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# A MODEL FOR FREQUENCY-DEPENDENT FRICTION IN ONEDIMENSIONAL UNSTEADY FLUID FLOWS

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***Abstract.*** *This work presents a consistent thermo-mechanical model based on the continuum theory of mixtures to describe the unsteady friction in one-dimensional fluid flows in deformable pipings. The real fluid flow is treated as a virtual structured motion of several constituents, all of them having the same equation of state. The governing equations are derived in a systematc and coherent thermodynamic basis. The constitutive parameters of the model are easily determined from the knowlegment of the actual velocity profile, what constitutes one of its advantages when compared to other existing models. The governing equations form a system of hyperbolic partial differential equations whose numerical approximation is carried out by using the method of characteristics with specified time interval. A comparison with experimental data reveals that the predictions of the proposed exhibit an excellent agreement for laminar flows.*

***Keywords****: Unsteady friction, transient fluid flows, continuum theory of mixture, method of characteristics.*

## 1. Introduction

Unsteady fluid flows in pipelines are commonly described by one-dimensional models inasmuch the pipeline extensions are several orders of magnitude greater than their diameters. Within this context, friction losses have often been estimated by formulae derived for steady state flow conditions in a procedure referred to as quasi-steady approximation. It is based upon the assumption that the head loss during transient conditions is equal to the head loss obtained for steady uniform flow, with an average velocity equal to the instantaneous transient velocity.

However, during unsteady flow conditions, shear stress at the pipewall is not in phase with the mean velocity and the velocity profile can be completely different from a uniform flow. Therefore, friction losses computed according to the quasi-steady approximation are inaccurate especially in fast transient laminar or turbulent flows. As a result, pressure peaks are erroneously predicted in both magnitude and phase. To overcome this difficulty, different models primarily based on the past history of the mean velocity and acceleration have been proposed (Zileke, 1967; Araya and Chaudry, 1997, .Bergant et al., 2001; Brunome et al, 2000, Vardy and Brown, 2003)

Although these approaches fairly give satisfactory results, they require the values of empirical parameters to be deduced from specific experiments. This paper presents a one-dimensional model based on the continuum mixture theory. The fluid is regarded as mixture of n kinematically independent continua, having all of them the same state laws. The equations governing the fluid flow in the pipe are obtained in a consistent thermo-mechanical basis. Constitutive expressions for the interaction forces among the pseudo-constituents are derived through a systematic way, which verifies automatically the second law of the thermodynamics. Simulations carried out for laminar transient flow are presented and compared with experimental data, showing an excellent agreement.

## 2. Balance equations

Since the radius-to-length ratios of pipes conveying fluids are very small, the motion of the fluid inside the pipe is usually described by one-dimensional models. To describe the frequency-dependent friction in transient fluid flows confined in pipes we propose in this paper a model based on the continuum mixture theory (Drew and Passman, 1998). For the time being, we assume that the fluid is a mixture of $n, n > 2,$ deformable constituents each of which is regarded as a continuum. As usual, the mixture is understood as a superposition of the $n$ constituents, each of them having its own independent motion in such a way that each spatial position $x$ along the pipe centerline, with $x \in L$ ($L$ being the pipe length) is simultaneously occupied by them at any time instant $t$ . The pipe is supposed to be an axi-symmetrical deformable shell, so that its internal diameter is represented by $D(x,t)$ and its cross-sectional area by $A(x,t)$, $A = pD^2/4$.Let $r_j(x,t)$ and $v_j(x,t)$ designate the spatial mass density of the *j-th* constituent within the mixture and its

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spatial velocity field, respectively. Let $[a,b]\subset L$ represent an arbitrary stretch along the pipe centerline giving rise to a control volume whose volume is given by the Riemann integral of $Adx$ between the limits $x=a$ and $x=b$. Within this context, the balance of mass principle for the mixture as a whole can be stated as:

$$\sum_{j=1}^{n}\frac{d}{dt}\int_{a}^{b} r_j A dx + \left[r_j A v_j\right]_{x=b} - \left[r_j A v_j\right]_{x=a} = 0 \qquad (1)$$

By assuming that the fluids form a non-reacting mixture, it can be proved that the point form of the balance mass principle for each constituent is expressed as:

$$\frac{\partial}{\partial t}(r_j A) + \frac{\partial}{\partial x}(r_j A v_j) = 0 \qquad (2)$$

The *j-th* constituent flowing inside the pipe is subjected to body force per unit of volume $b_j$ (gravity), surface force per unit of cross-sectional area $t_j$ and a reactive contact friction force per unit of lateral area $a_j$ acting on the pipewall-fluid interface, all of them along the *x*-direction. Thus, referred to an inertial frame of reference the balance of linear momentum principle for the whole mixture can be postulated as:

$$\sum_{j=1}^{n}\frac{d}{dt}\int_{a}^{b} r_j v_j A dx + \left[r_j v_j^2 A\right]_{x=b} - \left[r_j v_j^2 A\right]_{x=a} = \sum_{j=1}^{n}\left[t_j A\right]_{x=a} - \left[t_j A\right]_{x=a} + \int_{a}^{b}\left[b_j A - t_j\frac{\partial A}{\partial x} - a_j p D\right]dx \qquad (3)$$

whose counterpart point form for each constituent may be expressed as:

$$r_j\left(\frac{\partial v_j}{\partial t} + v_j\frac{\partial v_j}{\partial x}\right) = \frac{\partial t_j}{\partial x} - 4\frac{a_j}{D} + b_j - m_j \qquad (4)$$

in which $m_j$ is an internal interaction force per unit of cross-sectional area exerted by the other constituents on the *j-th* constituent. As long as Eqs. (3) and (4) hold, it can be easily proved that the balance of linear momentum for the mixture as whole (3) is equivalent to require that the sum of the internal forces must be zero everywhere:

$$\sum_{j=1}^{n} m_j = 0 \qquad (5)$$

By considering that the constituents of the mixture share a same absolute temperature $q$ which remains constant throughout its motion and that the constituents are non-heat conducting substances, the entropy production inequality can be written for the mixture as a whole as (Rajagopal and Tao, 1995):

$$\sum_{j=1}^{n}\frac{d}{dt}\int_{a}^{b} r_j s_j A dx + \left[r_j s_j A\right]_{x=b} - \left[r_j s_j A\right]_{x=a} \geq \frac{1}{q}\sum_{j=1}^{n}\int_{a}^{b}\left(r_j r_j A - p D q_j^L\right)dx \qquad (6)$$

in which $q$, $s_j$, $r_j$ and $q_j^L$ represent the absolute temperature, the specific entropy, the rate of heat production/absorption per mass and the lateral heat flux through the pipewall associated to the *j-th* constituent, respectively. If, within this same context, we postulate that the entropy production inequality for each constituent is given by (Drew and Passman, 1998):

$$\frac{d}{dt}\int_{a}^{b} r_j s_j A dx + \left[r_j s_j A\right]_{x=b} - \left[r_j s_j A\right]_{x=a} \geq \frac{1}{q}\left[\int_{a}^{b}\left(r_j r_j A - p D q_j^L\right)dx + m_j v_j\right] \qquad (7)$$

then, it is possible to prove, under the assumptions made so far, that counterparts local forms may be written as;

$$\frac{1}{q}\sum_{j=1}^{n} m_j v_j \geq 0 \tag{8}$$

for the mixture as a whole and as,

$$\frac{4}{D} a_j v_j \geq 0 \tag{9}$$

for each constituent. The above equations form a complete set of balance equations describing the fluid flow of the mixture as a whole and of each constituent. To them, we must add the constitutive behavior of the constituents, the internal interaction force among them and, finally the pipewall-fluid reactive force.

**3. Virtual structured mixture model**

The balance equations presented in the past section are quite general and apply to any kind of mixture flowing in a general conduit. However, to go a step further towards applications one must not only characterize the structure of the mixture but also provide constitutive relationships for the internal forces $m_j$ among the constituents as well as for the reactive contact friction force per unit of lateral area $a_j$ acting on the pipewall-fluid interface.

As a mixture model, we consider a virtual structured flow of concentric cylindrical shells each of them representing a constituent whose velocity and volumetric fraction are denoted by $v_j$ and $a_j$. If $R^0 = D^0 / 2$ stands for the internal radius of the pipe of the non-deformed pipe, we assume without losing generality and for the sake of simplicity that every cylindrical shell has a same thickness $\Delta R_j^0 = R^0 / n$, in which *n* is the number of constituents in the mixture. By considering that $\Delta R_0^0 = 0$ and $R_0^0 = 0$, the radius of each cylindrical shell (of each constituent) can be assessed through the following recurrence relationship:

$$R_j^0 = R_{j-1}^0 + \frac{\Delta R_{j-1}^0}{2} + \frac{\Delta R_j^0}{2} \quad , \quad j = 1,\ldots,n \tag{10}$$

With the radius and the thickness of each constituent it becomes possible to determine the volume fraction of each constituent in the mixture by computing the ratio of its volume to the total volume of the mixture resulting:

$$a_j = \frac{2R_j^0 \Delta R_j^0}{R^0 R^0} \tag{11}$$

It is worth noting that, by construction, the following restriction is automatically verified:

$$\sum_{j=1}^{n} a_j = 1 \tag{12}$$

The virtual feature of the mixture appears when, in addition, we admit that all the constituents are formed by a same barotropic fluid whose equation of state, relating its mass density $r$ to the pressure $p$ it is subjected to, is given according to:

$$r = r_0 \exp(p / K) \tag{13}$$

in which $r_0$ stands for the mass density at the reference condition of atmospheric pressure and $K$ represents the isentropic bulk modulus. By assuming that the surface force per unit of cross-sectional area $t_j$ is only due to the partial pressure of the constituents, then the above mentioned assumptions allows one to write for each constituent *j*:

$$t_j = a_j p \quad , \quad r_j = a_j r \tag{14}$$

so that the mean velocity of the flow is given by:

$$v = \sum_{j=1}^{n} r_j v_j \Big/ \sum_{j=1}^{n} r_j = \sum_{j=1}^{n} a_j v_j \tag{15}$$

The structured nature of the mixture intrinsically suggests that the interaction force among the constituents be of the form:

$$m_j = C_{j,j-1}(v_j - v_{j-1}) + C_{j,j+1}(v_j - v_{j+1}) \tag{16}$$

and the reactive contact friction force per unit of lateral area $a_j$ acting on the pipewall-fluid interface be zero except for the *n-th* constituent:

$$a_j = \begin{cases} 0, & \text{for} \quad j = 1,\ldots,n-1 \\ Cv_j, & \text{for} \quad j = n \end{cases} \tag{17}$$

in which $C_{j,j-1}$, $C_{j,j+1}$ and $C$ are all positive material constants. Since the interaction forces must satisfy restriction the restriction given by Eq. (5), these material parameters are chosen in such way that $C_{1,0} = 0$, $C_{n,n+1} = 0$ and $C_{j,j-1} = C_{j,j+1}$. It should be remarked that the past choices ensure that the entropy production inequalities given by Eqs. (9) and (8) for the each constituent and for the mixture as a whole is automatically verified.

The cross-sectional area variation is known to play a crucial role in the transient response of the fluid flow. Since the pipewall is subjected to small deformations, the cross-sectional area can be related to pipewall circumferential deformation $e_q$ according to (Freitas Rachid and Stuckenbruck, 1990):

$$A = A_0(1 + 2e_q) \tag{18}$$

in which $A_0$ stands for the cross-sectional area in the non-deformed configuration. If the pipewall behaves elastically and its radial inertia is neglected then,

$$e_q = \frac{y\, s_q}{E} = \frac{yR^0}{Ee}\, p \tag{19}$$

in which $e$ is the pipewall thickness, $E$ is the Young modulus and $y$ is a factor related to the way the pipe is anchored (Wylie and Streeter, 1993). Now, by assuming slightly compressible fluid flows ( $p/K << 1$ ) with low Mach numbers in the absence of gravitational effects Eqs. (2), (4), (12), (13), (14), (15), (18) and (19) can be combined to result the following system of equations:

$$\begin{aligned} &\frac{1}{r_0 a^2}\frac{\partial p}{\partial t} + \frac{\partial v}{\partial x} = 0 \\ &r_0 \frac{\partial v}{\partial t} + \frac{\partial p}{\partial x} + \frac{4}{D^0} a_n = 0 \\ &a_j r_0 \frac{\partial v_j}{\partial t} + a_j \frac{\partial p}{\partial x} + m_j + \frac{4}{D^0} a_j = 0, \quad \text{para} \quad j = 2,\ldots,n \end{aligned} \tag{20}$$

in which $a$ stands for the wave propagation velocity in the mixture inside the tube:

$$a = \sqrt{\frac{K/r}{1 + D^0 K y / Ee}} \tag{21}$$

Equations (20) form a hyperbolic partial differential equations system whose numerical approximation is carried out by using the characteristics method with a specified time interval (Wylie and Streeter, 1993 and Freitas Rachid and Stuckenbruck, 1990). Before simulating a specific problem, one must determine the coefficients $C_{j,j-1}$ (or $C_{j,j+1}$) and $C$. This is done by requiring that under steady-state regime the velocity of the *j-th* constituent be equal to the velocity observed in a developed laminar fluid flow of a viscous fluid whose viscosity is $m$ at a same radial position $R_j^0$. When it is performed the following result is obtained:

$$C = \frac{2mR^0}{R^0R^0 - R_n^0R_n^0} \quad , \quad C_{j,j+1} = \frac{4m}{R^0R^0}\frac{\sum_{j=1}^{n-1} 2R_j^0 \Delta R_j^0}{R_{j+1}^0R_{j+1}^0 - R_j^0R_j^0}, \quad \text{para} \quad j = 1,\dots,n-1 \qquad (22)$$

**4. Numerical results and experimental validation**

In order to verify the capability of the proposed model in properly describing the frequency-dependent friction the experimental data of Holmboe and Roleau (1967) were taken as reference. The experimental setup used by the authors consists of a reservoir-pipe-valve installation shown in Figure 1. The reservoir is a constant pressure tank from which a fluid of mass density $r = 878.4$ kg/m$^3$ and absolute viscosity of $m = 0.03484$ N.s/m$^2$ flows initially downstream the pipe towards the valve in steady-state regime. The pipe is horizontal and its length is $L = 36.09$ m. It is made of steel and has an internal diameter $D = 0.025$ m. The wave speed disturbances propagate in the in liquid-pipe system is reported to be $a = 1324$ m/s. The valve at the downstream end of the pipe is a block valve which is initially fully open. At these conditions the flow is laminar with a Reynolds number of 82 and with a fluid mean velocity $V_0 = 0.125$ m/s. The transient regime is produced by rapid valve closure, being the closure time less than 0.004 s. Pressure transducers located at the middle of the pipe length and at the valve are used to record the pressure signal along the time after the closure of the valve.

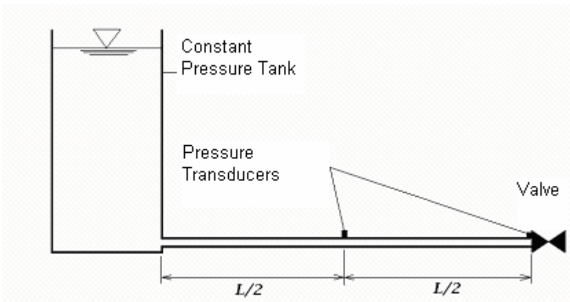


Figure 1 Experimental setup used by Holmboe and Roleau (1967).

Before establishing a comparison between the experimental data and the results predicted by the proposed model preliminary simulations were carried out in order to identify the appropriate number of constituents to be used in the analysis. For this purpose three numerical simulations were done by using $n = 5$, $n = 10$ and $n = 50$ constituents. The results obtained are depicted in Figure 2 which exhibits the normalized overpressure head at the valve against the normalized time. The overpressure head is normalized by using the Joukowsky head $aV_0/g$ whereas the time $t$ is normalized by the wave travel time from the tank to the valve $L/a$. The variable $H_0$ stands for the head at the valve at steady-state and is approximately zero since the block valve is initially fully open.

As it can be observed in Figure 2, the shape of the curves is drastically altered when the number of constituents is increased form five to ten. On the other hand, almost no significant variation is detected when we compare the curves associated with ten and fifty constituents. In fact, by employing a number of constituents greater than fifty brings no benefit in terms of precision. Thus, ten constituents are expected to be enough in order to accurately describe the unsteady friction phenomenon.

Having identified an adequate number of constituents to be used in the model, a comparison with experimental data is presented in Figure 3, for the pressure response at the valve (Figure 3 (a)) and at the mid point of the pipe length (Figure 3 (b)). To highlight the results predicted by the proposed model, it has also been include in these figures the predictions of the model proposed by Araya and Chaudry (1997). By inspecting the curves presented in both Figures 3 (a) and 3 (b) we note that both theoretical predictions present a fairly good agreement with the experimental one, at least for the first fluid cycle.

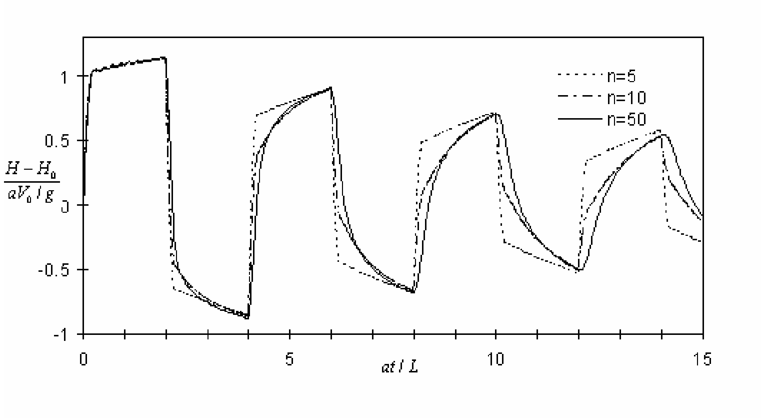
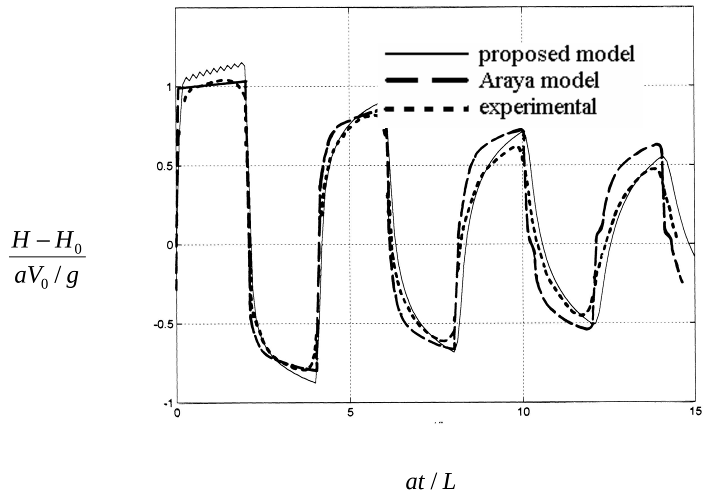
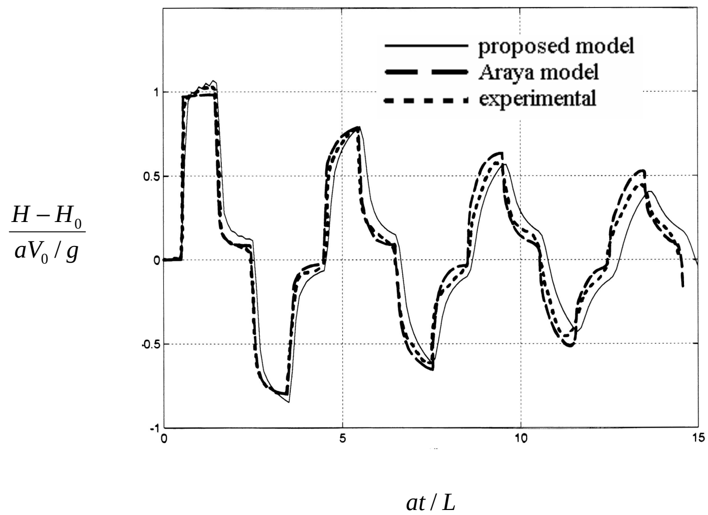



Figure 2 Normalized head at the valve against the normalized time for different number of constituents.

Nevertheless, beyond the first cycle it becomes apparent that the response predicted by the proposed model presents a much better agreement, not only in phase but also in amplitude, with the experimental data than the one of Araya. It is worth mentioning that the discrepancy observed in pressure amplitude in the half first cycle between the experimental data and the proposed model is due to the limitation of the one-dimensional theoretical model in properly reproducing the circumferential anchorage conditions of the pipe close to the valve. Such a phenomenon has also been observed in similar models when comparisons with experimental results were conducted (Freitas Rachid and Stuckenbruck, 1990). It is also important to remark that the pressure pulse attenuation due to viscous dissipation is not adequately described when the steady-state friction factor is used in the transient simulation (Wylie and Streeter, 1993).



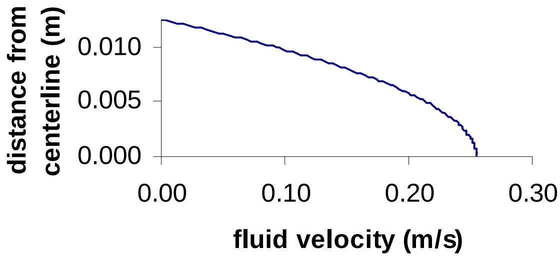

(a) Normalized pressure heads at the valve



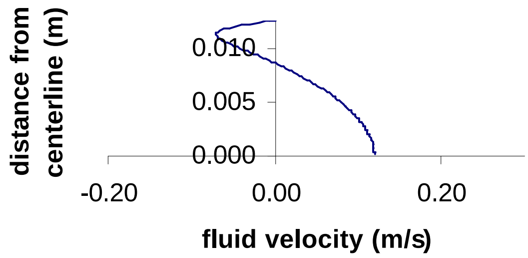
(b) Normalized pressure heads at the middle of the pipe

Figure 3 Normalized pressure heads for laminar transient flow at the valve (a) and at the middle of the pipe length (b) for the experimental data, Araya's model and the proposed model.

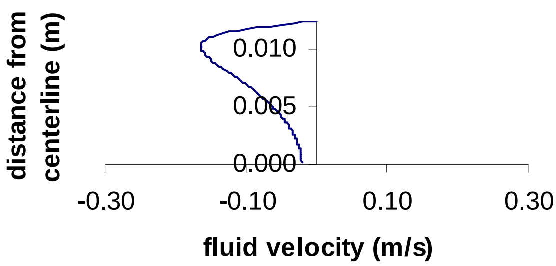
The model presented herein not only describes the pressure and mean velocity transient fluctuations but also allows the computation of the velocity profile at any time instant. A sequence at consecutive time instants of the velocity profile at the mid point of the pipe length is shown in Figure 4, at *t=0* Fig 4(a), at *t=L/a* Fig 4(b), at *t=2L/a* Fig 4(c) and at *t=4L/a* Fig 4(d). Figure 4(a) depicts the initial parabolic velocity profile typical of a laminar flow. When the pressure wave originated by the valve slam passes through the middle of the pipe for the first time, the velocity profile is suddenly altered and the mean flow velocity is significantly reduced. After this instant, the fluid velocity close to the pipe wall reverses direction (see Fig. 4 (b)) increasing energy dissipation. As the wave is reflected from the upstream tank passing again at the mid point of the pipe moving towards the valve, the flow direction has been almost totally reversed, producing higher velocity gradients close to the wall as illustrated in Fig.4 (c). This intense velocity gradient contributes even further to energy dissipation when the reflected wave passes. Note that Fig. 4 (c) illustrates a typical velocity profile when the flow is increasing in the reverse direction.



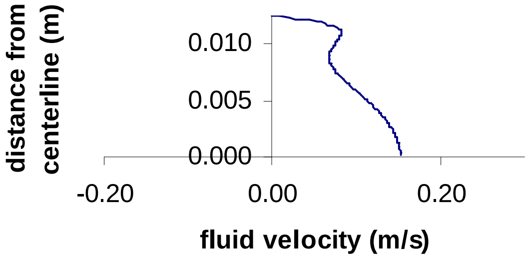

(a) *t=0*




(b) *t= L/a*




(c) ) *t=* 2*L/a*




(d) ) *t*= 4*L/a*

Figure 4. Velocity profiles at the middle of the pipe length: (a) at *t=0* , (b) at *L/a*, (c) at *2L/a* and (d) at *4L/a*.

Finally, it is shown in Fig. 4 (d) the profile velocity at *t=4L/a* when the flow is acquiring velocity in the forward direction again, completing the first fluid cycle of oscillation.

**5. Conclusions**

It has been presented in this paper a thermo-mechanical consistent frequency-dependent friction model for one-dimensional unsteady fluid flows in deformable pipes. In opposition to the models proposed by Brunome (2000), Araya and Chaudry (1997) and Zielke (1968), the main advantage of the proposed model is that it does not require complex identification of its constitutive parameters. The results obtained have shown the proposed model presents a better agreement with experimental data than those proposed by Araya and Chaudry (1997). The excellent results obtained enable us to extend the model to cope with turbulent fluid flows.

**6. Acknowledgements**

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