THE WIND CONSIDERED AS A NON-IDEAL EXCITATION IN MODEL OF A WIND TURBINE TOWER WITH IMPACT DAMPER

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Abstract. *We present simple mathematical models of a wind turbine tower. Here, the wind excitation is considered to be a non-ideal power source. In such a consideration, there is interaction between the energy supply and the motion of the supporting structure. If power is not enough, the rotation of the generator may get stuck at a resonance frequency of the structure. This is a manifestation of the so-called Sommerfeld Effect. In this model, at first, only two degrees of freedom are considered, the horizontal motion of the upper tip of the tower, in the transverse direction to the wind, and the generator rotation. Next, we add another degree of freedom, the motion of a free rolling small mass inside a chamber. Its impact with the walls of the chamber provides control of both the amplitude of the tower vibration and the width of the band of frequencies in which the Sommerfeld effect occur. Some numerical simulations are performed using the equations of motion of the models obtained via a Lagrangian approach.*

Keywords*: wind turbines; non-ideal power sources; non-linear dynamics; impact damper*

1. Introduction

In the last years, the production of wind turbines as alternative form to generate energy has grown very rapidly. The technological development has allowed the use of more slender and light towers increasingly. For example, recently it was inaugurated the biggest wind turbine of the world at the moment, supported by a 120 m high tower and generating 5 MW of power, sufficient to supply 4,500 households. The vibration analysis of those structures by the designers has been quite limited in scope. Some basic features of the analysis and the design of the prototype of a steel 1 MW wind turbine tower are presented in Lavassas (2003). Figure 1(a) shows the Wind Park in Mucuripe/Ceará/Brazil (2003), composed by four wind turbines E-40/600 kW with 40 m rotor diameter. Figure 1(b) gives an internal view of a conical steel tower during manufacturing.

In this paper, we intend to discuss and connect two interesting issues in nonlinear dynamics: a) non-ideal power sources, represented, in this case, by the in-board non-ideal generator being lead by an unbalanced rotor that induces structural vibrations; b) impact damper for controlling high-amplitude vibrations.

A simple (ideal) model supposes a periodic forcing coming from an external source that is not appreciably perturbed by the motion of the structure, as in Nordmark (1991). However, in practical situations, the dynamics of the forcing system cannot be considered as given a priori, and it must be taken as also a consequence of the dynamics of the whole system, see, for example, Warminski et al. (2001). In other words, if the forcing system has a limited energy source, as that provided by wind, its own dynamics is influenced by that of the oscillating system being forced, as in Kononenko (1969). This increases the number of degrees of freedom, and is called a non-ideal problem.

Vibro-impact systems have oscillating parts colliding with other vibrating components or rigid walls. A vibro-impact system of practical importance is an impact damper, the vibration of a primary system being controlled by the momentum transfer through collisions of the primary system with a secondary loose mass that bounces back and forth, as in Chaterjee et al. (1995a), Marhadi and Kinra (2005) and Duncan et al. (*in press*). Impact dampers have been used to control high-amplitude oscillations, such as those typically appearing in chaotic motion, and hence these vibro-impact systems can be regarded as devices for controlling chaos in mechanical engineering systems, as in Chaterjee et al. (1995b). In fact, impact dampers have been used to perform such tasks in cutting tools, turbine blades and chimneys. Chaterjee and co-workers (1995a, 1995b) studied the dynamics of impact dampers for both externally and self-excited oscillators. Their model consists of a damped oscillating structure and a point mass that collides with walls of a container mounted directly to the primary mass. It is through these seemingly random collisions that the momentum transfer is effective to decrease the amplitude of oscillations, in particular for situations where the dynamics is chaotic, either of a transient or stationary nature.

In terms of the vibrating model of Chaterjee et al. (1995a), the non-ideal system is obtained by replacing the external sinusoidal driving of the structure by a rotor attached to the structure, and fed by a motor, as in Warminski et al. (2001). The angular momentum of the rotor is partially imparted to the structure. The application of the non-ideal model to the gear rattling dynamics has been done in a recent paper by Souza et al. (2002).

In this paper, we aim to analyze a possible practical application. A wind turbine tower with an unbalanced non-ideal generator suffers the Sommerfeld Effect of getting stuck at resonance (energy imparted to the generator being used to excite large amplitude motions of the supporting structure). We intend to show that the impact damping may control the undesired phenomena without dissipation of energy. A parallel work interested on control of chaos is to be found in Souza et al. (2005).

This paper is organized as follows: in Section 2.1 we derive the system of ordinary differential equations describing the dynamics of a non-ideal structural system without impact dampers. The inclusion of the impact dampers, by means of a bouncing particle with momentum transfer to the vibrating structure, is left to Section 2.2, where the boundary conditions are worked out in order to yield numerical solutions for the equations of motion. Section 3 presents numerical results about controlling motion of the structure by means of its interaction with the impact dampers. Our conclusions are left to the final section.

Figure 1. (a) Wind Park in Mucuripe/Ceará (Brazil, 2003): Four wind turbines E-40/600 kW (40 m rotor diameter). (b) Internal view of a conical steel tower in manufacturing.

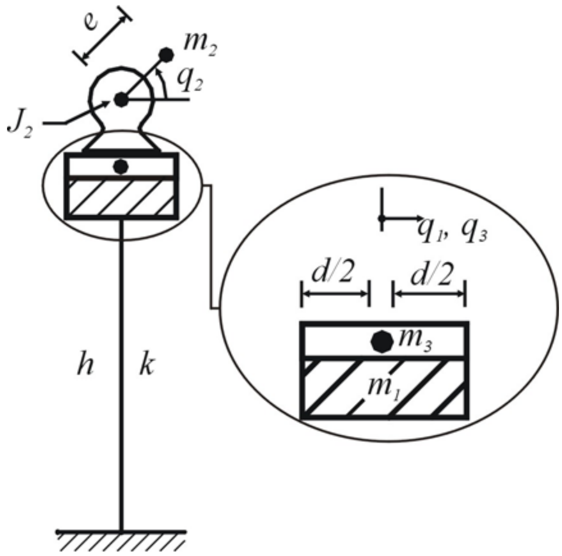

Figure 2. Model of a wind turbine tower.

2. The mathematical model

Our structure, as displayed in Fig. 2, is a simple cantilever tower, designed as a foundation for a wind turbine. The mass m_1 of the structure and rotor is supposed to be lumped at the tower upper tip. The column mass is considered not to change its height h as it undergoes lateral flexion. The column stiffness related to the horizontal motion is k. To simulate a possible unbalance of the motor, we include a small unbalance mass m_2 at distance e of the rotor axis (eccentricity). The moment of inertia of this rotor is J_2.

To derive our equations of motion, we use the Lagrange equations in the form

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_k}\right)-\frac{\partial L}{\partial q_k}=-\frac{\partial \Im}{\partial \dot{q}_k} \qquad k=1,2,\dots n \tag{1}$$

where $L = T - V$ is the Lagrangian function and
q_k → k-th generalized coordinate
T → kinetic energy
V → total potential energy
$\Im$ → dissipation function
n → number of degrees of freedom

As displayed in Fig. 2, in our model, q_1 is the horizontal motion of the tower and q_2 is the angular displacement of the generator rotor.

2.1. Uncontrolled model

We first derive the equations of motion for the uncontrolled model, that is, in the absence of the impacting mass inside the cavity.
The strain energy is

$$U=\frac{1}{2}kq_1^2\,, \tag{2}$$

the work of the conservative forces (only the weight of the unbalanced mass) is

$$W_c=-m_2 g\ e\ sin\, q_2\,, \tag{3}$$

g being the acceleration of gravity, and the total potential energy is

$$V=U-W_c\,. \tag{4}$$

The kinetic energy of our system, in terms of the velocities of the generalized coordinates, indicated by a dot over the symbols, is

$$T=\frac{1}{2}\left\{m_1\dot{q}_1^2+J_2\dot{q}_2^2+m_2\left[\left(\dot{q}_1-e\dot{q}_2 sin\, q_2\right)^2+\left(e\,\dot{q}_2\cos q_2\right)^2\right]\right\} \tag{5}$$

We introduce a Rayleigh dissipation function in order to model viscous structural damping and the mechanisms of introduction and dissipation (internal friction) of energy in the rotor, in the form

$$\Im=\frac{1}{2}(c\dot{q}_1^2+b\dot{q}_2^2)-a\dot{q}_2\,, \tag{6}$$

where
c viscous structural damping constant
a,b generator constants (given by the manufacturers).

Introducing (4)-(6) into (1), we obtain the following pair of nonlinear ordinary differential equations of motion for our uncontrolled model:

$$M\ddot{q}_1+c\dot{q}_1+kq_1=S(\ddot{q}_2 sin\, q_2+\dot{q}_2^2\cos q_2) \tag{7}$$

$$J\ddot{q}_2+b\dot{q}_2-a=S(\ddot{q}_1 sin\, q_2-g\cos q_2) \tag{8}$$

where

$$M = m_1 + m_2\,,\; S = m_2 e\,,\; J = J_2 + m_2 e^2 \tag{9}$$

2.2. Controlled model

Next, we introduce our impact control by means of a third point mass m_3 free to bounce back and forth inside a box attached to the top mass. This introduces an additional degree of freedom to our dynamical system, and accordingly we denote q_3 the displacement of this point mass, whose displacement is bounded by the walls of the gap. The additional equation of motion for this coordinate, assuming absence of friction, is the uncoupled second order homogeneous differential equation

$$\ddot{q}_3 = 0 \tag{10}$$

Its initial conditions are set equal to the displacement and velocity q_{3a} and $\dot{q}_{3a}$ in the time t_a immediately *after* each impact between the mass and the main structure of mass m_1. For any time t in the interval between a certain impact and the following one, we determine velocities and displacements by simple integration of Eq. (10), as follows:

$$\dot{q}_3 = \dot{q}_{3a} \tag{11}$$

$$q_3 = q_{3a} + \dot{q}_{3a}(t - t_a)\,. \tag{12}$$

The principle of conservation of linear momentum applies when either no forces act or when the sum of forces acting is zero. The latter encompasses a large number of related situations involving impact or collision. The key to the solution of many of these problems is that during impact, the only significant forces acting on the system are due to impact and cancel, occurring as they do in equal and opposite pairs.

Consider the dynamics during impact of our two masses. As the masses contact, each deform until maximum deformation occurs. The time interval from initial contact to maximum deformation is the *period of deformation*, and from the maximum deformation to the point of just separating, the *period of restitution*. The total time of impact, deformation plus restitution, is quite short, usually negligible for practical purposes. The physics of what happens during this period is quite evolved and need not be known for our purposes. We only need to consider the states just *before* and *after* material deformation, usually measured experimentally.

Let us consider inelastic collisions, with restitution coefficients $0 < r < 1$, defined as the ratio between the difference of velocities of the masses as they approach each other just *before* collision and the same difference as they separate one from the other just *after* impact is

$$r = -\frac{\dot{q}_{1a} - \dot{q}_{3a}}{\dot{q}_{1b} - \dot{q}_{3b}} = \frac{-(velocity\ of\ \ separation)}{(velocity\ of\ \ approach)}\,. \tag{13}$$

The limiting cases are the perfectly elastic impact (r = 1), when the velocities of separation and approach are equal and have opposite signs, and the perfectly plastic impact (r = 0), when the two masses are lumped together and have the same velocity after collision.

As we have two unknown velocities after impact to determine, we need another equation besides Eq. (13). It of course is the statement of the conservation of linear moment:

$$m_1\,\dot{q}_{1a} + m_3\,\dot{q}_{3a} = m_1\,\dot{q}_{1b} + m_3\,\dot{q}_{3b} \tag{14}$$

Re-arranging Eqs. (13) and (14) as a system of 2 linear algebraic equations we have

$$\begin{bmatrix} m_1 & m_3 \\ 1 & -1 \end{bmatrix} \begin{Bmatrix} \dot{q}_{1a} \\ \dot{q}_{3a} \end{Bmatrix} = \begin{Bmatrix} m_1\,\dot{q}_{1b} + m_3\,\dot{q}_{3b} \\ -r(\dot{q}_{1b} - \dot{q}_{3b}) \end{Bmatrix} \tag{15}$$

whose solutions are the sought for initial velocities to integrate the separate motions of the structure and the impacting mass after the shock:

$$\dot{q}_{1a} = \frac{(m_1 - r m_3)\dot{q}_{1b} + m_3(1+r)\dot{q}_{3b}}{m_1 + m_3} \tag{16}$$

$$\dot{q}_{3a} = \frac{(m_3 - r m_1)\dot{q}_{3b} + m_1(1+r)\dot{q}_{1b}}{m_1 + m_3} \tag{17}$$

It is interesting to check that for the perfectly elastic and perfectly plastic impact situations, the expected relations between resulting velocities are obtained from solutions (16) and (17).

3. Comments on numerical results

In this section we make some comments on our preliminary numerical simulation results. We used a standard 4th order Runge-Kutta algorithm. To that end, we must first manipulate Eqs. (7) and (8) to isolate the accelerations, solving the following algebraic system:

$$\begin{bmatrix} M & -S\sin q_2 \\ -S\sin q_2 & J \end{bmatrix} \begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} = \begin{Bmatrix} G_1 \\ G_2 \end{Bmatrix} \tag{18}$$

with

$$G_1 = -(kq_1 + c\dot{q}_1 + S\dot{q}_2^2 \cos q_2) \tag{19}$$

$$G_2 = -(b\dot{q}_2 - a + Sg\cos q_2) \tag{20}$$

The solutions are:

$$\begin{Bmatrix} \ddot{q}_1 \\ \ddot{q}_2 \end{Bmatrix} = \begin{Bmatrix} \dfrac{JG_1 + SG_2 \sin q_2}{\Delta} \\ \dfrac{MG_2 + SG_1 \sin q_2}{\Delta} \end{Bmatrix} \tag{21}$$

where

$$\Delta = MJ - S^2 \sin^2 q_2 \tag{22}$$

We have adopted the following numerical values for the structural parameters of our model: k = 1.229x10^6 N/m and M = 3.543x10^4 kg, leading to natural frequency ω = 5.646 rad/s; S = 50 kg.m; J = 225 kg.m^2; c = 4x10^3 N.s/m, equivalent to 1.0% of the critical damping.

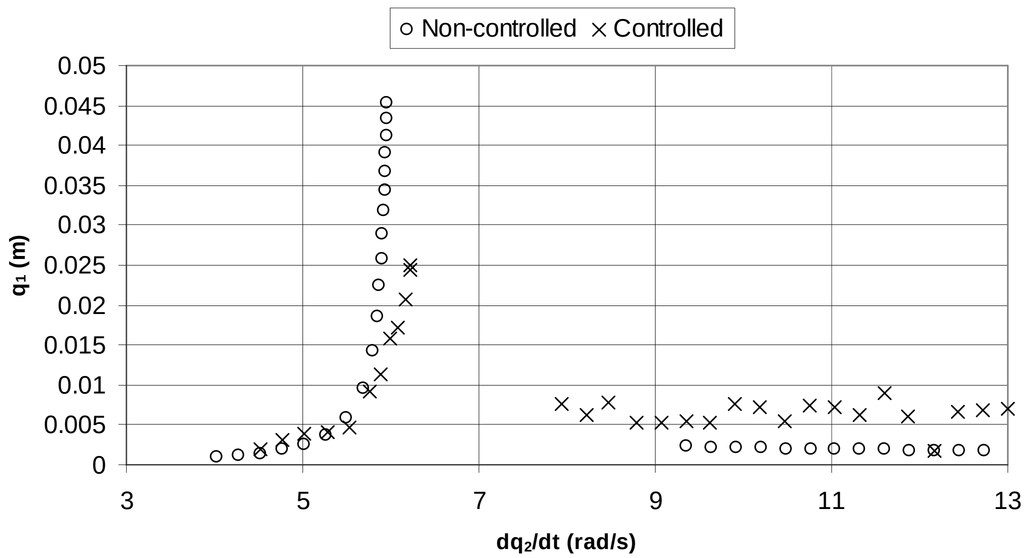

Figure 3. Horizontal displacement amplitudes of stable steady-state solutions vs. rotation speed of motor, rising energy levels in non-controlled and controlled models.

3.1. Non-controlled model

In this case, in the absence of the impact damping, we have obtained, as expected, the appearance of the so-called Sommerfeld Effect.

As a greater wind speed occurs, the rotation velocity of the generator usually also increases, reflecting in an increase of parameter *a*. But as we approach the resonance region, oscillations of the supporting structure raise rapidly consuming a large part of the energy we are imparting to the generator whose speed stops increasing at the same rate as before. If enough energy is not available, it can get stuck at resonance, not being able to reach higher regimes. If some more energy is given to the generator, jump phenomena may occur; the rotation of the motor change abruptly to higher values, no stable steady-state solutions possible inside the jumped interval of frequencies.

In Fig. 3 we display the above described behavior for generator parameters: *b*= 7 Nm/s and parameter *a* rising from 10 to 90. For a certain amount of available energy (wind speed), related to parameter *a*, we have a certain fixed amplitude of rotation velocity of the rotor and a certain fixed amplitude of horizontal displacement of the structure.

3.2. Controlled model

Now we introduce a freely bouncing mass m_3 = 500 kg inside a box, *d* = 0.03 m is the maximum gap between the mass and the walls of the box, as shown in Fig.2. Our preliminary numerical simulations clearly show that we can control the amplitude of the generator induced vibrations as well as the width of the band of frequencies inside which no stable solutions were possible due to the Sommerfeld Effect previously described. In other words, the energy imparted to the generator that was being used to induce large amplitude motions of the supporting structure in resonance is now used by the generator itself to reach higher rotation regimes, mitigating the undesired phenomena related to the non-ideal power source considerations, such as the generator getting stuck at resonance.

Figure 4 shows a sample response time history of horizontal motions of the structure for a same energy level with and without control. It is noticed a reduction about 45 % at the displacements in the controlled model, assuring the effectiveness of the impact damper.

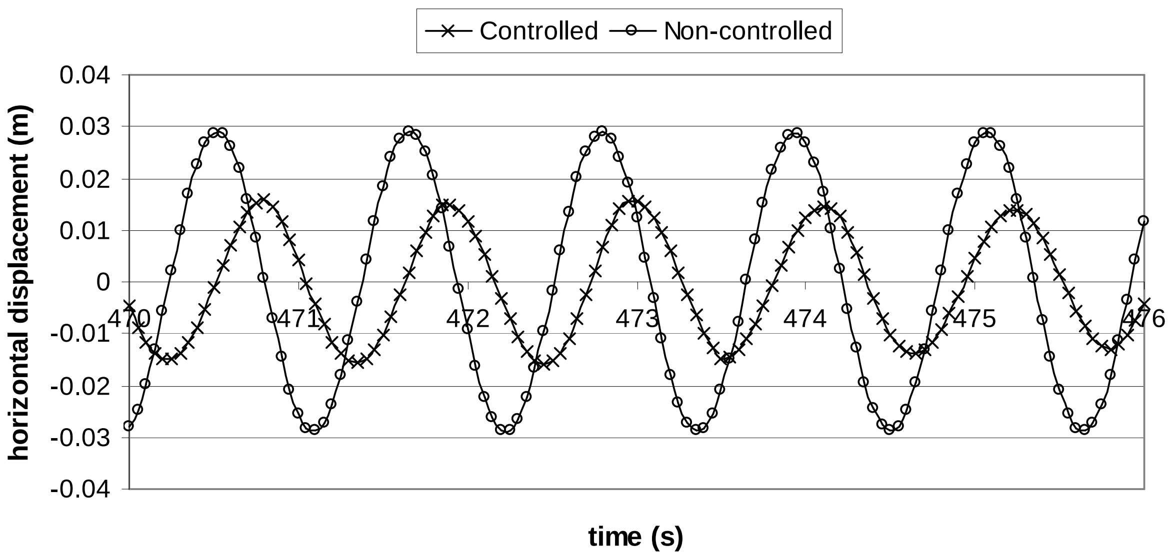

Figure 3. Sway displacement time response of stable steady-state solutions for a same energy level, controlled and non-controlled

It is important to notice that *impact dampers are not really dampers* in the usual way, that is, they do not dissipate energy or, at least, dissipate very little energy if the restitution coefficient *r* is kept as close to 1 as possible (perfectly elastic impact).

4. Conclusions

The purpose of this paper was to present a mathematical model for a non-ideal wind turbine tower controlled by an impact damper.

We have successfully discussed and connected two interesting issues in nonlinear dynamics: a) non-ideal power sources, represented, in this case, by the in-board non-ideal generator being lead by an unbalanced rotor that induces the motion of the structure; b) impact damper for controlling high-amplitude vibrations.

In this paper, we have analyzed a possible practical application: an unbalanced non-ideal generator supported by a tower that suffers the Sommerfeld Effect of getting stuck at resonance (energy imparted to the generator being used to excite large amplitude motions of the supporting structure). We have shown that the impact damping may mitigate the undesired phenomena without dissipation of energy.

In future work we intend to build a laboratory model of this foundation and impact damper to validate our present analysis. That would be an extension of experimental work our group has already performed on the Sommerfeld Effect in motor foundations as reported in Balthazar et al. (2004).

5. Acknowledgements

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5. Responsibility notice