + R\left(\omega_b - B_b \dot{A}_b \right) \cos\theta' \right] \vec{K}_b \end{aligned} \quad (9)$$

Before proceeding, one recalls that in the derivation of the Reynolds Equation, the terms of relative magnitude c_R/D or c_R/L, identified as $O\left(c_R/D\right)$ and $O\left(c_r/L\right)$ respectively, are disregarded (Hamrock, 1995). In order to make a similar magnitude analysis, the following set of variables is defined:

$$\begin{cases} \bar{p} = \dfrac{p}{\mu\Omega\left(\dfrac{R}{c_R}\right)^2}; \ \bar{y}_{C_{j,b}} = \dfrac{y_{C_{j,b}}}{c_R}; \ \bar{z}_{C_{j,b}} = \dfrac{z_{C_{j,b}}}{c_R}; \ \bar{\alpha}_{j,b} = \dfrac{2c_R A_{j,b}}{L}; \ \bar{\beta}_{j,b} = \dfrac{2c_R B_{j,b}}{L} \\ \bar{\zeta} = \dfrac{2\zeta}{L}; \ \bar{h} = \dfrac{h}{c_R}; \ r_c = c_R/D; \ r_{LD} = L/D; \ \Omega = \dfrac{\left(\omega_j + \omega_b\right)}{2\pi}; \ \Omega_j = \dfrac{\omega_j}{2\pi}; \ \Omega_b = \dfrac{\omega_b}{2\pi} \end{cases} . \quad (10)$$

After substitution in the left side of the Reynolds Equation (LHS), one finds

$$LHS = \Omega c_R \left[\frac{\partial}{\partial\theta'}\left(\bar{h}^3 \frac{\partial \bar{p}}{\partial\theta'} \right) + \left(\frac{D}{L} \right)^2 \frac{\partial}{\partial\bar{\zeta}}\left(\bar{h}^3 \frac{\partial \bar{p}}{\partial\bar{\zeta}} \right) \right], \quad (11)$$

and the calculation of $\vec{V}_b$ leads to

$$\begin{aligned}\vec{V}_b = & c_R \left[\frac{2c_R}{L}\left(\bar{\alpha}_b \dot{\bar{y}}_{C_b} - \bar{\beta}_b \dot{\bar{z}}_{C_b} \right) + \frac{D}{L}\left(\dot{\bar{\beta}}_b \sin\theta' - \dot{\bar{\alpha}}_b \cos\theta' \right) \right] \vec{I}_b + \left[c_R\left(\dot{\bar{y}}_{C_b} + \bar{\zeta}_b \dot{\bar{\alpha}}_b \right) - R\left(\omega_b - 4\left(\frac{c_R}{L} \right)^2 \bar{\beta}_b \dot{\bar{\alpha}}_b \right) \sin\theta' \right] \vec{J}_b \\ & + \left[c_R\left(\dot{\bar{z}}_{C_b} - \bar{\zeta}_b \dot{\bar{\beta}}_b \right) + R\left(\omega_b - 4\left(\frac{c_R}{L} \right)^2 \bar{\beta}_b \dot{\bar{\alpha}}_b \right) \cos\theta' \right] \vec{K}_b , \end{aligned} \quad ,(12)$$

where it is seen clearly that the terms $A_b \dot{y}_{C_b}$, $B_b \dot{z}_{C_b}$ and $B_b \dot{A}_b$ must be disregarded, so $\vec{V}_b$ can be written in the form

$$\vec{V}_b = R\left(\dot{B}_b \sin\theta' - \dot{A}_b \cos\theta' \right) \vec{I}_b + \left(\dot{y}_{C_b} + \zeta_b \dot{A}_b - R\omega_b \sin\theta' \right) \vec{J}_b + \left(\dot{z}_{C_b} - \zeta_b \dot{B}_b + R\omega_b \cos\theta' \right) \vec{K}_b . \quad (13)$$

Analogously, the calculation of $\vec{V}_j$ yields

$$\vec{V}_j = \left[r\left(\dot{B}_j \sin\alpha' - \dot{A}_j \cos\alpha' \right) + r\omega_j\left(A_r \sin\alpha' + B_r \cos\alpha' \right) \right] \vec{I}_b + \left(\dot{y}_{C_j} + \zeta_b \dot{A}_j - r\omega_j \sin\alpha' \right) \vec{J}_b + \left(\dot{z}_{C_j} - \zeta_b \dot{B}_j + r\omega_j \cos\alpha' \right) \vec{K}_b \quad (14)$$

Now $\vec{V}_b$ and $\vec{V}_j$ must be written in terms of the local coordinate system described in Fig.1; using the following relation,

$$\begin{Bmatrix} \vec{I}_b \\ \vec{J}_b \\ \vec{K}_b \end{Bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -\cos\theta' & \sin\theta' \\ 0 & -\sin\theta' & -\cos\theta' \end{bmatrix} \begin{Bmatrix} \vec{e}_\zeta \\ \vec{e}_\gamma \\ \vec{e}_\xi \end{Bmatrix}, \quad (15)$$

one obtains

$$\begin{aligned}\vec{V}_b = & R\left(\dot{B}_b \sin\theta' - \dot{A}_b \cos\theta' \right) \vec{e}_\zeta + \left[\left(\dot{y}_{C_b} + \zeta_b \dot{A}_b \right) \sin\theta' - \left(\dot{z}_{C_b} - \zeta_b \dot{B}_b \right) \cos\theta' - R\omega_b \right] \vec{e}_\xi \ \ldots \\ & - \left[\left(\dot{y}_{C_b} + \zeta_b \dot{A}_b \right) \cos\theta' + \left(\dot{z}_{C_b} - \zeta_b \dot{B}_b \right) \sin\theta' \right] \vec{e}_\gamma \end{aligned} \quad (16)$$

and

$$\begin{aligned}\vec{V}_j = &\left[r\left(\dot{B}_j\sin\alpha' - \dot{A}_j\cos\alpha'\right) + r\omega_j\left(A_r\sin\alpha' + B_r\cos\alpha'\right)\right]\vec{e}_\zeta + \ldots\\ &\left[\left(\dot{y}_{C_j} + \zeta_b\dot{A}_j\right)\sin\theta' - \left(\dot{z}_{C_j} - \zeta_b\dot{B}_j\right)\cos\theta' - r\omega_j\cos\left(\theta'-\alpha'\right)\right]\vec{e}_\xi \ldots\\ &-\left[\left(\dot{y}_{C_j} + \zeta_b\dot{A}_j\right)\cos\theta' + \left(\dot{z}_{C_j} - \zeta_b\dot{B}_j\right)\sin\theta' + r\omega_j\sin\left(\theta'-\alpha'\right)\right]\vec{e}_\gamma\end{aligned} \tag{17}$$

Focusing on the right hand side (RHS) of Eq. 1, the velocity components are

$$\begin{cases} u_j = \left(R - c_R\right)\left[\left(\dot{B}_j\sin\theta' - \dot{A}_j\cos\theta'\right) + \omega_j\left(A_r\sin\theta' + B_r\cos\theta'\right)\right]; \\ v_j = -\left[\left(\dot{y}_{C_j} + \zeta\dot{A}_j\right)\cos\theta' + \left(\dot{z}_{C_j} - \zeta\dot{B}_j\right)\sin\theta' - \omega_j\dfrac{\partial h}{\partial\theta'}\right]; \\ w_j = \left[\left(\dot{y}_{C_j} + \zeta\dot{A}_j\right)\sin\theta' - \left(\dot{z}_{C_j} - \zeta\dot{B}_j\right)\cos\theta' - \left(R - c_R\right)\omega_j\right]; \\ u_b = R\left(\dot{B}_b\sin\theta' - \dot{A}_b\cos\theta'\right); v_b = -\left[\left(\dot{y}_{C_b} + \zeta\dot{A}_b\right)\cos\theta' + \left(\dot{z}_{C_b} - \zeta\dot{B}_b\right)\sin\theta'\right]; \\ w_b = \left[\left(\dot{y}_{C_b} + \zeta\dot{A}_b\right)\sin\theta' - \left(\dot{z}_{C_b} - \zeta\dot{B}_b\right)\cos\theta' - R\omega_b\right] \end{cases}, \tag{18}$$

and using the nondimensional variables defined in Eq. (10), the terms shown in RHS result in

$$\begin{cases} \left(v_j - v_b\right) = -c_R\left[\left(\dot{\bar{y}}_r + \bar{\zeta}\,\dot{\bar{\alpha}}_r\right)\cos\theta' + \left(\dot{\bar{z}}_r - \bar{\zeta}\,\dot{\bar{\beta}}_r\right)\sin\theta' - 2\pi\Omega_j\dfrac{\partial\bar{h}}{\partial\theta'}\right]; \\ \dfrac{\partial\left(w_j + w_b\right)}{\partial\xi} = -\dfrac{c_R}{R}\left\{\left[\left(\dot{\bar{y}}_{C_j} + \dot{\bar{y}}_{C_b}\right) + \bar{\zeta}\left(\dot{\bar{\alpha}}_j + \dot{\bar{\alpha}}_b\right)\right]\cos\theta' + \left[\left(\dot{\bar{z}}_{C_j} + \dot{\bar{z}}_{C_b}\right) - \bar{\zeta}\left(\dot{\bar{\beta}}_j + \dot{\bar{\beta}}_b\right)\right]\sin\theta'\right\}; \\ \dfrac{\partial\left(u_j + u_b\right)}{\partial\xi} = 0; \left(w_j - w_b\right) = c_R\left[\left(\dot{\bar{y}}_r + \bar{\zeta}\,\dot{\bar{\alpha}}_r\right)\sin\theta' - \left(\dot{\bar{z}}_r - \bar{\zeta}\,\dot{\bar{\beta}}_r\right)\cos\theta' - 2\pi\Omega_j\right] - 2\pi R\left(\Omega_j - \Omega_b\right); \\ \left(u_j - u_b\right) = c_R\left\{\dfrac{D}{L}\left[\dot{\bar{\beta}}_r\sin\theta' - \dot{\bar{\alpha}}_r\cos\theta' + \omega_j\left(\bar{\alpha}_r\sin\theta' + \bar{\beta}_r\cos\theta'\right)\right]\ldots\right. \\ \qquad\left. -2\dfrac{c_R}{L}\left[\dot{\bar{\beta}}_j\sin\theta' - \dot{\bar{\alpha}}_j\cos\theta' + \omega_j\left(\bar{\alpha}_r\sin\theta' + \bar{\beta}_r\cos\theta'\right)\right]\right\} \end{cases} \tag{19}$$

where $\bar{h} = \dfrac{h}{c_R} \Rightarrow \bar{h} = 1 - \left(\bar{y}_r + \bar{\zeta}\,\bar{\alpha}_r\right)\cos\theta' - \left(\bar{z}_r - \bar{\zeta}\,\bar{\beta}_r\right)\sin\theta'$.

Since

$$\begin{cases} \dfrac{\partial\bar{h}}{\partial\theta'} = \left(\bar{y}_r + \bar{\zeta}\,\bar{\alpha}_r\right)\sin\theta' - \left(\bar{z}_r - \bar{\zeta}\,\bar{\beta}_r\right)\cos\theta'; \dfrac{\partial\bar{h}}{\partial\bar{\zeta}} = -\bar{\alpha}_r\cos\theta' + \bar{\beta}_r\sin\theta'; \\ \dfrac{d\bar{h}}{dt} = -\left(\dot{\bar{y}}_r + \bar{\zeta}\,\dot{\bar{\alpha}}_r\right)\cos\theta' - \left(\dot{\bar{z}}_r - \bar{\zeta}\,\dot{\bar{\beta}}_r\right)\sin\theta' \\ \dfrac{\partial}{\partial\bar{\zeta}}\left(\dfrac{\partial\bar{h}}{\partial\theta'}\right) = \bar{\alpha}_r\sin\theta' + \bar{\beta}_r\cos\theta'; \dfrac{\partial}{\partial\bar{\zeta}}\left(\dfrac{d\bar{h}}{dt}\right) = -\dot{\bar{\alpha}}_r\cos\theta' + \dot{\bar{\beta}}_r\sin\theta' \end{cases}, \tag{20}$$

and neglecting the $O\left(\dfrac{c_R}{L}\right)$ terms, the first and the last expressions shown in Eq. (19) become

$$\begin{cases} \left(v_j - v_b\right) = c_R\left(\dfrac{d\bar{h}}{dt} + 2\pi\Omega_j\dfrac{\partial\bar{h}}{\partial\theta'}\right) \\ \left(u_j - u_b\right) = c_R\dfrac{D}{L}\dfrac{\partial}{\partial\bar{\zeta}}\left(\dfrac{d\bar{h}}{dt} + 2\pi\Omega_j\dfrac{\partial\bar{h}}{\partial\theta'}\right) \end{cases}. \tag{21}$$

Substituting Eqs. (19), (20) and (21) in Eq. (1), yields

$$\frac{\partial}{\partial\theta'}\left(\overline{h}^3\frac{\partial\,\overline{p}}{\partial\theta'}\right)+\left(\frac{D}{L}\right)^2\frac{\partial}{\partial\overline{\zeta}}\left(\overline{h}^3\frac{\partial\,\overline{p}}{\partial\overline{\zeta}}\right)=12\left(\frac{1}{\Omega}\frac{d\,\overline{h}}{d\,t}+\pi\frac{\partial\,\overline{h}}{\partial\theta'}\right)-12\frac{r_C}{r_{LD}^2}\frac{\partial}{\partial\overline{\zeta}}\left[\frac{1}{\Omega}\frac{d\,\overline{h}}{d\,t}+2\pi\frac{\Omega_j}{\Omega}\frac{\partial\,\overline{h}}{\partial\theta'}\right]\frac{\partial\overline{h}}{\partial\overline{\zeta}}\cdots$$
$$-12r_C\overline{h}\left\{\left[\left(\dot{\overline{y}}_{C_j}+\dot{\overline{y}}_{C_b}\right)+\overline{\zeta}\left(\dot{\overline{\alpha}}_j+\dot{\overline{\alpha}}_b\right)\right]\cos\theta'+\ldots\right.$$
$$\left.\left[\left(\dot{\overline{z}}_{C_j}+\dot{\overline{z}}_{C_b}\right)+\overline{\zeta}\left(\dot{\overline{\beta}}_j+\dot{\overline{\beta}}_b\right)\right]\sin\theta'-\left(\dot{\overline{y}}_r+\overline{\zeta}\dot{\overline{\alpha}}_r\right)\sin\theta'-\left(\dot{\overline{z}}_r-\overline{\zeta}\dot{\overline{\beta}}_r\right)\cos\theta'-2\pi\Omega_j\right\} \quad (22)$$

The first term of the RHS in Eq. (22) is $O(1)$ and the third term is $O\left(\frac{c_R}{D}\right)$, so the latter can be disregarded. The intermediary term, $-12\frac{r_C}{r_{LD}^2}\frac{\partial}{\partial\overline{\zeta}}\left[\frac{1}{\Omega}\frac{d\,\overline{h}}{d\,t}+2\pi\frac{\Omega_j}{\Omega}\frac{\partial\,\overline{h}}{\partial\theta'}\right]\frac{\partial\overline{h}}{\partial\overline{\zeta}}$, is $O\left(\frac{r_C}{r_{LD}^2}\right)$ and can not be neglected "a priori" in the study of short bearings. On the other hand, it is clear that when $r_{LD}\geq 1$ the aforementioned term is $O\left(\frac{c_R}{D}\right)$ and its influence is irrelevant.

Therefore, the Reynolds Equation appropriate for the study of short tilted bearings is

$$\frac{1}{\mu}\left[\frac{\partial}{\partial\xi}\left(h^3\frac{\partial\,p}{\partial\xi}\right)+\frac{\partial}{\partial\zeta}\left(h^3\frac{\partial\,p}{\partial\zeta}\right)\right]=12\frac{dh}{dt}+6\left(\omega_j+\omega_b\right)\frac{\partial\,h}{\partial\theta'}-6R\frac{\partial}{\partial\zeta}\left[\frac{dh}{dt}+\omega_j\frac{\partial\,h}{\partial\theta'}\right]\frac{\partial\,h}{\partial\zeta}. \quad (23)$$

The term $6R\frac{\partial}{\partial\zeta}\left[\frac{dh}{dt}+\omega_j\frac{\partial\,h}{\partial\theta'}\right]\frac{\partial\,h}{\partial\zeta}$ in the RHS of Eq. (23) was first identified by Zachariadis(2000), and since it is proportional to $\frac{\partial\,h}{\partial\zeta}$, it was named the "Axial Wedge Effect" term. To the best of the authors´ knowledge, the inclusion of the Axial Wedge Effect term in the Reynolds Equation is an original contribution.

3. Numerical Results

Equation (23) will be solved numerically in order to evaluate the influence of the consideration of the "Axial Wedge Effect" in the calculation of the static equilibrium position of a tilted short bearing. Using the change of variable proposed by Vogelpohl (1937), $\Pi=\overline{h}^{3/2}\overline{p}$, the nondimensional form of Eq. (23) is

$$\frac{\partial^2\Pi}{\partial\theta'^2}+\frac{1}{r_{LD}^2}\frac{\partial^2\Pi}{\partial\overline{\zeta}^2}=\frac{1}{\overline{h}^{3/2}}\frac{12}{\Omega}\left[\frac{d\overline{h}}{dt}+\pi\,\Omega\frac{\partial\overline{h}}{\partial\theta'}-\frac{r_C}{r_{LD}^2}\frac{\partial\overline{h}}{\partial\overline{\zeta}}\left(\frac{d\overline{h}}{dt}+2\pi\Omega_j\frac{\partial\overline{h}}{\partial\theta'}\right)\right]+\ldots$$
$$\frac{3\Pi}{4\overline{h}^2}\left\{\left(\frac{\partial\,\overline{h}}{\partial\theta'}\right)^2+2\overline{h}\frac{\partial^2\overline{h}}{\partial\theta'^2}+\frac{1}{r_{LD}^2}\left[\left(\frac{\partial\,\overline{h}}{\partial\overline{\zeta}}\right)^2+2\overline{h}\frac{\partial^2\overline{h}}{\partial\overline{\zeta}^2}\right]\right\}. \quad (24)$$

After the determination of the pressure field, via a finite differences procedure, the resultant force on the journal is calculated using Simpson's numerical integration method. Afterwards, a nonlinear algebraic equation must be solved in order to find the equilibrium position of the journal as a function of its speed and static load. Several analyses and results presented in Zachariadis (1999, 2000 and 2004) attest to the validity of these numerical procedures.

The results presented in Figs. 4 and 5 refer to a plain cylindrical bearing with $r_{LD}=0.125$ and $r_c=1.0\times10^{-3}$, subject to relative angular misalignments $\overline{\alpha}_r$ and $\overline{\beta}_r$ that vary between 0.0 and 0.75. In order to calculate the static equilibrium positions, the angular speed of the journal was increased gradually between $0.0025\leq S_o\leq 0.2300$, where S_o is the Sommerfeld Number (Hamrock, 1995). The shape of the static equilibrium position locus of the journal is sufficiently described in Figs. 4 and 5, since it is possible to interpolate the results related to intermediary speeds. For example, in the case corresponding to $\overline{\alpha}_r=0.25$ and $\overline{\beta}_r=0.0$, the following values of S_o are considered: $S_o=0.0025, 0.0054, 0.0091, 0.0144, 0.0231, 0.0390, 0.0732, 0.1400$; likewise, for $\overline{\beta}_r=0.25$ and $\overline{\alpha}_r=0.00$, the values of S_o are: $S_o=0.0025, 0.0054, 0.0091, 0.0144, 0.0231, 0.0732, 0.1706, 0.1900, 0.2214$. It is seen that the consideration of the Axial Wedge Effect affects more significantly the relative static equilibrium position when the angular misalignment between bearing and journal occurs around the vertical Y axis (that is, the axis of the static load). Other relevant static properties, such as side leakage and friction are affected in a similar way. Consequently, the dynamic stiffness and damping coefficients of the bearing, used in linear vibration analyses of rotor systems, are also affected. Due to lack of space, these results will be presented in future works.

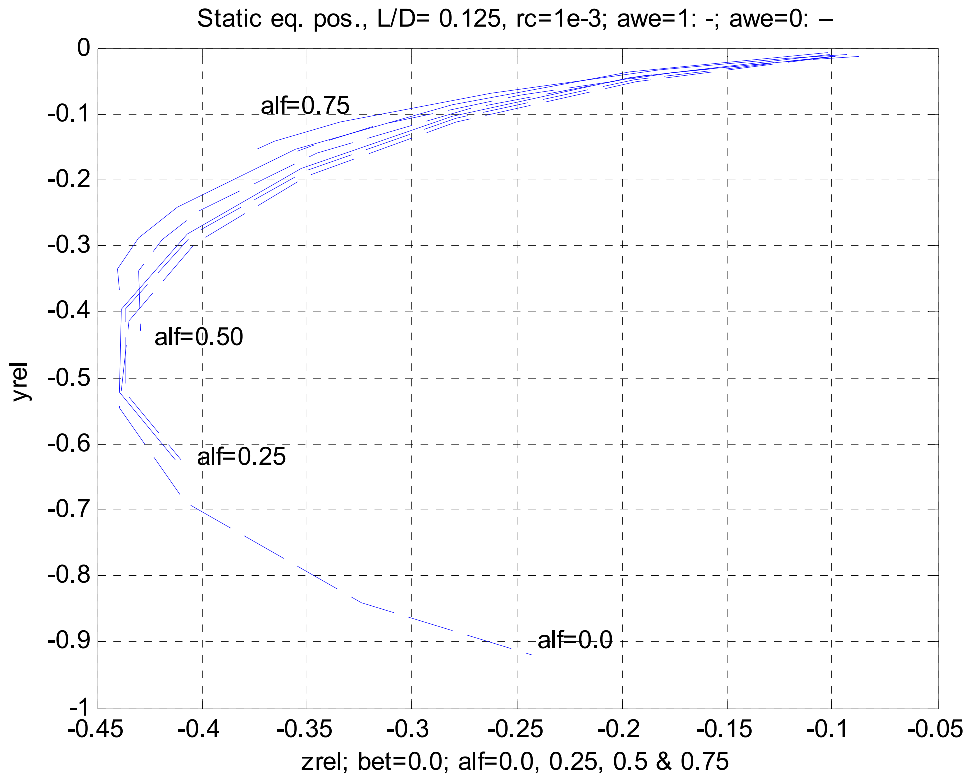

Figure 4. Static equilibrium position; $\overline{\beta}_r = 0$, $0 \le \overline{\alpha}_r \le 0.75$; awe=1 (solid lines) identify the results calculated with the consideration of the Axial Wedge Effect.

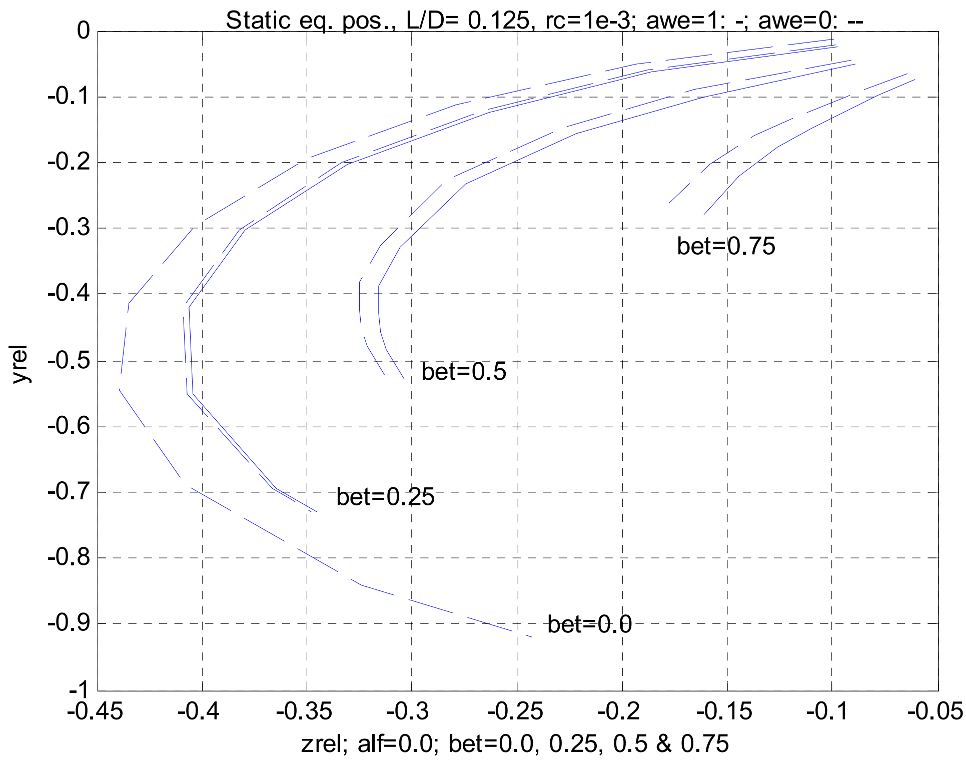

Figure 5. Static equilibrium position; $\overline{\alpha}_r = 0$, $0 \le \overline{\beta}_r \le 0.75$; awe=1 (solid lines) identify the results calculated with the consideration of the Axial Wedge Effect.

4. Conclusion

A properly conducted order of magnitude analysis performed during the derivation of the Reynolds Equation shows that, in the study of tilted short hydrodynamic journal bearings, one has to consider the term here called Axial Wedge Effect. This term was originally identified by the author and is associated with the variation of the clearance in the longitudinal direction of the bearing. In this paper, the Axial Wedge Effect was derived within the context of the hypotheses conventionally adopted during the derivation of the Reynolds Equation, that is, isothermal and laminar flow. Numerical results related to a plain cylindrical journal bearing show that the consideration of the Axial Wedge Effect affects significantly the static properties of short journal bearings subject to relative angular misalignment.

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