IRREVERSIBILITY OF THE PHASE CHANGE PROCESS IN DYNAMIC CAVITIES

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Abstract. *This work presents a continuum thermo-mechanical coherent model to describe isothermal flows of incompressible liquid within cavities filled with vapor and a non-condensable gas. The motions of the constituents inside the cavities are described with the aid of the continuum mixtures theory, while the liquid motion description is accomplished by the classical continuum theory. The concepts of the singular surface theory are used to provide the additional interfacial balance equations. Irreversible phase change transformations are accounted for in such a way that the interfacial entropy inequality is always satisfied, regardless the singular surface shape and the initial and boundary conditions. The model is used to investigate the influence of the irreversibility associated with the phase change process on the cavity dynamics when an external harmonic pressure pulse is introduced in the liquid far away from the cavity.*

Keywords*: Thermodynamics, phase change, irreversible process, singular surface theory.*

1. Introduction

The theoretical research interest in single cavity dynamics and cavitation goes back to the beginning of the twenty century, when Besant (1913) first investigated the pressure developed by a collapsing cavity. Almost at the same time, experimental interest arose from the finding that cavitation was associated with erosion, which was first reported on ship propellers (Silberrad, 1912). Soon after, Rayleigh (1917) has made the first consistent and relevant analysis in cavitation and cavity dynamics by solving the problem of the collapse of an empty cavity in an unbounded, incompressible, non-viscous liquid medium. The growth and collapse of the cavities in contact with solid surfaces might have damaging effects on the solid and so has been of great interest in mechanics.

Sixty years later, Plesset and Prosperetti (1977) have included in the analysis liquid viscosity and surface-tension effects by assuming, at first, that the medium filling the cavity was essentially a permanent non-condensable gas. Viscous, thermal and acoustic components of the growth and collapse of a bubble have been analyzed, when it was subjected to small-amplitudes and non-linear oscillations. Later on that and in another work (Plesset and Prosperetti, 1976), other effects such as the liquid compressibility, mass-diffusion processes across the bubble liquid interface, energy flow into and out of the bubble as well as the stability of the interface have been considered.

Although significant progress has been achieved along the years, a growing interest in such a problem has motivated further development in this subject due to its applications not only on mechanics but also in the biomedical area. From the 80’s to the present time, a large amount of theoretical and experimental works have been carried out, in which emphasis has been placed on phase change process (evaporation-condensation) and shock formation effects (Vuong et al and references therein., 1999).

In the great majority of these problems the cavity dynamics is analytically treated by the traditional thermostatic theory, which assumes there is no dissipation associated to the phase change process. However, it is well accepted that this approach may lead to unsatisfactory results (Freitas Rachid, 2003 and Silva and Freitas Rachid, 2002), what motivates the search for more physically realistic and mathematically coherent models.

The theoretical analysis of the phase change process in the context of irreversible thermodynamics has been originally carried out by Bornhorst and Hatsopoulos (1967), who have derived general rate equations across the non-equilibrium region existing at a phase interface. However, their study is limited to phase change problems involving only linear phenomenological equations without taking a third constituent (a non-condensable gas) into account.

This paper presents a logically consistent thermo-mechanical model to describe the isothermal cavity dynamics problem. The continuum mixture theory is used to describe the motions of the vapor and of a non-condensable gas inside the cavity, while the traditional continuum theory is used to model the Newtonian behavior of the liquid outside the cavity. Interfacial balance equations of mass, linear momentum as well as an entropy inequality are considered with the aid of the concepts of singular surface theory. The macroscopic dissipative effects of the liquid-vapor transformation

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are accounted for in such a way the interfacial entropy inequality is always satisfied. The potentiality of the model as well as its basic features are illustrated through a numerical example in which it is investigated the influence of the phase change process in the cavity dynamics.

2. Theoretical model

Consider a liquid flow in which there are cavities filled with its vapor along with a non-condensable gas. The liquid and the gas-vapor filled cavities occupy open and bounded regular regions of the space denoted by Ω_l and Ω_{g}, so that the liquid-cavity interface is represented by the set $\overline{\Omega}_l \cap \overline{\Omega}_{\mathrm{g}}$. The flow is supposed to be isothermal, so that all the three constituents have the same temperature at any spatial position $x \in \Re^3$ and time instant *t*. The thermo-mechanic behaviors of the vapor and the gas inside Ω_{g} are described with the aid of the Continuum Theory of Mixtures. In this theory, the constituents have independent superimposed kinematics meaning that any spatial position $x \in \Omega_{\mathrm{g}}$ is occupied by both vapor and gas at any time instant. To each constituent $a \in \Xi = \{g, v\}$ is associated a spatial mass density r_a and a spatial velocity field $\mathbf{v}_a$. For $a = g$, the properties of the constituent are associated with the air and for $a = v$ the properties are referred to the vapor. In the absence of a gravitational field and also by considering that the air and the vapor do not react with each other, the balance equations of mass and linear momentum for both constituents can be expressed as (Rajagopal and Tao, 1995):

$$
\begin{aligned}
&\frac{\partial}{\partial t} r_a + \nabla \cdot (r_a \mathbf{v}_a) = 0 \quad \text{in } \Omega_g \text{ for } a \in \Xi \\
&\frac{\partial}{\partial t}(r_a \mathbf{v}_a) + \nabla \cdot (r_a \mathbf{v}_a \otimes \mathbf{v}_a) = \nabla \cdot \mathbf{T}_a + \mathbf{f}_a \quad \text{in } \Omega_g \text{ for } a \in \Xi
\end{aligned}
\tag{1}
$$

In the above equations, $\mathbf{T}_a$ stands for the partial stress tensor, $\mathbf{f}_a$ the internal force of interaction between the two constituents and the symbol $\otimes$ denotes the usual tensorial product.

To ensure that the balance of linear and angular momentum be satisfied for the mixture as a whole, the internal force of interaction and the partial stress tensor are to subjected to the following restriction:

$$
\begin{aligned}
&\sum_{\substack{a \\ a \in \Xi}} \mathbf{f}_a = 0 \\
&\mathbf{T}_a = \mathbf{T}_a^T \quad \text{for} \quad a \in \Xi
\end{aligned}
\tag{2}
$$

It should be noticed that the last equation is far restrictive than that required by the classical Theory of Continuum Mixtures (Rajagopal and Tao, 1995). It is further assumed that both the air and the vapor behave as a compressible Newtonian fluid whose constitutive behavior is given by:

$$
\mathbf{T}_a = -p_a \mathbf{I} + \mathbf{S}_a \tag{3}
$$

in which p_a is the thermodynamic pressure and $\mathbf{S}_a = 2 m_a \mathbf{D}_a$ is the extra stress tensor due to motion. The dynamic viscosity of the air and the vapor is represented by m_a and $\mathbf{D}_a = 1/2\left(\nabla \mathbf{v}_a + (\nabla \mathbf{v}_a)^T\right)$ is the rate of strain of these constituents. As usual in Continuum Theory Mixture, we adopt the following form for the interaction force between the constituents:

$$
\mathbf{f}_v = -\mathbf{f}_g = k(\mathbf{v}_v - \mathbf{v}_g) \tag{4}
$$

in which k is a constitutive constant. The liquid occupies the domain Ω_l and is supposed to be incompressible and Newtonian so that the balance equations of mass and linear momentum which governs its motion are described by (Germain and Muller, 1995):

$$
\begin{aligned}
&\nabla \cdot \mathbf{v}_l = 0 \quad \text{em} \quad \Omega_l \\
&r_l \left(\frac{\partial \mathbf{v}_l}{\partial t} + \nabla \cdot (\mathbf{v}_l \otimes \mathbf{v}_l) \right) = \nabla \cdot \mathbf{T}_l \quad \text{in} \quad \Omega_l
\end{aligned}
\tag{5}
$$

in which $\mathbf{T}_l = -p_l\mathbf{I} + \mathbf{S}_l$ is the Cauchy stress tensor, with p_l being the non-thermodynamic pressure and $\mathbf{S}_l = 2m_l\mathbf{D}_l$ the extra stress tensor due to motion. The dynamic liquid viscosity is represented by m_l and $\mathbf{D}_l = 1/2\left(\nabla\mathbf{v}_l + (\nabla\mathbf{v}_l)^T\right)$ is the rate of strain of the liquid. The liquid density, which is a constant, is denoted by r_l.

The balance equations presented above can be applied to the domains Ω_g and Ω_l up to an interface but not across it. Particular forms of the balance equations should be used at an interface in order to take into account the sharp changes (discontinuities) in various variables. By considering that the interface is a singular surface whose motion is described by $x(\boldsymbol{x},t) = 0$, for all time instant and spatial position $\boldsymbol{x}$, then the normal component of the surface velocity $\mathbf{v}_i$ is given:

$$\mathbf{v}_i\,.\mathbf{n} = -\frac{\partial x/\partial t}{\|\nabla x\|} \tag{6}$$

in which $\boldsymbol{n}$ represents the outward unit vector to the interfacial surface. Since the three constituents are present at the interface, it becomes convenient to introduce the set $\Gamma = \{g,v,l\}$.

To complete the model description, local interfacial balance equations are derived next following the ideas presented by Ishii (1975). By neglecting the mass density of the interface, kinetic energy and momentum fluxes across it, molecular diffusion fluxes along it as well as surface sources such as radiation effects, it can be shown that the interfacial balance equations of mass, momentum and the second law of thermodynamics can be written for an isothermal field ($\left(\dot{q}_a = 0 \text{ e } \nabla q_a = 0\right)$, $q_a = q_i = q$), q being the absolute temperature, as (Ishii, 1975 and Drew and Passman, 1998):

$$\begin{gathered}
\sum_{\substack{a \\ a\in\Pi}} \dot{m}_a = 0 \quad \text{in} \quad \overline{\Omega}_l \cap \overline{\Omega}_\mathrm{g} \\
\sum_{\substack{a \\ a\in\Pi}} \left[\dot{m}_a \hat{\boldsymbol{v}}_a - \hat{\boldsymbol{T}}_a \boldsymbol{n}_a\right] + Hs\boldsymbol{n} = 0 \quad \text{in} \quad \overline{\Omega}_l \cap \overline{\Omega}_\mathrm{g} \\
\dot{D} := \sum_{\substack{a \\ a\in\Pi}} \dot{m}_a \left[\hat{g}_a + \tfrac{1}{2}(\hat{\boldsymbol{v}}_\mathrm{a} - \boldsymbol{v}_\mathrm{i})\cdot(\hat{\boldsymbol{v}}_\mathrm{a} - \boldsymbol{v}_\mathrm{i}) - \frac{1}{\hat{r}_a}\left(\hat{\boldsymbol{S}}_a \boldsymbol{n}_a\right)\cdot \boldsymbol{n}\right] - \sum_{\substack{a \\ a\in\Pi}} \left(\hat{\boldsymbol{S}}_a \boldsymbol{n}_a\right)\cdot\left((\boldsymbol{I} - \boldsymbol{n}\otimes\boldsymbol{n})(\hat{\boldsymbol{v}}_a - \boldsymbol{v}_i)\right) \geq 0 \quad \text{in} \quad \overline{\Omega}_l \cap \overline{\Omega}_g
\end{gathered} \tag{7}$$

in which $\dot{m}_a = \hat{r}_a\, \mathbf{n}_a\, .(\hat{\mathbf{v}}_a - \mathbf{v}_i)$ stands for the interfacial mass efflux of the a constituent, $\hat{g}_a = \hat{u}_a - \hat{s}_a\hat{q}_a + \hat{p}_a/\hat{r}_a$ represents the Gibbs free energy per unit mass, s is the surface tension (a constant) and H is the mean curvature of the singular surface. In the past equations, the superimposed hat means that the variable is evaluated at the interface. Since air and vapor are assumed to coexist at one side of the interface while liquid flows alone at the other side, it comes out that:

$$\mathbf{n}_g = \mathbf{n}_v = -\mathbf{n}_l = \mathbf{n} \tag{8}$$

Equation (7a) establishes that the total flux of mass through the interface must be equal to zero. Since we consider by hypothesis that the air can not be dissolved into nor comes out from the liquid, then it results that:

$$\dot{m}_g = 0 \tag{9}$$

The interfacial balance of momentum Eq. (7b) states that the flux of momentum through the interface is compensated by the surface traction. Finally, the second law of thermodynamics Eq. (7c) establishes a distinction between possible and impossible processes. Possible processes are those for which the inequality is satisfied while impossible ones are characterized by the violation of this inequality. The possible processes are classed as reversible and irreversible. In the reversible processes the equal sign prevails and in the irreversible ones the unequal sign holds strictly.

To ensure that the second law of thermodynamics is always verified, we assume as constitutive assumptions that the no-slip condition holds at the interface and also that the rate of liquid-vapor phase change is proportional to the difference of the Gibbs free energy of the vapor and of the liquid:

$$\begin{aligned}&(\mathbf{I}-\mathbf{n}\otimes\mathbf{n})\hat{\mathbf{v}}_g=(\mathbf{I}-\mathbf{n}\otimes\mathbf{n})\hat{\mathbf{v}}_v=(\mathbf{I}-\mathbf{n}\otimes\mathbf{n})\hat{\mathbf{v}}_l=(\mathbf{I}-\mathbf{n}\otimes\mathbf{n})\mathbf{v}_i\\&\dot{m}=\dot{m}_v=-\dot{m}_l=b\left[(\hat{g}_v-\hat{g}_l)+\tfrac{1}{2}\left((\hat{\mathbf{v}}_v-\mathbf{v}_i)^2-(\hat{\mathbf{v}}_l-\mathbf{v}_i)^2\right)+\frac{1}{\hat{r}_l}(\hat{\mathbf{S}}_l\mathbf{n})\cdot\mathbf{n}-\frac{1}{\hat{r}_v}(\hat{\mathbf{S}}_v\mathbf{n})\cdot\mathbf{n}\right]\end{aligned}\tag{10}$$

Equation (10a) states that the tangential velocities of all constituents (gas, vapor and liquid) at the interface are equal to the tangential velocity of the interface. By assuming that b is a non-negative constant ($b \geq 0$), it is easy to show that with the above constitutive assumptions the second law of the thermodynamics Eq. (7c) is always satisfied regardless the initial and boundary conditions as well as the geometric nature of the interface.

To finally complete the modeling, state equations are required for the air and vapor constituents. For the sake of simplicity, we consider in this paper that both air and vapor behave as an ideal gas. For such kind of behavior, the equations of state are

$$p_a = r_a R_a q \quad \text{for} \quad a \in \Xi, \tag{11}$$

and the Gibbs free energy of the liquid and the vapor are given by:

$$\begin{aligned}g_l &= -c_l q\,\mathbf{log}(q)\\ g_v &= -c_v q\,\mathbf{log}(q)+R_v q\,\mathbf{log}(r_v)+R_v q+L(q)\end{aligned}\tag{12}$$

in which the R_a, c_a and $L(q)$ represent the gas constant, the specific heat at constant volume and the latent heat associated with the liquid-vapor phase change, respectively. The latent heat is an affine function of the temperature.

Equations (1) along with Eqs. (3-12), completed by appropriated boundary and initial conditions, form a complete set of equations to describe the motion of air-vapor cavities in flows of incompressible liquids taking into account irreversible mechanisms associated with the isothermal phase change transformation. Whatever the form of the singular surface, it can be seen that at the mechanical equilibrium (i.e. $\dot{m}=0$, $\mathbf{v}_\mathrm{i}=0$ and $\hat{\mathbf{v}}_a=0$, for $a \in \Pi$) one must have from Eq. (10b) that $\hat{g}_v=\hat{g}_l$. By using Eqs. (11) and (12), this relationship shows that there exists, for each fixed temperature, an equilibrium vapor pressure p_{sv} - called the saturated vapor pressure - for which $p_v = p_{sv}(q)$.

3. Single spherical cavity

Aiming to evaluate the capability of the model described in the past section to properly describe liquid flows with air-vapor cavities taking into account irreversible phase change transformations, we consider, as a first attempt, the classical spherical cavity dynamic problem (Plesset and Prosperetti, 1977; Knapp and Hollander, 1948).

Consider a single spherical cavity filled with air and water vapor immersed in an unbounded incompressible liquid medium. The liquid far away from the bubble is at rest and its pressure, denoted by p_∞, can be varied in time according to a known function $p_\infty(t)$. No matter this input function looks like, the shape of the cavity is assumed to remain spherical with radius $R(t)$ centered at a fixed point, chosen as being the origin of our spherical frame of reference.

Under these circumstances, the interfacial singular surface motion described by the function $z(\mathbf{x},t)=0$ takes the form

$$z(r,t)=r-R(t) \tag{13}$$

and, consequently, the surface velocity is expressed as:

$$\mathbf{v}_i=\frac{\partial R}{\partial t}\mathbf{e}_r \tag{14}$$

in which r is the radial coordinate and $\mathbf{e}_r$ is the unit vector in the radial direction. Denoting by $v_l = v_l(r,t)$ the magnitude of radial component of the liquid velocity field, the integration of Eq. (5a) in the radial coordinate between the limits $r = R^+$ and $r \to \infty$ yields:

$$v_l=\frac{R^2}{r^2}\hat{v}_l \tag{15}$$

in which $\hat{v}_l = \hat{v}_l(t)$ stands for the liquid velocity at the interface (i.e., $r = R^+$). The application of the same procedure to Eq. (5b), taking into account Eq. (15), allows one to rewrite the balance of linear momentum for the liquid as:

$$\frac{d\hat{v}_l}{dt}R + \frac{3}{2}\hat{v}_l^2 = \frac{1}{r_l}\left[\frac{4m_l}{3R}\hat{v}_l + \hat{p}_l - p_\infty\right] \tag{16}$$

By neglecting the inertia of the air and the vapor, the viscous effects and also the interaction force between these constituents and carrying out the integration of Eq. (1a), for $a \in \Xi$, in the radial coordinate between $r = 0$ and $r = R^-$ we obtain:

$$\hat{v}_v = -\frac{R}{3\hat{r}_v}\frac{d\hat{r}_v}{dt} \quad , \quad \hat{p}_g = \hat{p}_{g0}\frac{R_0^3}{R^3} \tag{17}$$

in which $\hat{p}_{g0}$ and R_0 represents the air pressure and the radius of the cavity at the initial time instant $t = 0$, respectively. By employing the assumptions made so far, the interfacial balance of mass and linear momentum (Eqs. (7a) and (7b)) can be written as:

$$\begin{aligned} \dot{m} &= \hat{r}_v\left(\hat{v}_v - \dot{R}\right) = \hat{r}_l\left(\hat{v}_l - \dot{R}\right) \\ \hat{p}_l - \hat{p}_g - \hat{p}_v &= -\frac{2s}{R} - \dot{m}^2\left(\frac{1}{\hat{r}_l} - \frac{1}{\hat{r}_v}\right) - 4m_l\frac{\hat{v}_l}{R} \end{aligned} \tag{18}$$

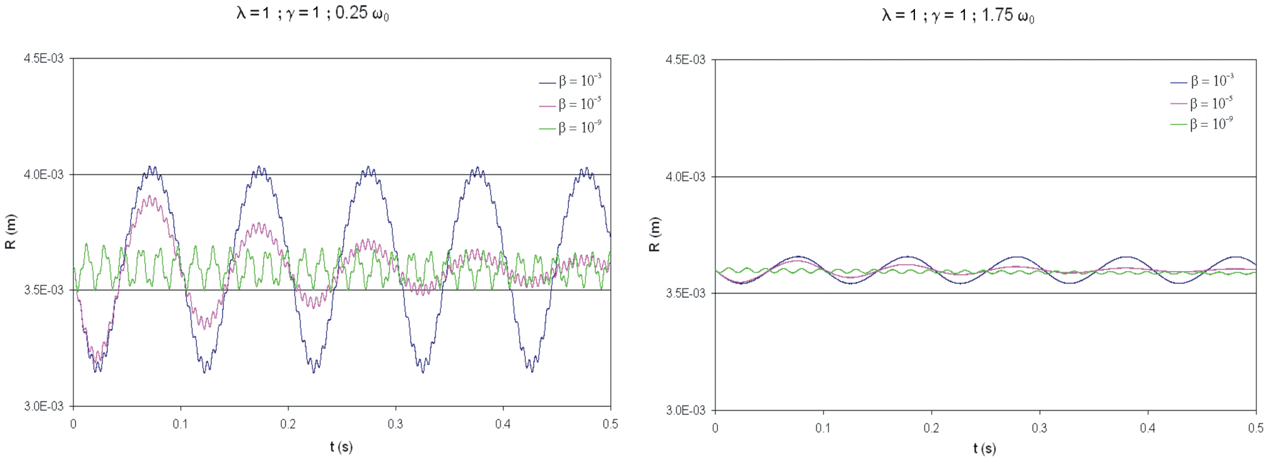
When the constitutive Eqs. (11) and (12) are evaluated at the interface they can be written as

$$\begin{aligned} \dot{m} &= \dot{m}_v = -\dot{m}_l = b\left[R_v q\,\mathbf{log}\left(\frac{\hat{p}_v}{\hat{p}_{vs}}\right) + \tfrac{l}{2}\left(\left(\hat{v}_v - \dot{R}\right)^2 - \left(\hat{v}_l - \dot{R}\right)^2\right) - 4gm_l\frac{\hat{v}_l}{R\hat{r}_l}\right] \\ \hat{p}_v &= \hat{r}_v R_v q \\ \hat{p}_g &= \hat{r}_g R_g q \end{aligned} \tag{19}$$

in which l and g are constants to which the values 0 (zero) and 1 (one) can be respectively assigned. These parameters were artificially introduced in Eq. (19) to allow the investigation of each of them separately on the system response. By analyzing Eq. (19a) bearing in mind that b, $b \geq 0$, is a material constant , which should be determined experimentally, it is possible to show that the second law Eq. (7c) is always satisfied, regardless the initial and boundary conditions as well as the geometry of the interface. Equations (16) to (19) form a complete set of nine equations for the nine unknowns $R, \hat{v}_l, \hat{v}_v, \hat{p}_l, \hat{p}_v, \hat{p}_g, \hat{r}_v, \hat{r}_g, \dot{m}$. It is worth mention that if we disregard the phase change process, what can be artificially done by setting $\dot{m} = 0$, the classical equation of the single bubble dynamics of Plesset and Prosperetti (1977) is recovered. In such a case, the vapor inside the cavity remains saturated at the saturated vapor pressure ($\hat{p}_v = \hat{p}_{sv} = const$) since there is no mass transfer through the bubble interface. As a result, the liquid and vapor velocities at the interface are always equal to the interfacial velocity; $\hat{v}_v = \hat{v}_l = \dot{R}$. The non-linear system of equations form by Eqs. (16-19) is used next to investigate the behavior of a spherical cavity submitted to an oscillating pressure field. To do so, the Newton-Raphason method was used to obtain a numerical approximate solution of the system of Eqs. (16) to (19).

4. Numerical results

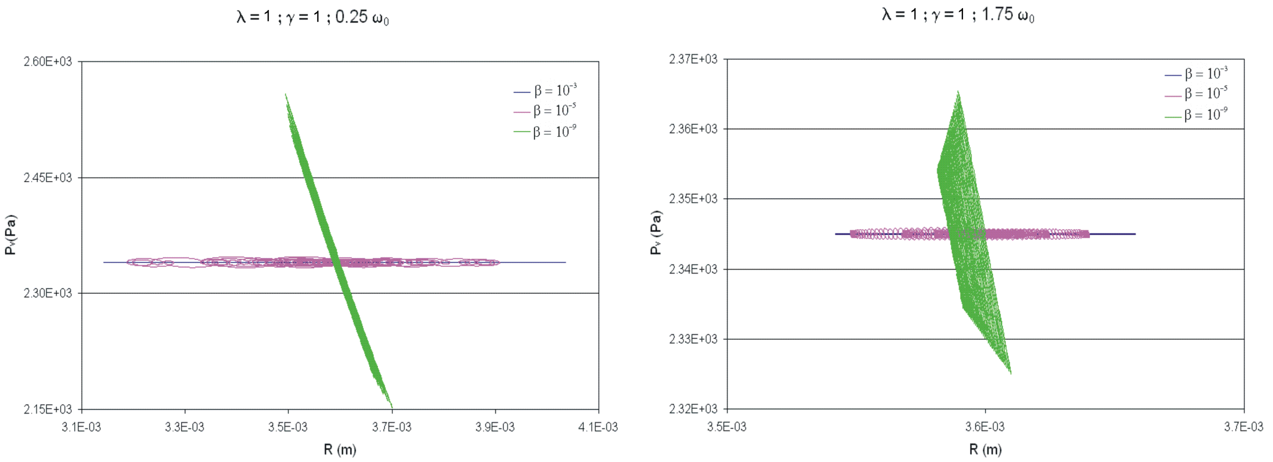
In order to assess the influence of the phase change process on the dynamics of a spherical cavity, we have employed the model described in the past section. To do so, we have considered a spherical cavity of water vapor and air having an initial radius $R = 0.0036m$. The vapor is at the beginning of the simulation at a saturated vapor pressure of $\hat{p}_v = \hat{p}_{sv} = 2340Pa$, which corresponds to a temperature $q = 25^{\circ}\mathrm{C}$, while the partial pressure of the air is $\hat{p}_g = 80\mathrm{Pa}$. The liquid pressure far away from the cavity p_∞, which is at static equilibrium at $t = 0$, is forced to oscillate according to $p_\infty = p_{eq} + 450 \cdot \mathbf{sin}[(\cdot\, w) \cdot t]\,\mathrm{Pa}$ around its equilibrium value $p_{eq} = 2379.44\,\mathrm{Pa}$ with frequency w. The surface tension, liquid viscosity and density at this temperature are $s = 0.073\text{N/m}$, $r_l = 998\text{kg/m}^3$ and $m_l = 1.01\times10^{-3}\text{Pa.s}$, respectively. The product $R_v q$ is equal to 135kJ/kg. The influence of phase change transformation on the bubble collapse is then investigated by attributing to the material constant b several different values: $b = 0$, $b = 10^{-3}$, $b = 10^{-5}$ and $b = 10^{-6}$ $\text{kg}^2.\text{m}^{-2}.\text{kW}^{-1}$. As it will be seen, when $b = 0$, the vapor inside the bubble is allowed to contract by raising its pressure, without transforming into liquid. On the other extreme, when a large value is assigned to it ($b = 10^{-3}$) the vapor remains almost all time at the saturated vapor pressure during the cavity collapse. Any value different from zero attributed to the material constant (such as, $b = 10^{-3}$, $b = 10^{-5}$ and $b = 10^{-6}$) gives the model a more realistic physical behavior since one should expect that the vapor be transformed into liquid during the cavity motion by dissipating some amount of energy. On physical grounds, this energy dissipation is associated with the irreversibility of the phase change process. Before going a step further, numerical simulations were carried out to determine an adequate time-step in order to achieve convergence. Time-steps of 10^{-6} s, 10^{-7} s and 10^{-8} s have been used and the results obtained have not shown any significant difference. Thus, time-steps of 10^{-7} s were used in the forthcoming simulations. In addition, different simulations were carried out by considering $l = 0$ $g = 0$, $g = 1$ $g = 0$, $l = 0$ $g = 1$ and $l = 1$ $g = 1$. The obtained results have also not differed among them, showing that neither the flux of interfacial kinetic energy nor the interfacial surface tension do affect the thermo-mechanical behavior of the cavity (Silva, 2004). Figure 1 presents the cavity radius variation against the time for two distinct frequencies for $l = 1$ and $g = 1$ and three different values of b. As it can be seen, different behaviors are noted according to the values assigned to b. For $b = 10^{-5}$ $kg^2 \cdot m^{-2} \cdot s^{-1} \cdot W^{-1}$,a more realistic response is observed since the phenomenon of attenuation is noticed throughout the history. It is also worth mentioning that the responses differ significantly with respect to the frequency of excitation. When the frequency is greater than the natural frequency w_0 ($w = 1.75 w_0$), the amplitudes of oscillation are drastically reduced whether compared to the casein which $w = 0.25 w_0$. It becomes evident that variations in b drastically change the physical response of system.



(a) (b)

Figure 1: Bubble radius histories for pressures excitations with frequencies: (a) of $0.25\,w_o$ and (b) of $1.75\,w_o$.

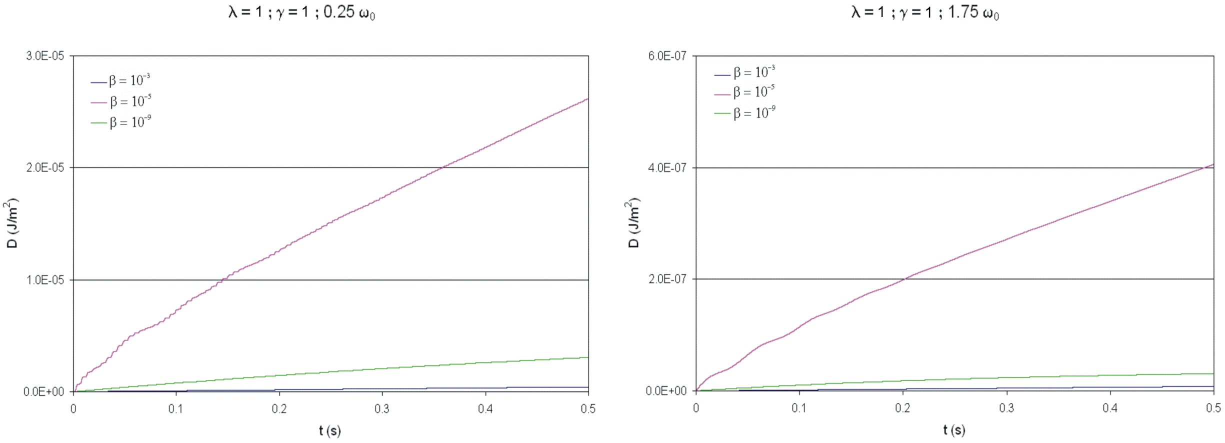
The difference exhibited by the system becomes more stringent when the vapor pressure is plotted against the cavity radius, as shown in Figure 2 for two different frequencies and for $b = 10^{-3}$, $b = 10^{-5}$, e $b = 10^{-9}$ $kg^2 \cdot m^{-2} \cdot s^{-1} \cdot W^{-1}$. Note that the y-axis scale is not the same in the two graphs.



(a) (b)

Figure 2: Vapor pressure against bubble radius for excitations with frequencies: (a) of 0.25 w_o and (b) of 1.75 w_o.

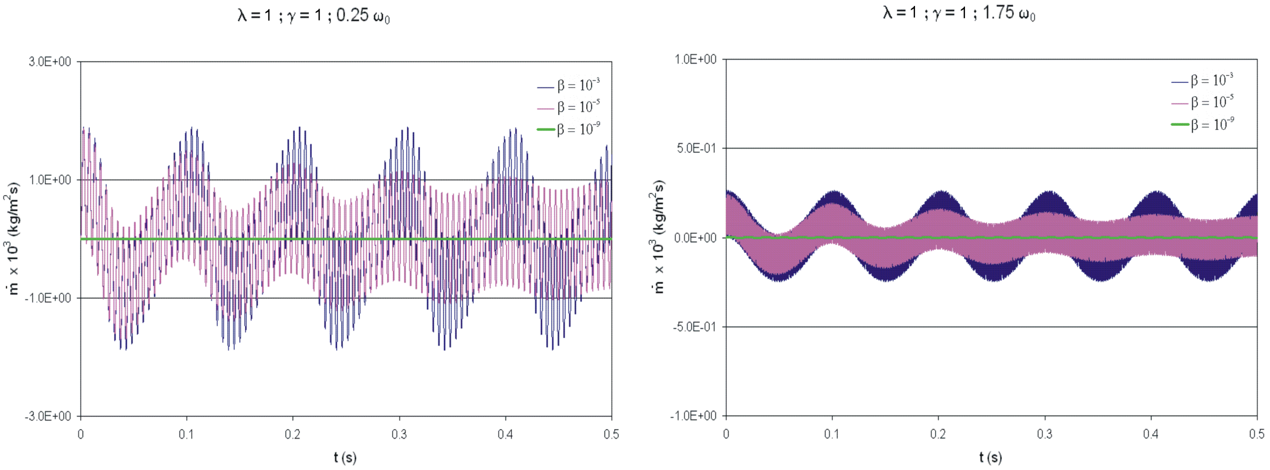
For $b = 10^{-3}\ kg^2 \cdot m^{-2} \cdot s^{-1} \cdot W^{-1}$ the vapor pressure remains almost constant during the whole period of simulation. The hysteretic loop observed for $b = 10^{-5}\ kg^2 \cdot m^{-2} \cdot s^{-1} \cdot W^{-1}$, whatever the frequency is, indicates that energy dissipation is greater than that noted for the other values of b. It can be seen easier with the aid of Figure 3 which presents the energy dissipation as a function of the time. For $b = 10^{-9}$ the cavity behaves as an elastic body by absorbing and then restituting the energy to the liquid medium. To allow an easier comprehension of the comments presented ahead involving the behavior of the vapor pressure (Figure 2) and the energy dissipation against the time(Figure 3), it is presented in Figure 4 the mass rate of phase change for the same values of b and frequencies presented before. By observing the Figures 2, 3 and 4 it can be noted that when $b = 10^{-3}\ kg^2 \cdot m^{-2} \cdot s^{-1} \cdot W^{-1}$ the phase transformation takes place with the vapor pressure almost constant and equal to the saturated vapor pressure. In addition, one can note that the energy dissipation involved in this process $b = 10^{-3}\ kg^2 \cdot m^{-2} \cdot s^{-1} \cdot W^{-1}$ is very small (see Figure 3) when compared to the cases in which $b = 10^{-5}\ e\ b = 10^{-9}\ kg^2 \cdot m^{-2} \cdot s^{-1} \cdot W^{-1}$. On the other hand, it can be seen that for a same frequency of excitation the case in which $b = 10^{-3}\ kg^2 \cdot m^{-2} \cdot s^{-1} \cdot W^{-1}$ is the one that presents the largest rate of liquid-vapor.



(a) (b)

Figure 3: Dissipated energy histories for pressures excitations with frequencies: (a) of 0.25 w_o and (b) of 1.75 w_o.

For $b = 10^{-9}\ kg^2 \cdot m^{-2} \cdot s^{-1} \cdot W^{-1}$ the phase change is almost negligible as well as the rate of energy dissipation, as it can be seen in Figures 4 and 3.



(a) (b)

Figure 4: Mass rate of phase change histories for pressures excitations with: (a) of 0.25 w_o and (b) of 1.75 w_o.

Finally, it is worth mentioning that any other value assigned to the material constant b within the interval (10^{-9}, 10^{-3}) gives the model a more realistic behavior since one should expect that time dependent liquid-vapor transformations takes place in a process in which energy dissipation is involved in. This dissipation of energy is of course associated with the irreversibility of the phase change process.

5. Final remarks

A theoretical model based on the Continuum Theory of Mixtures and on the Singular Surface Theory has been presented to describe the dynamics of cavities taking into account the irreversibility of the phase change process as well as the flux of interfacial kinetic energy and the interfacial surface tension. Numerical simulations carried out have shown that the last two terms do not alter the thermo-mechanical behavior of the cavity. However, the irreversibility of the phase change can introduce significant damp in its motion for frequencies of excitation below and above the natural frequency.

6. Acknowledgements

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8. Responsibility notice

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