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Nonlinear Response of An Oldroyd Fluid In Strong Flows

Farith Salas Absi
University of Brasilia, Department of Mechanical Engineering, Brasilia-DF, 70910 900, Brazil
farith78@yahoo.com

Francisco Ricardo Cunha, (*corresponding author*)
University of Brasilia, Department of Mechanical Engineering, Brasilia-DF, 70910 900, Brazil
frcunha@unb.br

Taygoara Felamingo Oliveira
University of Brasilia, Department of Mechanical Engineering, Brasilia-DF, 70910 900, Brazil
taygoara@unb.br

Abstract. *In this article we study the nonlinear response of a FENE-dumbbell fluid undergoing strong extensional flows. We start with a general description of the dimensionless governing equations for low Reynolds number. The elastic liquid is described by a pair of coupled constitutive equations related to the stress and the macromolecule conformation. The non-dimensional measure of the elastic stress defines the Deborah number, which is the ratio of the relaxation time of the material to the time for the liquid to flow through the region of interest. Two different asymptotic regimes are examined. The first corresponding to an elastic limit in which the extensional viscosity is a function of the Deborah number and the particle volume fraction. The second is an anisotropic regime with the extensional viscosity independent of Deborah number but strongly dependent on the extensibility of the macromolecule. The analysis presented in this work may explain at least from a phenomenological point of view why few ppms of macromolecules of high molecule weight or a small volume fraction of long fibers produce important attenuation of the pressure drop in turbulent flows.*

Keywords: *Extensional viscosity , Anisotropy , Macromolecule model*

1. Introduction

The addition of small amounts of polymer to a fluid can drastically enhance the extensional viscosity of the solution while just slightly affecting the shear viscosity (Hinch, 1977, Hinch, 1994). The large increase in the extensional viscosity of dilute polymer solutions can drastically reduce the pressure drop in flows through porous media, observed by Durst, Haas and Interthal (1982), and the drag in turbulent pipe flow, observed by a vast number of work (e.g. Scrivener et al., 1979; Min at al., 2003; Andreotti, Cunha and Sousa, 2003; Andreotti, 2004). For polymer solutions, however it is only recently that the technique of filament stretching at constant extension rates, that has been introduced by Tirtaatmadja and Sridhar (1993), allowed comparable extensional viscosity measurements.

Due to the important role of the extensional viscosity in these and many other flow phenomena, it is of great interest to be able to measure the extensional viscosity of dilute polymer solutions. However, it is difficult to create a well defined extensional flow field for dilute polymer solutions and then difficult to develop models describing the behavior of the macromolecules in the flow. The rheological properties of polymer solutions are strongly dependent upon the concentration regime of the polymer under study. In semi-dilute and concentrated systems, entanglement effects can have a predominant role in defining the behavior of the system. In dilute solutions the rheological properties of the system are a reflection of the mechanical properties of the individual polymer chains and the number of chains in the bulk solution. However, even in such dilute systems, the mechanical properties of the individual chains and, hence, the bulk solution are strongly dependent on the type of flow. In shear flow, the mechanical properties change gradually with the shear rate while in extensional flow the variations can be relatively sharp. Close to the critical strain rate, a coil stretch transition occurs where the coil quickly reaches its full stretched state. Such stretching significantly increases the hydrodynamic volume of the polymer chain, which leads to an increase in the extensional viscosity of the solution.

The rheological behavior of the common Oldroyd constitutive equation is well-known (e.g. Bird, Armstrong and Hassager, 1987). In steady simple shear flow, the effective viscosity is constant, the first-normal-stress difference increases quadratically with shear-rate, and there is no second-normal-stress differences. In steady pure straining motion, the extensional viscosity increases with strain-rate, becoming infinite at the finite critical Deborah number ($De \sim 1$). To understand the physics in this constitutive equation for elastic liquids, it is helpful to look at a micro-structural model which leads to an Oldroyd-like constitutive equation. Rather than studying the rheological performance and mathematical structure of the coupled flow equations, we retreat to some physical model of the microstructure which generates the constitutive equation. In this article it is considered a bead-and-spring model of polymer solutions.The simplest version of the bead-and-spring model (i.e. dumbbell model) introduced by by Kuhn and Kuhn (1945) and discussed in details by Hinch (1977) and Rallison and Hinch (1988) with a finite spring correction, i.e. a FENE (finitely extensible non-linear elastic) model. The original empirical FENE spring force was first proposed for dumbbells by Warner (1972).

We present scaling arguments based on a balance between viscous drag and a restoring Brownian force which lead to a time dependence partial differential equation for the conformation tensor of the macromolecule. In particular, this tensor gives an explicit information about the internal structure of the fluid. We then describe a highly anisotropic fluid flow with a nonlinear spring model which gives a finite limit of macromolecule extension and a nonlinear damper in the way suggested by Rallison and Hinch (1988). The present analysis has been attractive because deformable particles almost always become rod-like in shape when greatly deformed by the bulk flow, and this anisotropy may act as a stabilizing factor even in the flow of very dilute polymer solution.

2. Dimensional Analysis

Dilute polymer solutions (no overlapping of the macromolecules) behavior can be idealized as being a polymer chain composed of two beads representing drag forces, linked to a Hookean spring representing elastic forces (see figure 1). This configuration bead-and-spring is called a dumbbell (e.g. Bird, Armstrong and Hassager, 1987). In this model a macromolecule is composed of N rigid segments (each corresponding to an individual monomers) of length δ, with each individual manometer randomly oriented with respect to the adjoining segments. The equilibrium configuration distribution of the macromolecule is then given by a random walk of N steps each of length δ. In this model the mean end-to-end distance of the polymer, a is proportional to $N^{1/2}\delta$ as required by the central limit theorem, which is smaller than the length of the polymer chain by a factor of $N^{1/2}$. In figure (1a) the vector $\mathbf{r}$ between the centres of the two beads represents the end-to-end distance of the polymer molecule.

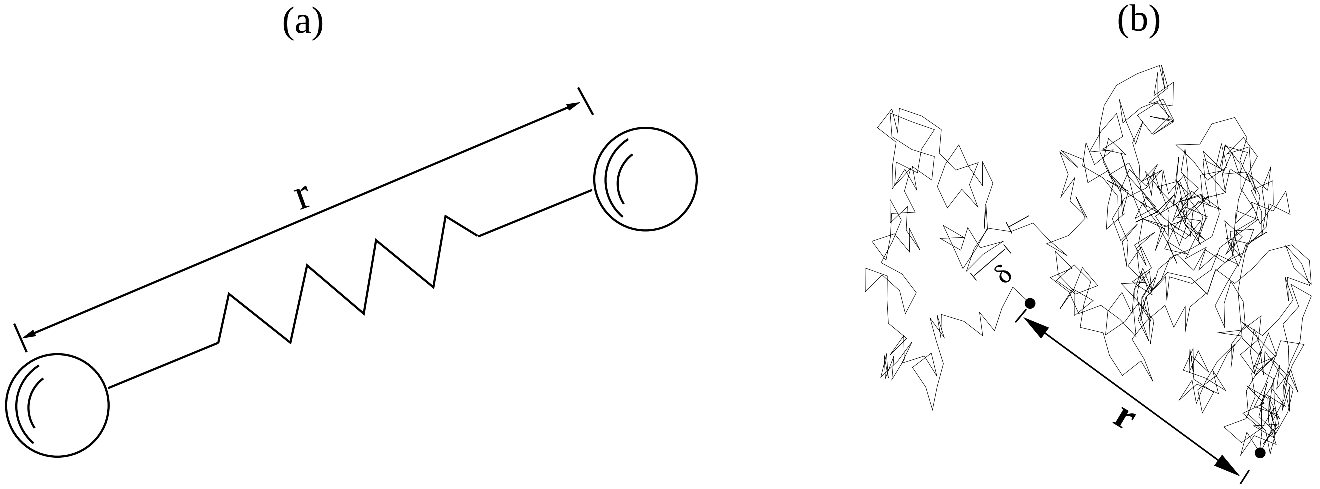

Figure 1. Sketch of a polymer molecule. (a) The elastic dumbbell model and (b) Polymer configuration corresponding to a chain of N individual manometer randomly orientated.

In a flow, the distribution of polymer configurations will be affected by the local velocity gradient. In an extensional flow the difference in the fluid velocity acting on the two halves of the polymer will cause the molecule to extend. The extension is opposed by Brownian motion which tend to restore the equilibrium distribution.

Let us examine the equilibrium condition between Brownian and viscous forces and consider for instance the simplest linear Hookean spring for modelling the restoring Brownian force. We express the elastic force as being $F_B = Gr$, where G denotes the spring constant given by $G = 3KT/a^2$ (e.g. Flory, 1969). Here a denotes the equilibrium length of the macromolecule (i.e. a typical value of $|\mathbf{r}|$ for the elastic regime where a linear spring model makes sense.), K is the Boltzmann constant and T the absolute temperature.

The hydrodynamic interactions between the molecule and the solvent is represented by the viscous drag on the beads. So, in a dumbell model the viscous force is defined as the force exerted over an isolated spherical particle of ratio a by the solvent with viscosity μ. At low Reynolds number, the expression is given by the Stokes law, namely: $F_V = 6\pi\mu a dr/dt$. At the thermodynamic equilibrium state, the balance between the viscous and the restoring forces acting on the macromolecule leads to

$$\frac{dr}{dt} = \frac{G}{C_v} r, \tag{1}$$

where $C_v = 6\pi\mu a$. Integrating the Eq. (1), one obtains:

$$r(t) = r_0 \exp\left(\frac{t}{\tau}\right), \tag{2}$$

where $\tau = C_v/G = 2\pi\mu a^3/KT$ denotes the relaxation time of an extended macromolecule to the randomly-coiled state. Since N is equal to the molecular weight of the polymer (M) divided by the molecular weight of a single monomer (M_i), $N = M/M_i$ it results that the macromolecule relaxation time is proportional to $M^{3/2}$.

2.1 Physical Parameters

In defining the Reynolds number for a polymer solution, we need to define carefully what is meant by he viscosity μ, since both shear and extensional viscosity vary wit shear-rate. Conventionally μ is taken to be the shear viscosity in the limit of small shear rates. Most polymeric fluids are highly viscous and so, we will consider the limit where Re is small. For the flow of a fluid density ρ and viscosity μ past a body of length scale a with velocity scale u_c, Re is defined to be $Re = \rho u_c a/\mu$.

In the context of elastic liquids a fundamental parameter is the Deborah number, that is defined as the ratio of the relaxation time and a characteristic time of the flow, i.e. $De = \tau/t_f$, where τ denotes the relaxation time of the polymer. The relaxation time measures the ability of the macromolecules to recoil to their equilibrium condition after stretching. The flow time scale t_f is defined as the ratio of a velocity U and a length a, characteristics of the flow. For $De << 1$ means weak flow and the fluid behaves closely to a Newtonian fluid. In this limit, Brownian motion prevents the polymer from becoming highly extended, and the macromolecule distribution remains close to the randomly coiled distribution. We denote this limit as being the elastic regime, with $r \sim a$. For large values of Deborah (i.e. $De \sim 1$), however, the flow is strong enough to overcome the Brownian relaxation and so the polymer may become greatly extended, so that the mean end-to-end distance becomes large compared to its values in the randomly-coiled distribution, i.e. $r \gg a$. This second regime will correspond to the anisotropic limit of the macromolecule. For strong flows $De \sim 1$ the macromolecule stretching produces anisotropy in the flow and non-linear effects as normal-stress-difference.

A second important dimensionless parameter is the extensibility of the polymer, L defined as being the ratio of the full stretched length ($\ell = N\delta$) to the average length corresponding to the end-to-end distance in the randomly coiled state of the macromolecule, a, i.e. $L = N\delta/a$. In the present work while Deborah can be interpreted as a measure of the polymer elasticity on the flow, L will denote a measure of the flow anisotropy created by the macromolecule orientation. We will examine the effect of both contribution to the flow.

The final dimensionless group is a measure of the effective volume fraction of the macromolecule evaluated at the randomly coiled state, namely $\phi = 4\pi a^3 n/3$, where n is the number of molecules N_m per unit of volume , $n = N_m/V$. For the solution to be dilute, ϕ must be much less than unit, therefore the polymer contribution to the fluid stress will be negligible unless the polymer become highly extended.

3. Mathematical Model

The balance equations are the conservation of mass and of momentum

$$\nabla \cdot \mathbf{u} = 0, \qquad \nabla \cdot \mathbf{\Sigma} = \mathbf{0} \tag{3}$$

where inertia is neglected.

3.1 Material Description

In general, the stress tensor for an elastic liquid is given by the following constitutive equation

$$\mathbf{\Sigma} = -p\boldsymbol{I} + \boldsymbol{\sigma} \tag{4}$$

with the deviatoric stress $\boldsymbol{\sigma}$ given by the following expression

$$\boldsymbol{\sigma} = 2\mu_s(\phi, \dot{\gamma})\mathbf{D} + \overline{\boldsymbol{\sigma}}_B, \tag{5}$$

where p is the mechanical pressure, $\boldsymbol{I}$ is the unit tensor, $\mu_s(\phi, \dot{\gamma})$ is the shear viscosity as a function of the volume fraction ϕ and the shear rate $\dot{\gamma}$, $\overline{\boldsymbol{\sigma}}_B$ is the elastic stress contribution due to the presence of macromolecules and $\mathbf{D}$ is the rate of the strain tensor, $\mathbf{D} = (\nabla\mathbf{u} + \nabla\mathbf{u}^T)/2$.

Next we derive an equation for $\overline{\boldsymbol{\sigma}}_B$ which correlates the average microscopic behavior of macromolecules with the macroscopic response of the fluid.

3.2 Nonlinear Relation Stress-Rate of Strain

Since we are treating with dilute polymer solutions, the shear viscosity may be considered as being a linear function of the volume fraction only, i.e. $\mu_s(\phi, \dot{\gamma}) \approx \mu(1 + c\phi)$. For a dilute aqueous solution of polyacrylamide-PAMA, the best fitting constant is found to be $c = 1.4 \times 10^4$ (Andreotti, Cunha and Sousa, 2003).

Now, the non-Newtonian average contribution stress due to the macromolecules is given by a volume average over a volume V sufficiently large to contain a meaningful number of macromolecules, that is

$$\overline{\boldsymbol{\sigma}}_B = \lim_{V\to\infty} \frac{1}{V}\int_0^V \boldsymbol{\sigma}_B d\mathbf{x} = n\langle \boldsymbol{\sigma}_B \rangle, \tag{6}$$

where $\langle\rangle$ represents an ensemble average.

According to the scaling analysis presented in §1, the elastic restoring force can be written as

$$\mathbf{F_B} = G\mathbf{r}, \tag{7}$$

and the associated stress tensor for one macromolecule is given by the dyadic $\boldsymbol{\sigma}_B = G\mathbf{rr}$

Now, taking the average over N_m macromolecules within a sufficiently large volume V, one obtain

$$\langle \boldsymbol{\sigma}_B \rangle = G\left(\frac{1}{N_m}\sum_{s=1}^{N_m} \mathbf{rr}\right). \tag{8}$$

Since $n = N_m/V$, $\overline{\boldsymbol{\sigma}}_B = n\langle\boldsymbol{\sigma}_B\rangle = nG\langle\mathbf{rr}\rangle$.

We define the conformation tensor or the moment of inertia tensor of the deformable macromolecule as being

$$\boldsymbol{B}(t) = \langle \mathbf{rr}\rangle = \frac{1}{N_m}\sum_{s=1}^{N_m}\mathbf{rr}. \tag{9}$$

Note that $\boldsymbol{B}$ is by definition symmetric and positive definite tensor.

Rewriting the Eq. (5) in terms of the conformation tensor it leads to:

$$\boldsymbol{\sigma} = 2\mu_s(\phi)\mathbf{D} + nG\boldsymbol{B}(t) \tag{10}$$

3.3 Time Developing of the Conformation Tensor

The closure problem of the constitutive equation, Eq. (10) requires a time evolution equation for the conformation tensor $\boldsymbol{B}(t)$. Turning back to the scaling analysis given in §(2), the balance between elastic and viscous forces gives

$$6\pi\mu a\frac{d\mathbf{r}}{dt} + G\mathbf{r} = 0. \tag{11}$$

Now, multiplying both sides of the Eq. (11) by $\mathbf{r}$, subtracting the rigid body translation and adopting a frame of reference rotating and deforming with the macromolecule, after averaging one obtains:

$$3\pi\mu a\frac{\delta\mathbf{B}}{\delta t} + G\mathbf{B} - KT\mathbf{I} = 0. \tag{12}$$

Here $\delta/\delta t$ is the upper convective time derivative or the Oldroyd derivative that is material frame indifference. It is simply the rate of change of $\mathbf{B}$ seen by an observer translating and deforming with the macromolecule

$$\frac{\delta\mathbf{B}}{\delta t} = \frac{D\boldsymbol{B}}{Dt} - \nabla\boldsymbol{u}\cdot\boldsymbol{B} - \boldsymbol{B}\cdot\nabla\boldsymbol{u}^T, \tag{13}$$

Now, writing Eq. (12) in terms of the Oldroyd derivative Eq. (13) and after rearranging the equation, we find

$$\frac{D\boldsymbol{B}}{Dt} = \nabla\boldsymbol{u}\cdot\boldsymbol{B} + \boldsymbol{B}\cdot\nabla\boldsymbol{u}^T - \frac{2}{\tau}\left[\mathbf{B} - (a^2/3)\mathbf{I}\right] \tag{14}$$

where D/Dt denotes the translational material derivative. Here, the first term on the right-hand side in equation (14) represents the stretching of the macromolecule by the flow, and the second term represents the relaxation of the polymer due to Brownian motion.

It should be important to note that by few algebraic manipulation, taking the Oldroyd derivative of the stress Eq. (10) and using the microstructure equation (12) it is straightforward to show that the pair of constitutive equation given by Eqs. (10) and (14), reduces to the most common Oldroyd B fluid, Bird, Armstrong and Hassager (1987), namely:

$$\hat{\boldsymbol{\sigma}} + \lambda\frac{\delta\hat{\boldsymbol{\sigma}}}{\delta t} = 2\mu(\phi)\left(\boldsymbol{D} + \lambda\frac{\delta\boldsymbol{D}}{\delta t}\right), \tag{15}$$

with $\hat{\boldsymbol{\sigma}} = \boldsymbol{\sigma} - nKT\boldsymbol{I}$ and the material constant $\lambda = \tau/2$.

In this work we will keep the pair-of-constitutive equations model in order to explore the explicit dependence of the microstructure given by the behavior of the second moment tensor $\mathbf{B}$.

3.4 Non-linear Bead-and-Spring Model.

A dumbbell model with a drag correction and a non-linear spring force which gives a finite limit to the spring extension is considered. A Hookean spring may produces unlimited elongation of the dumbbell. Such a behavior is not only unrealistic but can also lead to numerical difficulties as unlimited values of the stress tensor. These difficulties can at least be partially eliminated by limiting the extensibility of the macromolecule Rallison and Hinch (1988). Changing the linear spring law to one with a finite extension, i.e. $Gf(r)$, yields to a dumbbell-FENE model, where $f(r)$ is the nonlinear spring law. Actually the correct spring law for a random-chain model was proposed by Flory (1969) as being $f(r) = (\ell/3r)F^{-1}(r/\ell)$, where $F(z) = coth(z) - 1/z$ is the Langevin function. Because this law is difficult to work mathematically, we use instead a simpler law with the same qualitative behavior such as first proposed by Warner (1972)

$$f(r) = \frac{l^2}{l^2 - r^2} \quad \text{with} \quad r^2 = tr(\boldsymbol{B}). \tag{16}$$

Note that as r tends to the rigid fiber limit ℓ, the polymer reaches its limits of maximum extensibility, whereas in the case of small distortions of the macromolecules $r \sim a$ (i.e. randomly-coiled state), $f(r) \sim 1$ (i.e. linear spring law would be appropriated). In addition, the FENE dumbbell model described above needs also to take account of the variation in the hydrodynamic drag for $r \sim \ell$. In this respect this model is strictly valid only for small values of r. As the polymer expands, the size of the object on which the frictional force of the fluid acts increase. The viscous drag increases with the largest linear dimension and so the size of the beads should increase roughly with $r = tr(\mathbf{B})^{1/2}$ (Hinch, 1977).

After incorporating into the model the corrections discussed above, the constitutive equations for the stress tensor and the conformation tensor are written respectively as being

$$\boldsymbol{\sigma} = 2\mu_s(\phi)\mathbf{D} + nGf(r)\boldsymbol{B}(t), \tag{17}$$

$$\frac{d\boldsymbol{B}}{dt} = \nabla\boldsymbol{u}\cdot\boldsymbol{B} + \boldsymbol{B}\cdot\nabla\boldsymbol{u}^T - \frac{2af(r)}{\tau[tr(\mathbf{B})]^{1/2}}\left[\mathbf{B} - (a^2/3)\mathbf{I}\right]. \tag{18}$$

Eqs. (17) and (18) can be made dimensionless with appropriate scales. Using u_c and a as the reference speed and the reference length respectively, the flow is characterized by the Deborah number $De = 2\pi\mu a^2 u_c/KT$; for a simple shearing motion with shear rate $\dot{\gamma}$ the characteristic velocity $u_c = a\dot{\gamma}$, whereas for a pure extensional flow with rate of strain $\dot{\epsilon}$, $u_c = a\dot{\epsilon}$. As mentioned before the other parameter of interest are the particle volume fraction ϕ and the extensibility of the polymer $L = \ell/a$. The dimensionless constitutive equations expressed in terms of the stress $\widetilde{\boldsymbol{\sigma}}$ and the conformation tensor $\widetilde{\boldsymbol{B}}$ of the polymer are respectively

$$\widetilde{\boldsymbol{\sigma}} = \widetilde{\mu}(\phi)\widetilde{\mathbf{D}} + \frac{9\phi f(R)}{2De}\widetilde{\mathbf{B}} \tag{19}$$

$$\frac{d\widetilde{\mathbf{B}}}{d\widetilde{t}} = \widetilde{\nabla}\widetilde{\mathbf{u}}\cdot\widetilde{\mathbf{B}} + \widetilde{\mathbf{B}}\cdot(\widetilde{\nabla}\widetilde{\mathbf{u}})^T - \frac{2f(R)}{De[tr(\widetilde{\mathbf{B}})]^{1/2}}\left(\widetilde{\mathbf{B}} - \frac{\mathbf{I}}{3}\right) \tag{20}$$

where $\widetilde{\boldsymbol{\sigma}} = a\boldsymbol{\sigma}/\mu u_c$, $\widetilde{\mu} = \mu_s/\mu$, μ is the solvent viscosity, $\widetilde{\mathbf{B}} = \mathbf{rr}/a^2$, $R = r/a$ and the spring function in terms of the dimensionless quantities R and L is given by $f(R) = L^2/(L^2 - R^2)$. By examining the right-hand side term in the equation (20), it becomes clear that for large Deborah, the contribution for the polymer relaxation is negligible compared to the contribution of the first term responsible for the stretching of the polymer. One can see that this is possible only if one considers high relaxation time macromolecule compared to the time scale of the flow.

4. Results: Two Asymptotic Limits of the Fluid System

A physical interpretation of the role of the anisotropy and the elasticity on a flow due to the presence of additives can be gained by examining two limiting cases of the equation (20) for steady extensional flows. In a steady uniaxial extensional with strain-rate $\dot{\epsilon}$ the velocity field is given by, $\mathbf{u} = (\dot{\epsilon}x_1, -\frac{1}{2}\dot{\epsilon}x_2 - \frac{1}{2}\dot{\epsilon}x_3)$ with the dimensionless extensional viscosity being calculated as below

$$\frac{\mu_e}{\mu} = \frac{2\widetilde{\sigma}_{11} - \widetilde{\sigma}_{22} - \widetilde{\sigma}_{33}}{6} \tag{21}$$

In the above equation we denote by 1, 2 and 3 the streamwise, transverse and spanwise directions, respectively. The definition for μ_e given in equation (21) is normalized by using Trouton's viscosity which corresponds to the extensional viscosity of a Newtonian fluid experimenting a steady extensional flow, $\mu_e/\mu = 3$.

4.1 The Elastic Regime

In the case of elastic limit, when $f(R) = 1$, $De \sim 1$ and $R \sim L$, the governing equation (20) of $\mathbf{B}$ evolution reduces simply to

$$\frac{d\widetilde{B}_{11}}{d\widetilde{t}} = 2\left[(1 - \frac{1}{De})\widetilde{B}_{11} + \frac{1}{3De}\right], \quad \frac{d\widetilde{B}_{22}}{d\widetilde{t}} = \frac{2}{3De} - (1 - \frac{2}{De})\widetilde{B}_{22}, \quad \widetilde{B}_{33} = \widetilde{B}_{22}. \tag{22}$$

Integrating we find for the stationary solution $B_{11} = (1 - De)^{-1}/3$ and $B_{22} = 2(De + 2)^{-1}/3$. A prediction of the extensional viscosity for this elastic limit is obtained by inserting $\widetilde{B}_{11}$ and $\widetilde{B}_{22}$ into the stress equation (19) and using (21), which yields

$$\mu_e/\mu = \mu_s + \frac{3\phi}{2}(De + 2)^{-1}(1 - De)^{-1}. \tag{23}$$

Thus, at leading order

$$\mu_e/\mu = \mu^* + \frac{3}{8}\phi De, \tag{24}$$

where $\mu^* = \mu_s + 3\phi/4$ represents a modified shear viscosity as a function of the particle volume fraction. The result indicates that in a dilute polymer solution for which $\phi \ll 1$ under weak flow ($De \ll 1$), the Non-Newtonian contribution $O(\phi De)$ on the extensional viscosity is negligible effect and the behavior of dilute polymer solutions in this regime is approximately Newtonian and so is of less interest.

4.2 The Anisotropic Regime

Providing that $R \sim L \gg 1$, in the steady extensional flow the extension of the macromolecule may be considered predominantly in the 11 - direction. At high Deborah numbers (i.e. strong flows, $De \sim 1$), $\widetilde{B}_{11}$ will be order L^2, whereas $\widetilde{B}_{22}$ and $\widetilde{B}_{33}$ will be of order unit. Making the approximations $R^2 = tr(\widetilde{\mathbf{B}}) \approx \widetilde{B}_{11}$ and $\widetilde{\nabla}\widetilde{\mathbf{u}} \cdot \widetilde{\mathbf{B}} = \widetilde{\mathbf{B}} \cdot (\widetilde{\nabla}\widetilde{\mathbf{u}})^T \approx B_{11}\mathbf{e_1}\mathbf{e_1}$, and taking the trace of equation (20) it reduces to

$$\frac{dR}{dt} = R - \frac{2f(R)}{R^2 De}(R^2 - 1). \tag{25}$$

Defining the equilibrium extension of the macromolecule R_L as being the saturated value of R for a $De \sim 1$. Then R_L satisfies

$$R_L - \frac{2f(R_L)}{R_L^2 De}(R_L^2 - 1) = 0 \tag{26}$$

At equilibrium, the component of the dimensionless non-Newtonian stress in the 11-direction given by the equation (19) corresponds to the magnitude of the dimensionless extensional viscosity, namely

$$\frac{\mu_e}{\mu} = \frac{9\phi}{2De} R_L^2 f(R_L) = \frac{9\phi}{2} \frac{R_L^5}{R_L^2 - 1}. \tag{27}$$

The result indicates that in the limit of highly extended polymer the extensional viscosity is independent of the polymer relaxation time (i.e. the Deborah number), and when R_L tends to $L \gg 1$ (i.e. the rigid fiber limit) the extensional viscosity is proportional to ϕL^3, having the same scaling predicted by slender body theory for the extensional viscosity of a suspension of rigid rods (Batchelor, 1970).

Now, we can also use this calculation in order to estimate the Deborah number De_L corresponding to the asymptotic limit $R_L \to L$ for a given $L \gg 1$. Writing $R_L = cL$, with c being a constant very close to unit and inserting this condition into equation (26) we obtain

$$De_L \sim \frac{1}{c(1 - c^2)} L^{-1} \tag{28}$$

Equation (28) yields $De_L \approx 0.6$ for $c = 0.99$ and a typical value of $L = 80$. Therefore, in a polymer solution for which $\phi \ll 1$ but $\phi L^3 > 1$, the presence of polymer molecules will produce a potentially strong effect due to the anisotropy produced by the highly extended particles in flows with $De \sim 1$.

5. Final Remarks

In this article we have generated a pair of constitutive equations for describing anisotropic fluids through scaling arguments based on a balance between viscous drag and restoring Brownian forces. By this way the physics involved in the constitutive equations of elastic liquids has been better understood. We have shown theoretical results for extensional viscosity in terms of Deborah number De, the volume fraction ϕ and the extensibility of the polymer L. The studies have revealed that, once Deborah number reaches an critical value around $De_c = 1$, the macromolecules behave closely to rigid fibres capturing the same scaling given by viscous slender body theory of rigid rods. That is $\mu_\epsilon/\mu \sim= \phi L^3$, independent of Deborah number. In this nonlinear flow regime the macromolecule elasticity becomes irrelevant and the flow depends mainly on the anisotropy produced by the alignment of the highly stretched macromolecules with the flow. The present studies have suggested that this anisotropy should greatly dominates retardation or elastic effect in strong flows like turbulence. Thus, close to the rigid rod limit, the anisotropy produced by the extended macromolecule would be the key mechanism to explain dilute polymer solution giving turbulent drag reduction.

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