

# FRICIONAL AND ACCELERATING PRESSURE DROPS EFFECTS ON HORIZONTAL OIL WELL PRODUCTIVITY INDEX

## Wellington Campos

Petróleo Brasileiro S.A., Rua General Canabarro, 500, Maracanã, Rio de Janeiro, RJ  
wcampos@petrobras.com.br

## Rogério Campos Aguiar

Universidade Federal do Norte Fluminense, Rod. Amaral Peixoto, Km 163, Av. Brenand s/n, Imboacica, Macaé, RJ  
raiugar.fjpf@petrobras.com.br

## Divonsir Lopes

Petróleo Brasileiro S.A., Rua General Canabarro, 500, Maracanã, Rio de Janeiro, RJ  
divonsir@petrobras.com.br

**Abstract.** *This work quantifies the effects of the frictional and accelerating pressure drops on the overall productivity index of an oil well. The study comprises the horizontal section of an oil well, from a nodal point at its bottom to a nodal point at its entrance. To accomplish the study, a mathematical model is developed based upon balance equations written for the flowing conserved quantities of mass and momentum. A few simplifying hypotheses are adopted. For instance, the flow is assumed steady-state and fully developed and the horizontal section is considered fully horizontal. The result is a coupled system of ordinary differential equations for the oil flow rate and the flowing pressure inside the horizontal well bore. The mathematical model is solved numerically for proper boundary conditions and for flows above the bubble point. The results for flow rate and flowing pressure along the well are plotted and analyzed. An inflow performance relationship (IPR) is plotted for the nodal point chosen at the entrance of the horizontal section.*

**Keywords:** *mathematical model, horizontal well, inflow performance relationship, IPR, frictional pressure drop.*

## 1. Introduction

Oil production and water injection in horizontal wells through gravel packing and stand alone screens are common practices in many fields. In such completions, as the oil is produced by entering the horizontal well bore laterally, the flow rate along the well increases from bottom to the screen packer. This brings about an increase in pressure drop, with an increase in the well bore pressure, a decrease in the differential pressure or “drawn down”, and a final decrease in flow rate.

In addition, the oil that enters the well laterally needs impulse to be accelerated along the well. This will bring about further decrease in flow rate. The consequence of all this is a decrease in the well global productivity index.

Those two effects are related with the length of the horizontal section. Consequently, the cost of drilling additional length should be balanced with the revenue from a decreasing production rate from deeper or more distant zones. In another words, the length of the horizontal section should be optimized for maximum performance.

Here we develop a mathematical model for calculating this change in flow rate as a function of the governing parameters, namely, the well bore geometry, the fluid and wall physical properties and the controlling parameters. More specifically, the model incorporates variables such as the well bore diameter, the oil viscosity and density, the friction factor, the laminar and turbulent flow regimes, and others. The well head pressure can be controlled through an adjustable choke. This well head pressure plus the hydrostatic pressure exerted by the vertical column of fluid also acts against production. But in this work only the two factors cited above will be examined

## 2. Problema description

Figure 1 shows a horizontal well production system delivering the oil to a stationary production unit, a FPSO in this case. The oil is produced from the production interval comprised by the horizontal section AB. It is assumed that no oil enters through the bottom at B. As we go from B to A, i.e. from bottom to entrance of the horizontal well, the flow rate increases due to more and more contribution along the horizontal section. Beyond the entrance nodal point A it is assumed that no additional contribution to the production stream exists.

A choke valve at the FPSO can be actuated to set a well head pressure to control production. In addition to frictional pressure drop along the entire production system and well head pressure, the production stream has to overcome gravity, i.e. the hydrostatic pressure along the tubing string and production riser. This will add to friction in opposition to production. Complicating details include the fact that, as the production stream rises to the mud line and to the surface, pressure decreases, causing associated gas to come out of solution, this way ensuing a multiphase flow. Nevertheless, the well system can be divided into two sub-systems by affixing a nodal point at the entrance of the horizontal section, point A

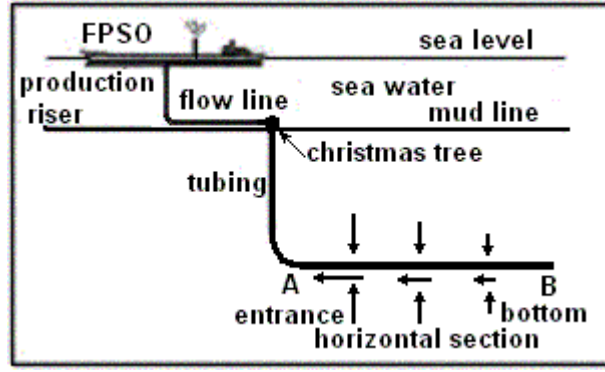


Figure 1. Schematic of a horizontal well.

The Inflow Performance Relationship (IPR) can then be determined for the sub-system AB and a Vertical Performance Relationship (VPR) can be determined for the sub-system from A to the FPSO, i. e. the flow in the tubing, flow line and production riser, by using some correlation for multiphase flows (Orkiszewski, 1967 or Beggs & Brill, 1973). This two relationships, the IPR and the VPR, can, later on, be combined to determine the point of operation of the production system, i.e. the flow rate and the bottom hole pressure at the nodal point A.

The focus of this work is on the horizontal section AB, so as to understand the phenomena going on in there. For that, a simple mathematical model is developed that will take care of the phenomena from A to B. This mathematical model is then implemented in a computer program to enable numerical results to be extracted and analyzed. In this paper the effects of the heterogeneities are taken into consideration, where by heterogeneities it is understood the intercalation of sandstone and shale intervals along the horizontal section (Fig. 3).

A number of studies concerning this subject can be found in the literature. Cho (2003), Folefac et al. (1991), Jin (1976), Penmartcha et al. (1997) and Seines et al. (1993), among others have considered the problem of pressure drop along a horizontal section of the well and its effect on well performance. Some of them have addressed the problem of determining the optimum length of a horizontal well. In all these works the formation heterogeneities are not considered.

## Mathematical Model

Figure 2 presents a closer view of section AB. Oil is produced along the well and contribute to increase the flow rate. It is assumed that the flow in the horizontal section is single phase. The pressure at the nodal point A will be kept at  $p_0$ . Oil flows from right to left with increasing flow rate, due to the production  $dq$ . The coordinate  $x$  has origin at B. To calculate the produced flow rate it is introduced the concept of specific productivity index,  $j$ , defined by

$$j = \frac{1}{p_R - p_{wf}} \frac{dq}{dx} \quad (1)$$

where  $p_R$  is the formation pressure,  $p_{wf}$  is the well bore pressure, the difference  $(p_R - p_{wf})$  being the differential pressure or “drawdown”.  $dq$  is the infinitesimal oil flow rate in the infinitesimal length  $dx$ , so that  $dq/dx$  is the production rate gradient along the well bore. Notice that these variables are functions of the coordinate  $x$ .

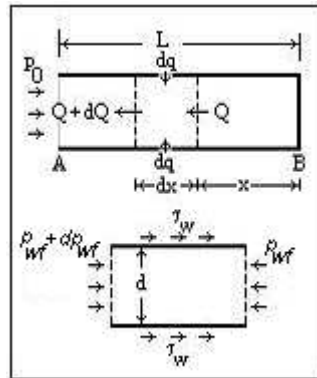


Figure 2. Schematic of the horizontal section AB showing an element of infinitesimal length  $dx$ .

The formation pressure  $p_R$  will be assumed to be constant in each permeable zone along the well bore. A mass balance on the control volume shown in Fig. (2) gives

$$\frac{dQ}{dx} = \frac{dq}{dx} = j(p_R - p_{wf}) \quad (2)$$

where  $p_{wf}$  is the pressure at the middle of each infinitesimal element.

Figure (2) also shows the forces acting upon the infinitesimal control volume so as to enable one to calculate a momentum balance for the element. This will reveal a further relationship for the phenomenon under study. Here one has to be more careful, since the fluid that are entering laterally through the well bore wall do not possess any momentum in the direction of the well bore axis. Therefore, its x-momentum is zero. To derive a momentum balance equation for the element, we consider an infinitesimal time interval  $dt$ , during which the balance of momentum must be fulfilled.

*momentum in – momentum out + forces impulses = change of momentum inside the control volume = 0*

$$\rho Q dt \frac{Q}{\pi d^2/4} - \rho(Q + dQ) dt \frac{Q + dQ}{\pi d^2/4} + p_{wf} \frac{\pi d^2}{4} dt - (p_{wf} + dp_{wf}) \frac{\pi d^2}{4} dt - \tau_w \pi d dx dt = 0 \quad (3)$$

where  $d$  is the well bore diameter and  $\tau_w$  is the wall shear stress and  $\rho$  is the oil density. Notice how the mass that enters laterally do not contribute directly to this momentum balance equation. After disregarding small higher order terms, Equation (3) gives

$$-\frac{\rho Q}{\pi d^2/4} \frac{dQ}{dx} - \frac{\pi d^2}{4} \frac{dp_{wf}}{dx} - \pi d \tau_w = 0 \quad (4)$$

As a simplification, it is assumed that the wall friction stress is given by the relationship valid for fully developed steady-state pipe flow

$$\tau_w = f \frac{\rho \bar{V}^2}{2} = \frac{d}{4} \frac{dp_{fric}}{dx} \quad (5)$$

where  $f$  is the friction factor,  $\bar{V}$  is the average velocity in the element, namely,  $\bar{V} = (Q + dQ/2)/(\pi d^2/4)$ . Equations (4) and (5) give

$$\frac{dp_{wf}}{dx} = -\frac{dp_{fric}}{dx} - \frac{2\rho Q}{A^2} j(p_R - p_{wf}) \quad (6)$$

where  $A = \pi d^2/4$  is the sectional area of the well bore. Equation (6) shows up that the pressure inside the well increases from A to B and that the increase is due to two effects. The first effect is the traditional frictional pressure drop. The second effect is the accelerating pressure drop, and the latter comes from the impulse that should be given to the produced fluid so that it catches up with the main flow along the well, that is traveling with average velocity  $Q/A$ .

The calculation of the frictional pressure drop is approximated with the use of the Fanning friction factor and is given by

$$\frac{dp_{fric}}{dx} = \frac{2f\rho\bar{V}^2}{d} \quad (7)$$

where  $\bar{V}$  is the average velocity in the element, namely,  $\bar{V} = Q/A$ ,  $\rho$  is the oil density and  $d$  is the well bore diameter. The Fanning friction factor is calculated for laminar flow by

$$f = \frac{16}{Re} \quad (8)$$

and for turbulent flow, by the well-known Colebrook equation

$$\frac{1}{\sqrt{f}} = -4 \log_{10} \left( \frac{0,269\varepsilon}{d} + \frac{1,261}{R_e \sqrt{f}} \right) \quad (9)$$

where  $R_e$  is the Reynolds number,  $R_e = \rho \bar{V} d / \mu$ . Here enters the effect of the oil absolute viscosity  $\mu$ . But, as the Colebrook formula needs an iterative solution, the following approximation

$$\frac{1}{\sqrt{f}} = -4 \log_{10} \left\{ \frac{\varepsilon/d}{3,7} - \frac{5,02}{R_e} \log_{10} \left[ \frac{\varepsilon/d}{3,7} - \frac{5,02}{R_e} \log_{10} \left( \frac{\varepsilon/d}{3,7} - \frac{13}{R_e} \right) \right] \right\} \quad (10)$$

is adopted. The advantage of this approximated formula is that  $f$  can be calculated explicitly with an error of less than 0.5% in relation with the results of Colebrook formula for a large range of the Reynolds number (see Table I).

Table I – Comparison of friction factor values for  $\varepsilon/d=0,0025$ .

$R_e$ dimensionless	Colebrook formula friction factor, $f_c$ dimensionless	approximated formula friction factor, $f_{zs}$ dimensionless	relative error $100  f_c - f_{zs} /f_c$ %
4000	0.010266	0.010256	0.095
10000	0.008140	0.008129	0.134
100000	0.005627	0.005626	0.022
200000	0.005353	0.005354	0.005

The lateral influx of fluid into the horizontal well should have an influence in the resulting friction factor. This is so if it is realized that the transport of momentum is enhanced across the pipe cross-section, contributing to increase turbulence. Here we disregard this influence. For an account of the magnitude of this effect report to the work of Su and Gudmundsson (1993) and to the work of Yalniz and Ozkan (2001).

For concluding the derivation of the mathematical model, it is observed that the problem rests on solving the coupled system of differential equations

$$\begin{cases} \frac{dQ}{dx} = j(p_R - p_{wf}) \\ \frac{dp_{wf}}{dx} = -\frac{dp_{fric}}{dx} - \frac{\rho Q}{A^2} j(p_R - p_{wf}) \end{cases} \quad (11)$$

where the appropriated boundary conditions are  $Q=0$  at  $x=0$  and  $p_{wf}=p_0$  at  $x=L$ . This last boundary condition can be related to the well head pressure if additional work is performed to model the vertical flow from A to the surface.

## Numerical Model

To solve the problem described by the system of ordinary differential equations, Eq. (11), a simple finite difference method is used. The horizontal interval has been discretized into a certain number of subintervals. Then, finite difference is used. The discretized equations are

$$\begin{cases} Q_{i+1} - Q_i = j_{i+1/2} (p_R - p_{wi+1/2}) \Delta x \\ p_{wi+1} - p_{wi} = -\frac{dp_{fric}}{dx} \Big|_{i+1/2} \Delta x - \frac{\rho Q_{i+1/2}}{A^2} j_{i+1/2} (p_R - p_{wi+1/2}) \Delta x \end{cases} \quad (12)$$

where  $i = 0 \dots n$  and the value at  $i+1/2$  is calculated as an average, for instance

$$j_{i+1/2} = \frac{j_i + j_{i+1}}{2} \quad (13)$$

The boundary conditions are  $Q_0 = 0$  at the bottom (point B) and  $p_n = p_0$  at the entrance (point A), where  $p_0$  is some specified value.

The solution is obtained iteratively from the explicit Eqs. (8). First the pressure  $p_0$  is assumed for all the grid points. Then the flow rates  $Q_i$  are calculated from bottom to entrance for all the grid point, starting up with the boundary

condition  $Q_0 = 0$ . Then the pressure is calculated from entrance to bottom, starting up with the boundary condition  $p_n = p_0$  at the nodal point A. The program iterates until there is no more appreciable changes from a step to the next. A criterion of convergence may be used, for example, from time step  $k$  to time step  $k+1$  the flow rate at the nodal point A should satisfy

$$\left| \frac{Q_n^{(k+1)} - Q_n^{(k)}}{Q_n^{(k+1)}} \right| \leq 0.0001 \quad (14)$$

Eventually, a bisection between two successive calculated flow rates can be used to accelerate and guarantee convergence, i.e.

$$Q_i^{(New)} = \frac{Q_i + Q_{i-1}}{2}, \quad i = 1 \dots n \quad (15)$$

### Case Study

In what follows a field case will be analyzed. To give an idea of how to proceed, a part of a logging chart is shown (Fig. 3). This chart has been analyzed by a geologist, who has colored the sand intervals in the right track. It can be noticed that the horizontal section is heterogenous, i.e., made up of a series of sand intervals interrupted by shale intervals.

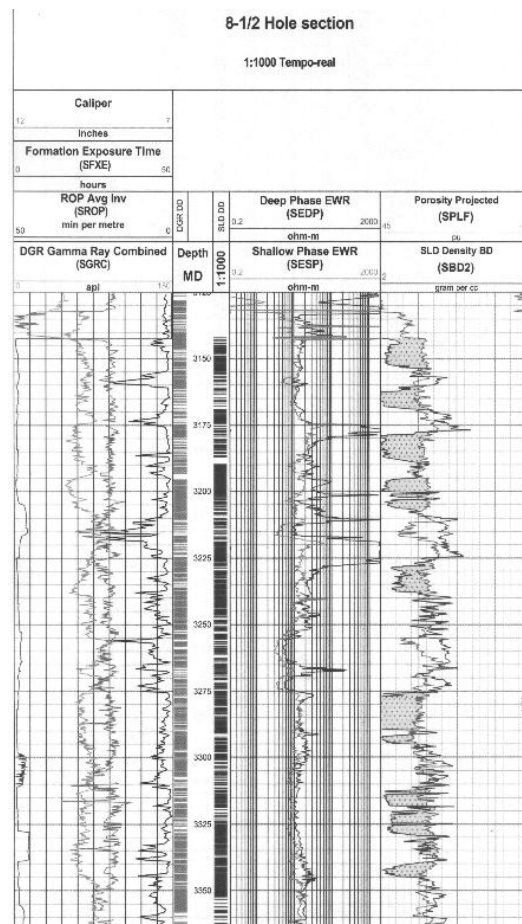


Figure 3. Part of a logging profile showing up the permeable intervals interrupted by non-permeable intervals. The section shown goes from a depth of 3125 m to approximately 3350 m. The remaining part of this profile goes to 3825 m, so that the measured depth will be related to the coordinate  $x$  by  $D_M = 3825 - x$ .

Figure (4) is derived from Fig. (3) by identifying the sandstone and shale intervals. The former are permeable and exhibit a value of  $j > 0$ . The latter are impermeable and exhibit a value of  $j = 0$ . As a simplification, all the sandstone intervals will be assigned the same  $j$ -value. Since the sandstone intervals are from the same horizon, this is not a very bad simplifying assumption. Nevertheless, the work can be easily adapted if one wants to attribute different specific productivity indexes,  $j$ , for each interval.

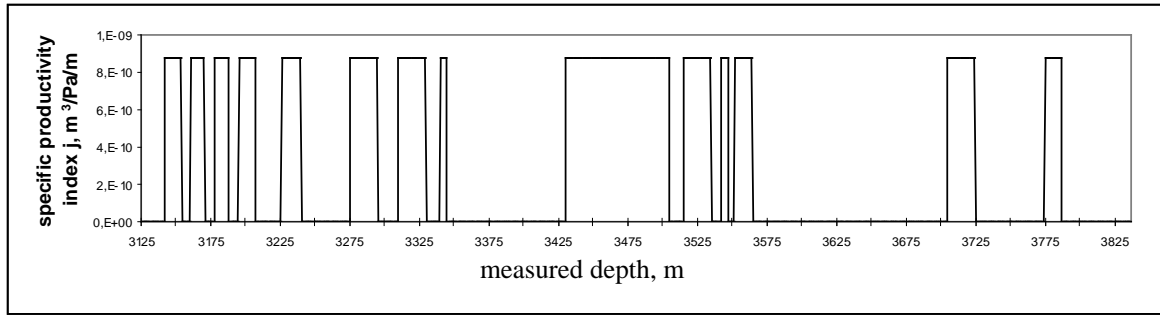


Figure 4. Distribution of sandstone and shale intervals based on the logging profile. The horizontal section goes from a measured depth of 3125 m to 3825 m. The sandstones are permeable ( $j > 0$ ). The shale intervals are impermeable ( $j = 0$ ).

Table II – Data used in the simulation for the field case.

Variable	SI	Field units
Length of horizontal section, L	700.00 m	700.00 m
Depth of nodal point A, $D_0$	3125.00 m	3125.00 m
Well total measured depth, $D_{M\max}$	3825.00 m	3825.00 m
Horizontal section diameter, d	0.1246 m	4.892 in
Formation pressure, $p_R$	26 460 kPa	270 kgf/cm <sup>2</sup>
Specific productivity index, j	$2.63 \times 10^{-11} \text{ (m}^3/\text{s)/(Pa}\cdot\text{m)}$	0.0300 (bbl/day)/(psi /m)
API density	25° API	25° API
Oil density, $\rho = 141.5/(131.5 + \text{API})$	904 kg/m <sup>3</sup>	56.4 lbm/ft <sup>3</sup>
Oil absolute viscosity, $\mu$	0.010 Pa s	10 cp
Cross-sectional area, A	0.0121264 m <sup>2</sup>	18.7959 in <sup>2</sup>
Relative roughness	0.0025	0.0025
Number of grid points, n	1000	1000
Cell length, dx	0.712 m	0.712 m

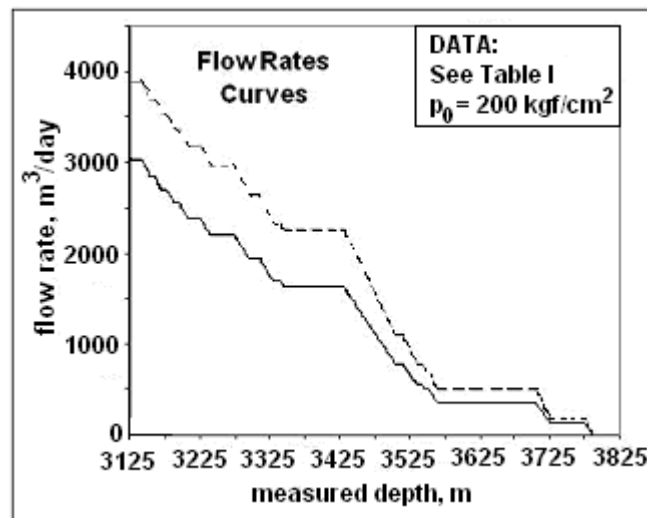


Figure 5. Flow rate increase along the horizontal section for a given value of the entrance nodal point pressure of  $p_0 = 19600 \text{ kPa}$  ( $200 \text{ kgf/cm}^2$ ).

Table II provides additional data used in the simulation of this field case. The data are presented in SI and field units. Even then these data correspond to a real well, some of the data has been modified to simulate more representative wells worldwide.

Figure 5 shows the flow rate increase along the horizontal section for a given value of the entrance nodal point pressure of  $p_0 = 19600 \text{ kPa}$  ( $200 \text{ kgf/cm}^2$ ). The dashed line shows how flow rate would grow up from bottom B to

entrance A if no frictional or accelerating pressure drop did existed. The full line shows another flow rate curve, but this time with the effects of the frictional and accelerating pressure drops. Notice how flow rate stays constant in front of the shale interval, since there is no contribution from these impermeable formations, and how it grows up in front of sandstone formations, since there will be substantial contributions in front of these permeable interval. The flow rate at the entrance nodal point drops from 3869 m<sup>3</sup>/dia to 3030 m<sup>3</sup>/dia on account of these effects, an apparent damage ratio of 1.67.

Figura 6 shows the percent decrease of the “drawdown”, the driving force for production, on account of these pressure drops effects, also for  $p_0=19600$  kPa (200 kgf/cm<sup>2</sup>). As shown, by about half the horizontal wellbore length, the “drawdown” is about 10% lower. The deeper ones go, the lower will be the “drawdown” and the lower will be the contribution of the permeable formations. On the other hand, the “drawdown” rate of change is more pronouncedly close to the nodal point A, due to the higher flow rate, that would bring about higher pressure drops.

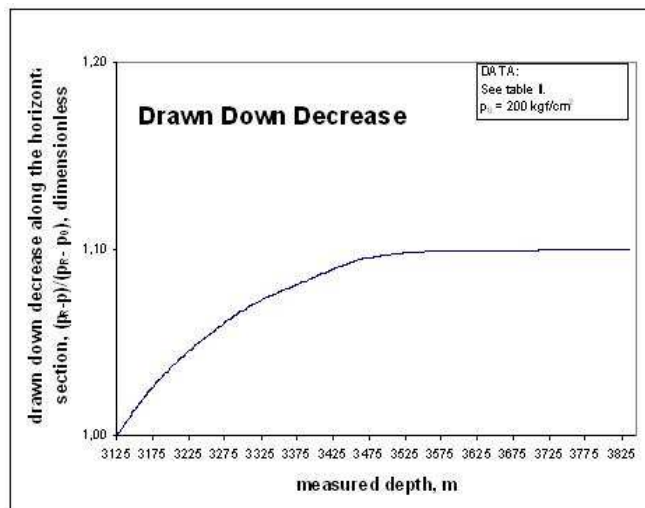


Figura 6. Percentual decrease of the “drawdown”, the driving force for production, on account of pressure drops effects.

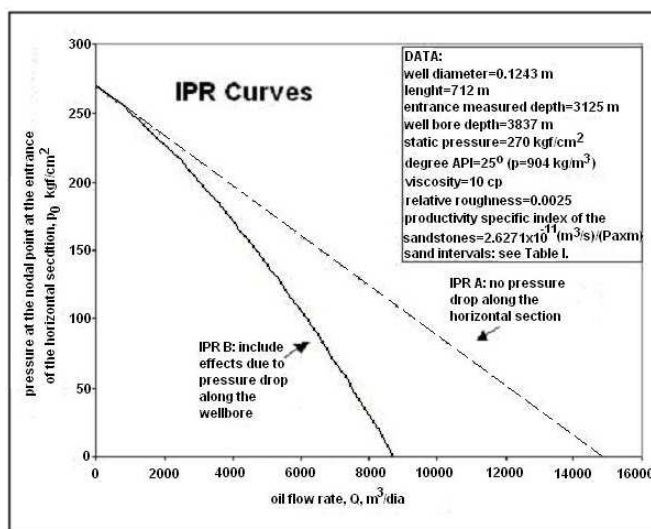


Figure 7. Curves of IPR for the cases with and without the pressure drop effects considering the nodal point A.

Figure 7 shows a very important result from these simulations, the full line IPR curve, or “Inflow Performance Relationship” curve, with the effects of these pressure drops properly considered. The dashed line shows how the IPR would appear if no frictional or accelerating pressure drop did existed. The Inflow Performance Relationship (IPR) characterizes the formation response to wellbore conditions in terms of well flowing pressure  $p_0$ . When the well flowing pressure equals the formation static pressure, flow rate is zero. As the flowing pressure inside the wellbore decreases, the “drawdown” increases, and flow rate will increase. In our case the wellbore flowing pressure increases from the entrance to the bottom of the well. The IPR will characterize not just the formation response, but the system formed by formation and wellbore.

So, the pressure at the nodal point A is used as the well flowing pressure in the calculation of the overall productivity index. This overall productivity index is related to the inclination of the IPR curve and it decreases as flow rate increases. The intersections with the horizontal axis gives the flow rate for zero “drawdown”. These are, theoretically, the maximum flow rates that could be attained, if it were possible to bring the bottom hole pressure down to zero. These maximum flow rates are known as the formation potentials. As seen, the potential without pressure drops effects of 14735 m<sup>3</sup>/dia decreases to about 8735 m<sup>3</sup>/dia on account of the pressure drop effects. This represents an apparent damage ratio of 1.71, i.e. the maximum flow rate would be 1.71 larger if these effects could be eliminated. It is known that they cannot be totally eliminated, but only partially decreased. The IPR curve shall be used together with an Outflow or Vertical Performance Curve (OPR or VPR), that will model the two-phase flow along the tubing to the surface, to find out the production flow rate of the well or point of operation.

As a limitation, it should be kept in mind that, if the wellbore flowing pressure is lower than the saturation or bubble point pressure, gas will ensue in the porous media with an additional effect in the IPR curve. Vogel (1968) IPR may be used to specify the  $j$ -values. Moreover, if the inside bottom hole pressure is below the bubble point, multiphase flow should be considered along the horizontal section (Taitel and Dukler, 1976, Dukler, 1969, Wallis, 1969).

## Conclusions

A mathematical model for the production of a horizontal well, including the effects of frictional and accelerating pressure drops has been developed. The model is constituted of two ordinary differential equations and proper boundary conditions.

This mathematical model has been solved for a field practical case to provide values for flow rate, flowing pressure and an IPR curve. This curve can be used in nodal analysis of production systems. The IPR curve is not linear and the overall productivity index decreases as flow rate increases.

It is shown that, for the case under study, an apparent damage of 1.71 is obtained in terms of potentials. Also, it is observed that the “drawdown” pressure decreases substantially with depth, but more pronouncedly close to the horizontal well entrance due to the larger flow rate effects.

An extension of this work to a transient analysis system, possibly with a modeling of the porous media flow around the horizontal section, would be of great help for analyzing productivity and injectivity transient tests.

## References

- Beggs, H. D. and Brill, J. P., 1973, “A Study of Two-Phase Flow in Inclined Pipes,” *Journal of Petroleum Technology*, May 1973., pp. 607-617.
- Cho, H., 2003, “Integrated Optimization on a Long Horizontal Well Length”, *SPE 83669, SPE Reservoir Evaluation & Engineering*, April 2003, pp. 81-88.
- Dikken, B. J., 1990, “Pressure Drop in Horizontal Wells and Its Effect on Production Performance”, *SPE 19824, Journal of Petroleum Technology*, November, 1990, pp. 1426-1433.
- Folefac, A. N., Archer, J. S., Issa, R. I. and Arshad, U., 1991, “Effect of Pressure Drop Along Horizontal Well bores on Well Performance”, *SPE 23094*, presented at the Offshore Conference held in Aberdeen, 3-6 September, 1991, pp. 549-560.
- Jin, A. K., 1976, “Accurate Explicit Equation for Friction Factor”, *Journal of Hydraulic Division*, ASCE, 1976, pp. 674.
- Orkiszewski, J., 1967, “Predicting Two-Phase Pressure Drops in Vertical Pipe,” *Journal of Petroleum Technology*, June, 1967, pp. 829-836.
- Penmatcha, V. R., Arbabi, S. and Aziz, K., 1997, “Effects of Pressure Drop in Horizontal Wells and Optimum Well Length”, *SPE 37494*, presented at the SPE Production Operations Symposium, Oklahoma City, Oklahoma, 9-11 March, 1997.
- Seines, K. et al., 1993, “Considering Wellbore Friction Effects in Planning Horizontal Wells”, *Journal of Petroleum Technology*, October 1993, pp. 994.
- Su, Z. and Gudmundsson, J. S., 1993, “Friction Factor of Perforation Roughness in Pipes”, *SPE 26521*, presented at the 68<sup>th</sup> Annual Technical Conference of the Society of Petroleum Engineers held in Houston, Texas.
- Yalniz, M. U., Ozkan, E., 2001, “A Generalized Friction-Factor Correlation to Compute Pressure Drop in Horizontal Wells”, *SPE Production & Facilities*, pp. 232-239.
- Vogel, J. V., 1968, “Inflow Performance Relationships for Solution-Gas Drive Wells”, *Journal of Petroleum Technology*, Jan. 1968, pp. 83-92.
- Taitel, Y. and Dukler, A. E., 1976, “A Model for Predicting Flow Regime Transitions in Horizontal and Near Horizontal Gas-Liquid Flow,” *AIChE J.*, 22 (1), Jan. 1976, pp. 47-55.
- Dukler, A. E., 1969, “Gas- Liquid Flow in Pipelines,” *American Gas Association, API, Vol I, Research Results*, May 1969.
- Wallis, G. B., 1969, *One Dimensional Two-Phase Flow*, McGraw-Hill, New York, 1969.

## 5. Responsibility notice

The author(s) is (are) the only responsible for the printed material included in this paper.