NUMERICAL EVALUATION OF ANGRA I AUXILIARY FEEDWATER SYSTEM RELIABILITY BY THE METHOD OF SUPLEMENTARY VARIABLES

José Luiz Fernandes
Federal Center of Technology Celso Suckow– CEFET-RJ - Brazil
jlfernandes@cefet-rj.br

Marcos de Oliveira de Pinho
Federal Center of Technology Celso Suckow– CEFET-RJ - Brazil
depinhogalois@aol.com

Antonio Carlos Marques Alvim
Nuclear Engineering Programme – COPPE / UFRJ - Brazil
alvim@con.ufrj.br

Paulo Fernando Ferreira Frutuoso e Melo
Nuclear Engineering Programme – COPPE / UFRJ - Brazil
frutuoso@con.ufrj.br

Abstract. *This paper presents the method of supplementary variables for calculating the availability of nuclear safety systems subject to aging. Particularly, the availability of the auxiliary feedwater system of a typical PWR plant was calculated. In this context, it has been necessary to study systems of partial and ordinary first order differential equations in order to represent a system subject to aging. Numerical solutions were obtained by the method of finite differences using Euler's implicit method and a new method named iterative method. A variant of this last method, called Iterative + Euler was also developed. The integrals appearing in the system equations were solved by the trapezoidal method and the numerical results were obtained using a computer code previously written. The integrals appearing in the system equations were solved by the trapezoidal method and the numerical results were obtained using a computer code previously written. This numerical solution is very helpful for considering the life extension of nuclear plants, an issue currently under discussion. For systems under aging, like the auxiliary feedwater system (SAAA), the supplementary variable method obtains numerically the availability and the failure probability for various times and component ages. The best among the methods presented is the implicit Euler.*

Keywords: *Numerical Solution, Supplementary Variables Method, Aging*

1. Introduction

The reliability theory comprehends a group of mathematical and statistical methods, which, through the study of the laws of occurrences of failure, aim at the prediction, analysis, prevention and mitigation of the failures throughout the time (Pinho et al., 1996). It is verified that a lot of nuclear power stations (Angra I) in the world are in the aging period. Therefore, the International Agency of Atomic Energy (IAEA) and CNEN (National Commission of Nuclear energy) study criteria both for increasing the useful life of the plants and also to develop specific programs of maintenance and one of the pillars of that study is the determination of the failure probabilities under aging condition.

According to Blashe (1994) and O'Connor (1998), a system can be defined as being a certain group of discrete elements (components or sub-systems) interconnected or in dynamic interaction, organized and associated as a function of an objective, this referred group being object of a control. The whole system, independent of its nature, is subject to failures. Failure is the interruption of the aptitude of a system to accomplish a certain function, and is said to be present in a system once it is no more in conditions of fulfilling its function. Failures in equipments, for instance, can represent large economical and human losses, presenting, in many cases, significative trade-off for the institutional image of the companies that uses them.

As such, the main parameter in the modeling of reliability of equipment (system) is the failure rate. Failure rate means the number of failures in an interval of time. When equipment is under aging, the failure rate is growing. Therefore, for the components which are no more in the useful life period, the use of an exponential model, implying constant failure rate, may not be adequate, since it will overestimate the reliability. Therefore, the consideration of component aging involves the adoption of models which take into account failure rates which are growing in time, as, for instance, the Weibull and the lognormal distributions.

Frequently, non repairable components present failures as a function of time. The curve that describes the probability distribution of this type of component is the so called bathtub curve. This curve, as shown in the fig. 1, has three different time periods (Blashe, 1994).

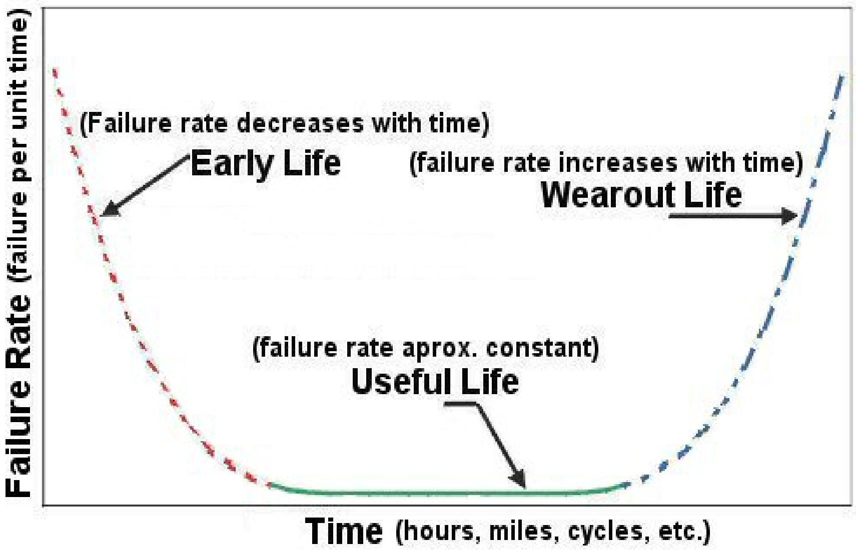


Figure 1. Bathtub Curve.

Mode I – period of precocious failure (infant mortality or early life): eventual period, in the beginning of the life of a component, in which the failure is high and decreasing with the time.

Mode II – Constant failure rate period (useful life): eventual period of the life of a component, in which the failure rate establishes itself into a sensibly constant value, this failure being due to random phenomena.

Mode III - Period of failure due to wear (aging or wearout life): eventual period of the life of a component during which the failure rate increases, rapidly, in comparison with that of the precedent period. It corresponds to the period of aging of the component, in which is submitted to wear, where failure results from physicochemical alterations of the materials.

The auxiliary feed-water system (AFWS) has two motor-driven pumps (A and B) and a turbine-driven pump (T). Each motor-driven pump supplies 50% of the water needs for the steam generators and the turbine-driven pump supplies 100% of the flow need. The system operates on average 4 hr a month, and the pump specifications and data were described in Amaral Netto (1999). The motor-driven pump A has a failure rate equal to 1.604 x 10^{-3} hr^{-1} and a repair time of 6.5 hr. The failure times of motor-driven pump B follow a Weibull distribution with a shape parameter equal to 1.322, and a scale parameter equal to 795 hr, and a mean repair time equal to 6.8 hr. The turbine-driven pump T has a failure rate equal to 5.547 x 10^{-4} hr^{-1} and average repair time of 4.9 hr. The analysis of the system functions has led to the definition of the following system states (Oliveira, 2001).

- <u>*State 1*</u> - $\left(T, A, B\right)$. The three pumps are working.
- <u>*State 2*</u> - $\left(T, A, \overline{B}\right)$. Only pump B is failed.
- <u>*State 3*</u> - $\left(A, B, \overline{T}\right)$. Only pump T is failed.
- <u>*State 4*</u> - $\left(T, B, \overline{A}\right)$. Only pump A is failed.
- <u>*State 5*</u> - $\left(T, \overline{A}, \overline{B}\right)$. Only pump T is working. This state is due to the failure of pump A in state 2.
- <u>*State 6*</u> - $\left(T, \overline{B}, \overline{A}\right)$. Only pump T is working. This state is due to the failure of pump B in state 4.
- <u>*State 7*</u> - $\left(\overline{T}, \overline{B}, \overline{A}\right)$. All pumps are failed and this is a system failure state.

The dynamic methodologies in the reliability and risk analysis studies are necessary for equipments that are under aging, where the age of the component becomes an important variable. So, two methods which consider aging; namely, the method of stages (which approximates the distributions by using fictitious states), and the supplementary variables method (Pinho et al., 1996 and 1999), that add auxiliary variables, in order to turn the problem into a Markovian problem (Cox, 1967, 1955 and 1995a), are commonly used.

In this work the modeling of maintenance rate (Pinho, 2000) follows the strategy *as good the new*. In other words, the maintenance presupposes that the equipment will be in good condition (identical to a new one). Work developed by using the *as bad as old* criterion, developed by Oliveira (2001), results in a strategy that leads to smaller availability, when compared to results presented in Pinho (2000).

2. Sthocastic Model

A stochastic process is a family of random variables observed in different times t, that is, this process is indexed by the parameter t and defined in a specific probability space. A state is a value assumed by a random variable, and the space of state of a stochastic process is the group of all the possible values that the random variable can assume. A special type of stochastic process whose probability of being in a state in the future is only determined by the present state is called a Markov process. With the inclusion of the Markov property, the problems are simplified considerably, since the knowledge of the present is uncoupled with the past and the future. A stochastic Markovian process with discrete state space of and with discrete time is referred to as a Markov chain (Cox, et. el., 1965; Ross, 1996).

When a system follows a constant failure rate, its failure times follow an exponential distribution. For the case in which the system is not governed by the exponential distribution, the process is non-Markovian, because the probability of going from an initial state *i* to a final state *j* does not depend only on both these states. To analyze these processes, one must cast them into Markovian processes. If two or more exponentially distributed states are combined, the resulting state will not be exponentially distributed. In other words, the model will not be Markovian anymore. For the solution of the stochastic problem the method of supplementary variables has been chosen, Cox, *et. al*. (1965), Ross (1996). The analytical solution of the resulting linear system in some cases may be difficult (or even impossible) to obtain. Because of that, it becomes necessary to use numerical methods to find approximate solutions. These numerical methods are described in Pinho *et. al*. (2004) and Pinho (2000). Figure 2 shows the representative outline of the proposed stochastic problem.

Pinho et al. (2004) presented a numerical solution for the supplementary variables model to evaluate the availability of the auxiliary feed-water system of a nuclear power plant under aging, according to Fig. 2. In this context, it was necessary to study systems of partial and ordinary differential equations of first order to represent the systems under aging. Numerical solutions were obtained by the method of finite differences using the implicit method of Euler and a new method, named iterative method. A variant of this last method, named Iterative + Euler was developed.

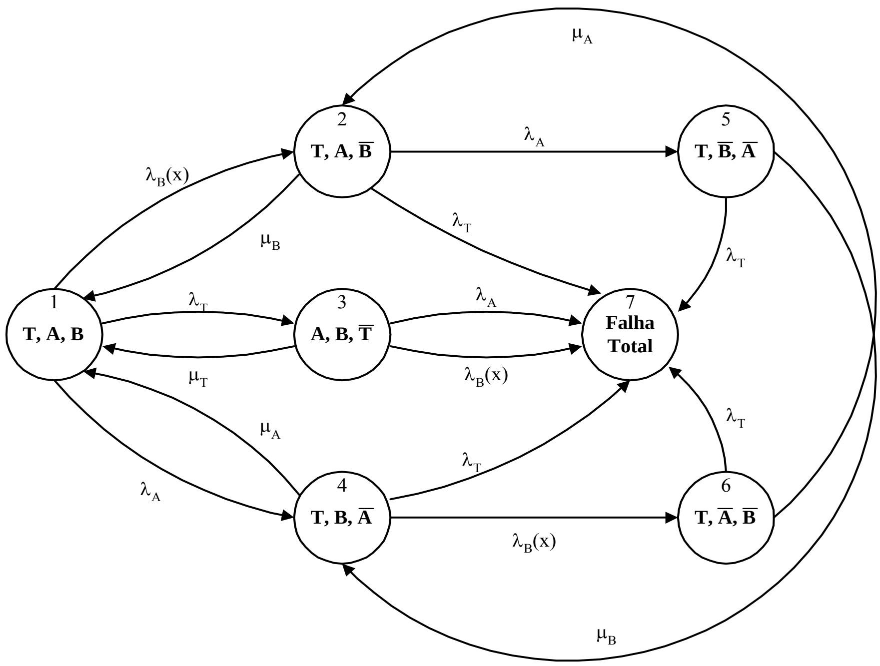


Figure 2. State diagram for auxiliary system for feed water (SAAA).

Equations (1) to (7) below represent the system model and the initial conditions are shown in equations (8) to (13) (Pinho, 2000 e 2004).

$$\frac{\partial p_1}{\partial x} + \frac{\partial p_1}{\partial t} = -\left[\lambda_B(x) + \lambda_T + \lambda_A\right] p_1(x,t)\,, \tag{1}$$

$$\frac{\partial p_2}{\partial x} + \frac{\partial p_2}{\partial t} = -\left[\lambda_A + \lambda_T + \mu_B\right] p_2(x,t), \tag{2}$$

$$\frac{\partial p_3}{\partial x} + \frac{\partial p_3}{\partial t} = -\left[\lambda_B(x) + \lambda_A + \mu_T\right] p_3(x,t), \tag{3}$$

$$\frac{\partial p_4}{\partial x} + \frac{\partial p_4}{\partial t} = -\left[\lambda_T + \lambda_B(x) + \mu_A\right] p_4(x,t), \tag{4}$$

$$\frac{\partial p_5}{\partial x} + \frac{\partial p_5}{\partial t} = -\left[\lambda_T + \mu_B\right] p_5(x,t), \tag{5}$$

$$\frac{\partial p_6}{\partial x} + \frac{\partial p_6}{\partial t} = -\left[\mu_A + \lambda_T\right] p_6(x,t), \tag{6}$$

$$\begin{aligned}\frac{dP_7}{dt} &= \int_0^\infty \left(\lambda_A + \lambda_B(x)\right) p_3(x,t)\,dx + \\ &+\lambda_T \int_0^\infty \left[p_2(x,t) + p_5(x,t) + p_4(x,t) + p_6(x,t)\right] dx\end{aligned} \tag{7}$$

$$p_1(0,t) = \mu_B \int_0^\infty p_2(x,t)dx + \mu_A \int_0^\infty p_4(x,t)dx + \mu_T \int_0^\infty p_3(x,t)dx, \tag{8}$$

$$p_2(0,t) = \int_0^\infty \lambda_B(x)\, p_1(x,t)dx + \mu_A \int_0^\infty p_6(x,t)dx, \tag{9}$$

$$p_3(0,t) = \int_0^\infty \lambda_T\, p_1(x,t)dx, \tag{10}$$

$$p_4(0,t) = \int_0^\infty \lambda_A p_1(x,t)dx + \mu_B \int_0^\infty p_5(x,t)dx, \tag{11}$$

$$p_5(0,t) = \int_0^\infty \lambda_A p_2(x,t)dx, \tag{12}$$

$$p_6(0,t) = \int_0^\infty \lambda_B(x)\, p_4(x,t)dx. \tag{13}$$

Developing the above equations, one obtains the discretized AFWS system as shown in eq. (14):

$$\frac{p_K^{i+1,j+1} - p_K^{i-1,j+1}}{2\Delta x} + \frac{p_K^{i,j+1} - p_K^{i,j}}{\Delta t} = -\Lambda_{K\ i}\, p_K^{i,j+1}, \forall K = 1...6 \tag{14}$$

However, the system is coupled only by the boundary conditions, that is: the $p_1..p_6$ calculations are coupled via p1(0,t)... p6(0,t). In this way, the matrix of this system will not be tri-diagonal. To overcome that difficulty, the following strategy was adopted at *i* = 1, as shown in equation (15):

$$\frac{p_K^{2,j+1} - p_K^{\ 0,j+1}}{2\Delta x} + \frac{p_K^{1,j+1} - p_K^{1,j}}{\Delta t} = -\Lambda_{K\ 1}\, p_K^{1,j+1}, \forall K = 1...6 \tag{15}$$

In equation (15) p(0,j+1) was approximated by p(0,j), so that the system could be uncoupled. After the calculation of the p(i,j+1), for i = 1,..., X, the linear system formed by the p(0,j+1) Eq. (7 to 13) was solved. A variation of the Euler's implicit method is to iterate in $p^{i,j}$, so that equation (16) can be cast into equation (16), where $p0_k^{i,j}$ is an initial estimate for $p_K^{i,j}$.

$$\frac{p_K^{2,j+1} - p0_k^{0,j+1}}{2\Delta x} + \frac{p_K^{1,j+1} - p_K^{1,j}}{\Delta t} = -\Lambda_{K1} p_K^{1,j+1}, \forall K = 1...6 \tag{16}$$

The algorithm for iterative method is described as follows:

1) An estimated initial vector p0ki,j is supplied, in accordance to the initial conditions,
2) $p0_k^{0,j+1}$ is calculated by equations (8) to the (13)
3) $p_k^{1,j+1}$, $p_k^{2,j+1}$, are calculated by equation (16)
4) $p_k^{i,j+1}$, $p_k^{i-1,j+1}$, $p_k^{i+1,j+1}$, are calculated by equation (15)
5) If $\| p_k^{i,j+1} - p0_k^{i,j+1} \|$ < tolerance, repeat the process for the next level of time if not,
6) Set $p0_k^{i,j+1} = p_k^{i,j+1}$ and
7) Go back to step 2

A variant of this algorithm was also developed, where the initial estimate (step 1) was obtained by the Implicit Euler Method, in the hope of accelerating the convergence of the process.

3. Results and Discussion

The results for a simulation considering a 20-year aging period and an average operating time of 0.1 years, for the auxiliary feed-water system are presented in tables 1 and 2. Table 3 shows the result of a simulation for a 40-year period and an average operating time of 0.2 years. In these tables, the data refer to discretization (*x*), time (*t*), the average availability, and maximum failure probability for the Lax, implicit Euler and the iterative method proposed in this work.

Table 1. Simulation 1 for a 20-year aging period with average operating time *m* of 0.1 years (part 1)

Method	Discretization (Age - years)	Time (years)	Mean Availability (%)	Maximum failure probability (%)
Lax	$\Delta x = 6.67.10^{-3}$	$\Delta t = 2.20.10^{-4}$	96.80	5.81
Euler Implicit	$\Delta x = 6.67.10^{-3}$	$\Delta t = 2.20.10^{-4}$	97.05	5.18
Iterative	$\Delta x = 6.67.10^{-3}$	$\Delta t = 2.20.10^{-4}$	96.97	5.98
Iterative+Euler	$\Delta x = 6.67.10^{-3}$	$\Delta t = 2.20.10^{-4}$	96.96	5.18

Table 2. Simulation 2 for a 20-year aging period with average operating time *m* of 0.1 years (part 2)

Method	Discretization (Age - years)	Time (years)	Mean Availability (%)	Maximum failure probability (%)
Lax	$\Delta x = 5.00.10^{-3}$	$\Delta t = 1.10.10^{-4}$	---	---
Euler Implicit	$\Delta x = 5.00.10^{-3}$	$\Delta t = 1.10.10^{-4}$	97.62	4.66
Iterative	$\Delta x = 5.00.10^{-3}$	$\Delta t = 1.10.10^{-4}$	97.67	4.97
Iterative+Euler	$\Delta x = 5.00.10^{-3}$	$\Delta t = 1.10.10^{-4}$	97.67	4.98

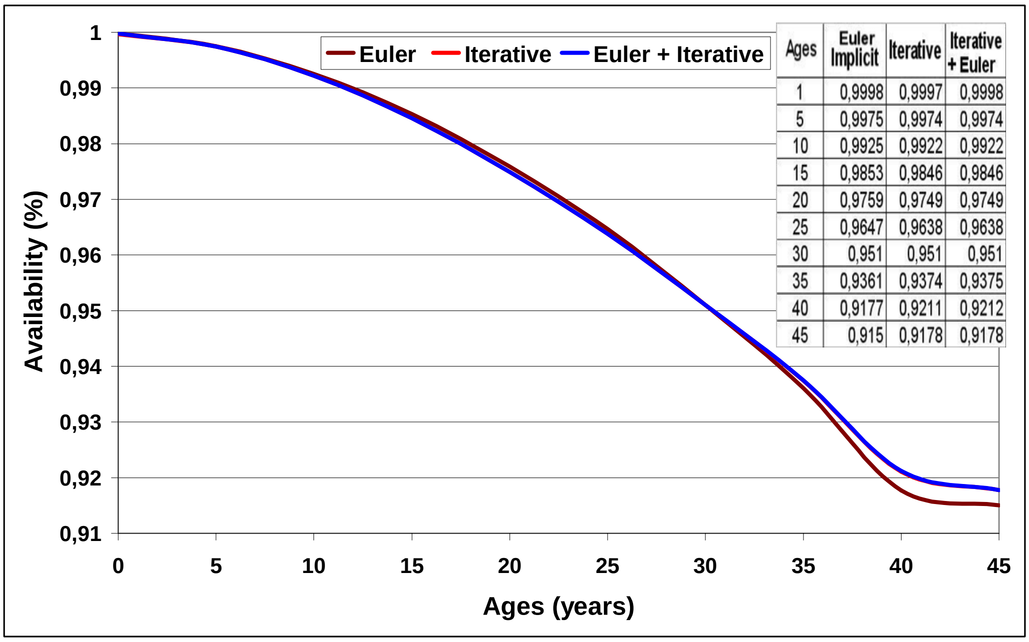
Table 3. Simulation 3 for a 40-year aging period with an average operating time m of 0.2 years.

Method	Discretization (Age - years)	Time (years)	Mean Availability (%)	Maximum failure probability (%)
Lax	$\Delta x = 1.00.10^{-2}$	$\Delta t = 2.20.10^{-4}$	---	---
Euler Implicit	$\Delta x = 1.00.10^{-2}$	$\Delta t = 2.20.10^{-4}$	92.35	14.91
Iterative	$\Delta x = 1.00.10^{-2}$	$\Delta t = 2.20.10^{-4}$	92.31	14.93
Iterative+Euler	$\Delta x = 1.00.10^{-2}$	$\Delta t = 2.20.10^{-4}$	92.31	14.93

According to tables 2 and 3, the results obtained from the Lax's method are not reasonable, since a stability condition was not derived for the AFWS problem modeling, and for the values of *x* and *t* presented the stability was violated. We are still developing research on this subject. The numerical differences between the mean availabilities and the maximum failure probabilities in tables 1 and 2 are due to decrease in the values of *x* and *t*. With implicit Euler's method it is possible to work with fewer points, because it is unconditionally stable. It can be verified in tables 1 and 3 that the best precision was obtained by using the iterative method.

We hope that better results will be obtained if the numeric integration is performed by the 1/3 Simpson's method, instead of the repeated trapezoidal method. This improvement of the program is necessary since the results for the failure probability and the availability for the 40 year-old period indicate that the best solution from the engineering point of view would be pump B replacement after 20 years of use. This issue is currently under investigation.

Results of a simulation considering ages of 1, 5, 10, 15, 20, 25, 30, 35, 40 and 45 years for the were also obtained, see Figure 3. Figures 4 to 7 show the results for the failure probability for the ages 30, 35, 40 and 45 years, obtained by the Euler, iterative and Euler + Iterative numerical methods, respectively.



Ages	Euler Implicit	Iterative	Iterative + Euler
1	0,9998	0,9997	0,9998
5	0,9975	0,9974	0,9974
10	0,9925	0,9922	0,9922
15	0,9853	0,9846	0,9846
20	0,9759	0,9749	0,9749
25	0,9647	0,9638	0,9638
30	0,951	0,951	0,951
35	0,9361	0,9374	0,9375
40	0,9177	0,9211	0,9212
45	0,915	0,9178	0,9178

Figure 3. AFWS availability.

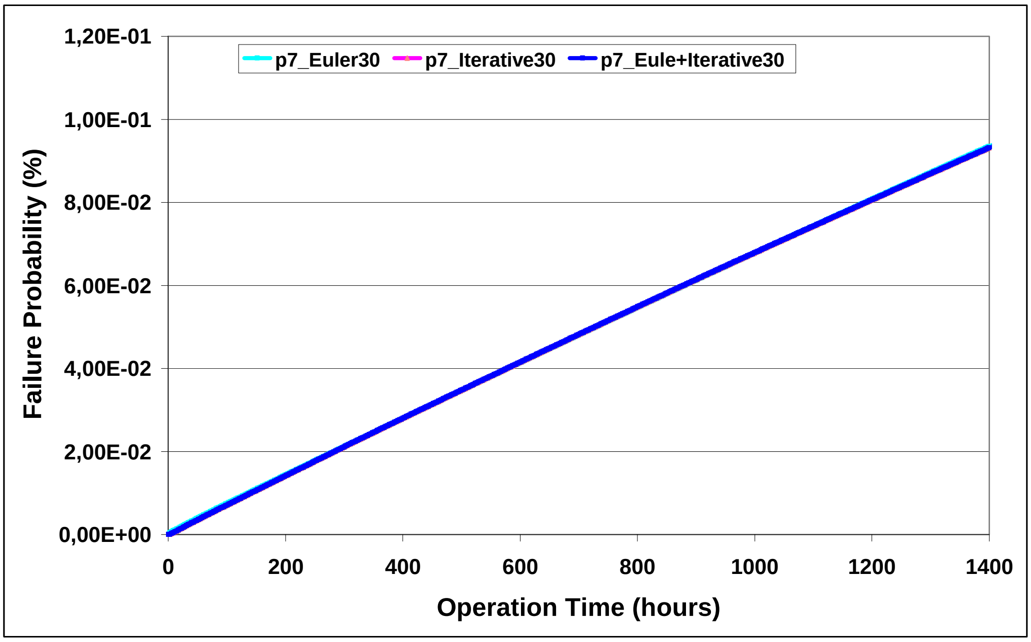


Figure 4. Failure probability for an operation time of 30 years.

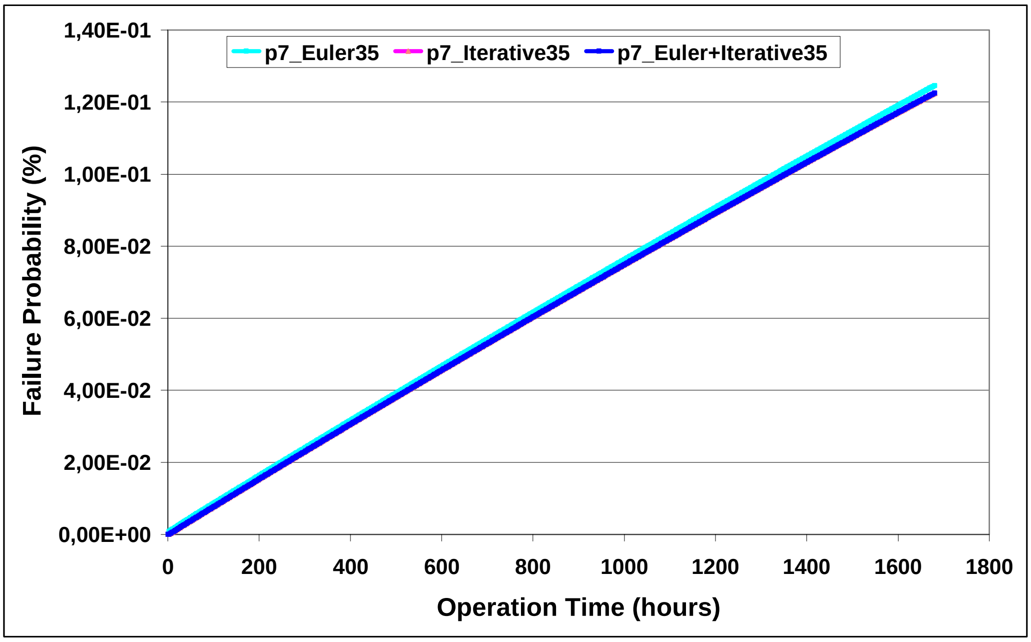


Figure 5. Failure probability for an operation time of 35 years.

The results of the numerical simulation presented in figures 3 to 7 were obtained by considering the discretization (x=5.00.10^{-3}, t=1.10.10^{-4}), the mean availability and the maximum failure probability (p7) by the Euler's, Iterative and Euler's + Iterative numerical methods.

It should be noticed from Fig. 3 that the results are practically the same for all the tested methods for the 35 year age, when the iterative and Euler's + iterative methods predict an availability slightly larger than the one given by the Euler's method. This difference is not significant (0.2%) and should be explained by the truncation error between the Euler's, iterative and iterative + Euler's numerical methods (which have the same truncation error). As the Euler's implicit method is computationally faster, we concluded that it is the method that should be used to model problems of aging of components, such as the one under discussion here.

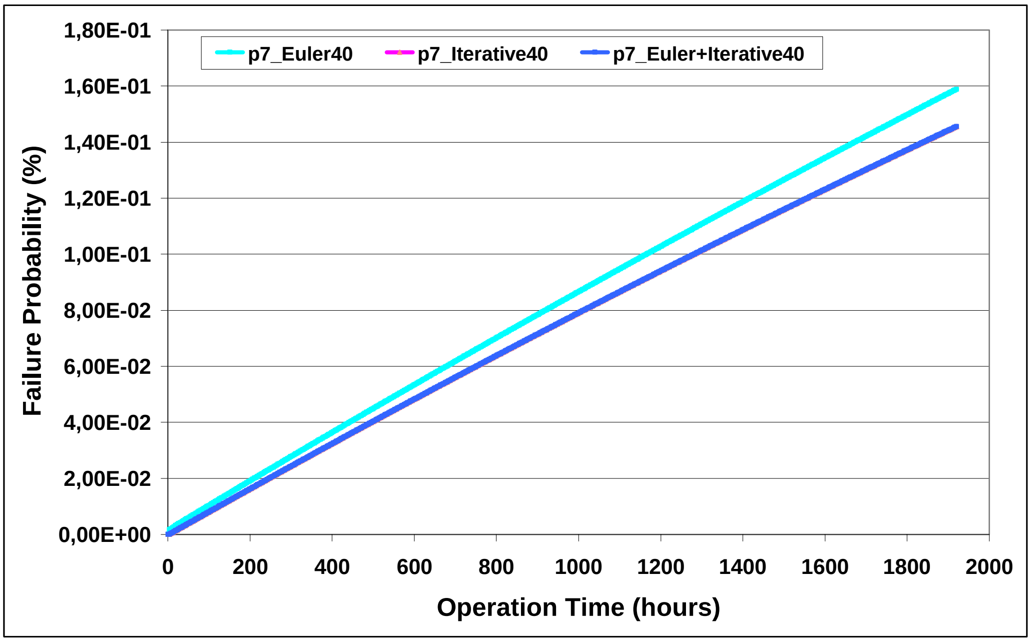


Figure 6. Failure probability for an operation time of 40 years.

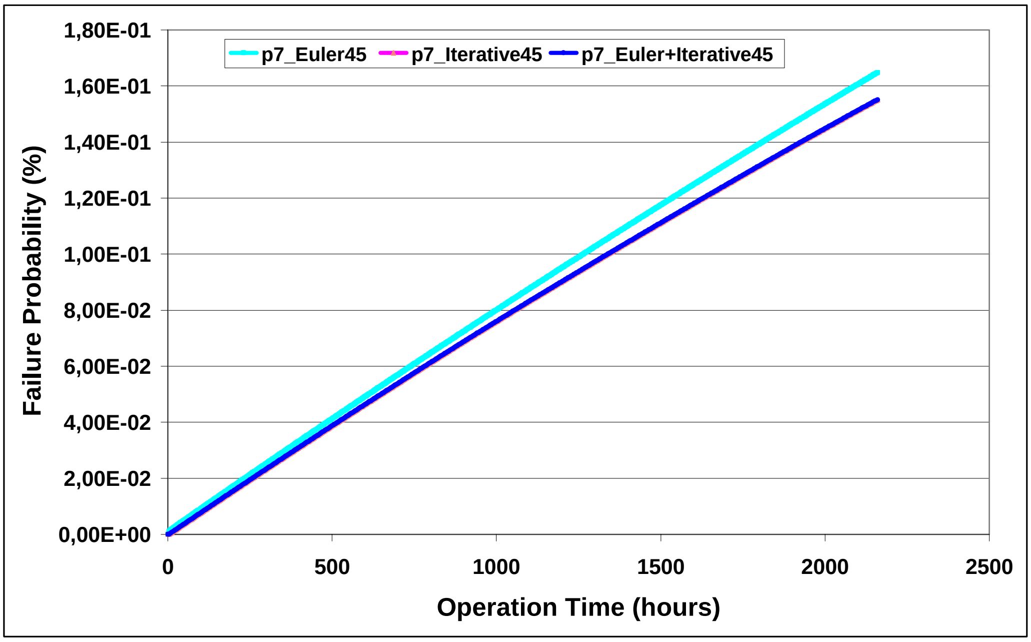


Figure 7. Failure probability for an operation time of 45 years.

Regarding the behavior of the failure probability with age and time of operation, Figs. 4 to 7 indicate a behavior that agrees with the one obtained for the availability. In other words, the method of Euler predicts larger failure probabilities than the iterative and the Euler's + iterative methods, which implies a smaller availability for the Euler's method, as previously obtained. The differences regarding the failure probabilities are also small and are attributed to the different truncation errors between the Euler's, the iterative, and the Euler's + iterative methods.

4. Conclusions

For systems undergoing aging, such as the auxiliary feed-water system (AFWS) modeled in this work, the method of supplementary variables allows to numerically obtain the availability and the failure probability for several times of operation and ages of components. The numerical method that is recommended is the Euler's implicit method, since it is computationally faster than the other methods tested in this work.

5. References

Amaral Netto, J. D., 1999, "Simulation of discrete events guided to objects for the analysis of the unavailability of dynamic systems", PhD Thesis (*in Portuguese*), COPPE – UFRJ, Nuclear Engineering Program – COPPE/UFRJ-Brazil.

Blashe, M. K. & Shrivastava, B. A. (1994) "Defining Failure of Manufacturing & Equipment", Proceeding Annual Reliability and Maintainability Symposium, p. 69-75.

Cox, D. R., Miller, H. D., 1965, "The Theory of Stochastic Processes", Chapman and Hall.

Cox, D. R., 1955a, "The Analysis of Non-Markovian Stochastic Processes by the Inclusion of Supplementary Variables", Proceedings of the Cambridge Philosophical Society, N° 51, pp. 431-441.

Cox, D. R., 1955b, "A Use of Complex Probabilities in the Theory of Stochastic Processes", Proceedings of the Cambridge Philosophical Society, N° 51, pp. 313-319.

Melo, P. F. F., Alvim, A. C. M, Silva, F.C., 1998, "Sensitivity Analysis on the Accident Rate of a Plant Equipped with a Protective Channel by Generalized Perturbation Methods", Annals of Nuclear Energy, Vol. 25, N° 15, pp. 1191-1208.

O`Connor, P. D. T., 1998, "Practical Reliability Engineering", New York: Wiley.

Oliveira, E. de A. O., 2001, "Use of supplementary variables and Inversion of Laplace transformation in a new model for the calculation of reliability of systems subject to aging and with repair without replacement". PhD Thesis (*in Portuguese*), COPPE – UFRJ, Nuclear Engineering Program – COPPE/UFRJ- Brazil.

Pinho, M. O., Noriega, H.C, Alvim, A. C. M., Frutuoso e Melo, P. F. F., 1996, "Reliability of a channel of protection considering aging for the method of the supplemental variables", (*in Portuguese*), Equipment Conference of Technology, Rio de Janeiro - Brazil, V.1. pp. 207 – 210.

Pinho, M. O., Noriega, H.C, Frutuoso e Melo, P. F. F. F., Alvim, A. C. M., 1999, “Availability of a Component Subject to an Erlangian Failure Model under Wearout by Supplementary Variables”. Journal of the Brazilian Society of Mechanical Sciences, V. XXI, n.1, pp.109 – 122.

Pinho, M. de O. de, 2000, “About the application of systems of partial and ordinary differential equations of first order to the reliability of systems of safety under aging”, PhD Thesis (*in Portuguese*), COPPE – UFRJ, Nuclear Engineering Program – COPPE/UFRJ- Brazil.

Pinho, M. O., Melo, P. F. F. de, Alvin, A. M., Fernandes, J. L., 2004, “Numerical calculation of the availability of Angra I auxiliary feedwater system by the supplementary variables method”, (*in Portuguese*) VII Modeling Computational Congress, Rio de Janeiro - Brazil, University –Polytechnic Institute of New Fryeburg, December, 10p.

Singh, C., Billinton, R., 1977, “System Reliability Modeling and Evaluation”, Hutchinson.

Singh, J., Murati, K., 1984, “Reliability of a Fertilizer Production Supply Problem”, Proceedings of the 3rd Annual Conference of Indian Society for Theory of Probability and its Applications, Willey Eastern, New Delhi, pp. 92-94.

Singh, J., 1989, “A Warm Standby Redundant System with Common Cause Failures”, Reliability Engineering and System Safety, Vol. 26, pp. 135-141.

6 Responsibility notice

The authors are the only responsible for the printed material included in this paper.