NON-LINEAR BURGERS´ EQUATION SOLUTION FOR TWO DIMENSIONAL LID-DRIVEN CAVITY

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Abstract. *Non-linear Burgers´ equation has no real flow meaning but it is a mathematical model of hydrodynamics. This would be important in the study of turbulence, shock waves among other kind of flows. The non-linear term in the Burgers´ equation contributes a wave of appreciable amplitude traveling in some direction. This wave eventually dissipates, and the non-linear solution tends to the same form as the linearized solution, though with a smaller amplitude. The non-linear term steepens the velocity gradient; however, because of the damping effect, no discontinuity occurs.*

The generalized integral transform technique (GITT) is a powerful tool to solve second order partial differential equations that has been used to solve equations with special difficulties such as non-separable equation coefficients, variable boundary condition coefficients, irregular domains, non-linearities, and so on. In this work we use the GITT to solve the non-linear Burgers´ equation for two-dimensional lid-driven cavity.

Keywords: *Burgers´ equation, generalized integral transform, non-linear.*

1. Introduction

Burgers equation is non-linear partial differential equation of diffusion-convection kind. It is considered as a simplified form of Navier-Stokes equations, for the cases when the pressure gradient is null. This model defines a non-solenoidal velocity field, e.g., a field with non-null divergence. The non-linear term in Burgers´ equation gives rise to a wave that moves in some direction. This wave eventually dissipates and the non-linear solution tends to the same shape as the linearized solution, but with smaller amplitude.

In this work, the Burgers´ equation solution is developed by applying the Generalized Integral Transform Technique (GITT) to unsteady fluid flows of Newtonian fluids inside two-dimensional lid-driven cavity. Numerical results obtained for Reynolds 100, 400 and 1000 are compared to those ones obtained by using finite difference method (FDM). The integral transform (GITT) is a hybrid analytical-numerical method that consists in transforming a given system of partial differential equations into a coupled infinite system of ordinary differential equations, which after being truncated to a number of equations can be solved numerically. This technique is a powerful tool that allows solving a variety of complex diffusive and convective-diffusive problems. Hydrodynamical or thermal purely diffusive problems, modeled by linear elliptical partial differential equations, defined over irregular domains, e. g., those ones in which it is not possible to find an orthogonal coordinate system that matches the domain boundary geometry, were solved by Aparecido and Cotta (1987, 1990b), Aparecido, Cotta and Ozisik (1989), Cotta, Leiroz and Aparecido (1992), and Aparecido, Viera and Campos-Silva (2000, 2001). Convective-diffusive problems involving hydrodynamically developed and thermally developing convection are mathematically modeled by second order partial differential equations of parabolic type. Such kinds of problems were solved by Aparecido (1997), Aparecido and Cotta (1990a, 1990c, 1992), Aparecido and Ozisik (1998), and Maia, Aparecido and Milanez (2000, 2001).

A work very used as reference is due to Ghia et al. (1982), in which the authors used the streamline and vorticity formulation to simulate the solution of Navier-Stokes equation for a square lid-driven cavity and Reynolds numbers up to 10000.

The problem geometry and coordinate system is shown in Figure 1, along with the domain height, L_y, the domain width, L_x, and boundary conditions

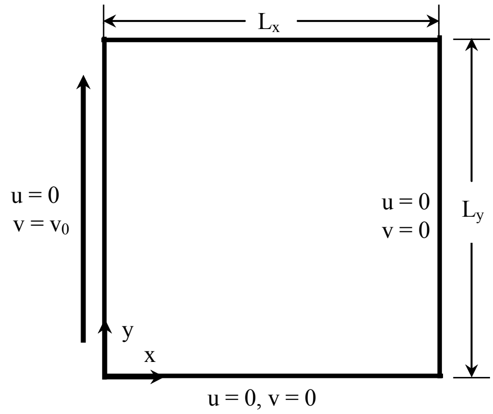


Fig. 1: Geometry and coordinate system for a lid-driven cavity.

2. Mathematical Formulation

Burgers´ equation defined in the domain $\Omega = \Omega_{xy} \otimes \Omega_t$, with $\Omega_{xy} \in \Re^2$ and $\Omega_t = (t_0, t_1)$, subjected to boundary and initial conditions, respectively, and considering the following dimensionless variables:

$$U(X,Y,\tau) \equiv \frac{u(x,y,t)}{v_0}; \qquad V(X,Y,\tau) \equiv \frac{v(x,y,t)}{v_0}; \qquad Y \equiv \frac{y}{l_y}; \qquad X \equiv \frac{x}{l_y}; \qquad L_x = \frac{l_x}{l_y}; \qquad L_y = 1; \qquad \tau = \frac{v_0\, t}{l_y}$$

can be written as follows:

$$\frac{\partial U(X,Y,\tau)}{\partial \tau} = \frac{1}{Re}\left[\frac{\partial^2 U(X,Y,\tau)}{\partial X^2} + \frac{\partial^2 U(X,Y,\tau)}{\partial Y^2}\right] - U(X,Y,\tau)\frac{\partial U(X,Y,\tau)}{\partial X} - V(X,Y,\tau)\frac{\partial U(X,Y,\tau)}{\partial Y} \tag{1a}$$

$$\frac{\partial V(X,Y,\tau)}{\partial \tau} = \frac{1}{Re}\left[\frac{\partial^2 V(X,Y,\tau)}{\partial X^2} + \frac{\partial^2 V(X,Y,\tau)}{\partial Y^2}\right] - U(X,Y,\tau)\frac{\partial V(X,Y,\tau)}{\partial X} - V(X,Y,\tau)\frac{\partial V(X,Y,\tau)}{\partial Y} \tag{1b}$$

$$U(X,Y,\tau) = 0, \quad \tau = 0; \qquad \text{for} \quad 0 \le X \le L_x \quad \text{and} \quad 0 \le Y \le L_y; \tag{2a}$$

$$V(X,Y,\tau) = 0, \quad \tau = 0; \qquad \text{for} \quad 0 \le X \le L_x \quad \text{and} \quad 0 \le Y \le L_y; \tag{2b}$$

$$U(X,Y,\tau)\big|_{X=0} = 0, \qquad U(X,Y,\tau)\big|_{X=L_x} = 0, \qquad 0 \le Y \le L_y; \tag{3a,b}$$

$$U(X,Y,\tau)\big|_{Y=0} = 0; \qquad U(X,Y,\tau)\big|_{Y=L_y} = 0, \qquad 0 \le X \le L_x; \tag{3c,d}$$

$$V(X,Y,\tau)\big|_{X=0} = 0, \qquad V(X,Y,\tau)\big|_{X=L_x} = 0, \qquad 0 \le Y \le L_y; \tag{3e,f}$$

$$V(X,Y,\tau)\big|_{Y=0} = V_0 = 1, \quad V(X,Y,\tau)\big|_{Y=L_y} = 0, \qquad 0 \le X \le L_x; \tag{3g,h}$$

for $\tau > \tau_0$.

The problem presented above involving the Burgers´ equation is solved by using the Generalized Integral Transform Technique (GITT).

3. Applying GITT to Transform the Problem in Y Direction

In according with the methodology to apply GITT, described by Cotta (1993), it is considered the following auxiliary Sturm-Liouville eigenvalue problems (Aparecido, 1997) associated to the velocities $U(X,Y,\tau)$ and $V(X,Y,\tau)$, and its boundary conditions, as follow:

$$\frac{d^2\Psi_i(Y)}{dY^2} + (\mu_i^u)^2\Psi_i(Y) = 0, \qquad 0 \le Y \le L_y; \qquad\qquad \Psi_i(0) = 0 \quad \text{e} \quad \Psi_i(L_y) = 0; \tag{4a,b}$$

$$\frac{d^2\Phi_i(Y)}{dY^2}+(\mu_i^v)^2\Phi_i(Y)=0\,, \qquad \Phi_i(0)=0 \quad \text{e} \quad \Phi_i(L_y)=0\,. \tag{5a,b}$$

The orthonormal eigenfunctions $\Psi_i(Y)$ and $\Phi_i(Y)$; the normalization integrals A_i^u and A_i^v, and the eigenvalues μ_i^u e μ_i^v, are expressed, respectively, by:

$$\Psi_i(Y)=A_i^u\,sen(\mu_i^u Y)\,, \qquad A_i^u=\sqrt{\frac{2}{L_y}}\,, \qquad \mu_i^u=\frac{i\pi}{L_y}\,, \quad i=1,2,3,\ldots,\infty\,; \tag{6}$$

$$\Phi_i(Y)=A_i^v\,sen(\mu_i^v Y)\,, \qquad A_i^v=\sqrt{\frac{2}{L_y}}\,, \qquad \mu_i^v=\frac{i\pi}{L_y}\,, \quad i=1,2,3,\ldots,\infty\,. \tag{7}$$

In according with the eigenvalue problem defined above, it is obtained the transform-inverse pairs for $U(X,Y,\tau)$ and $V(X,Y,\tau)$, regarding the y-axis, as follow:

$$\tilde{U}_i(x,\tau)=\int_0^{L_y}\Psi_i(Y)U(X,Y,\tau)dY\,; \qquad U(X,Y,\tau)=\sum_{j=1}^{\infty}\tilde{U}_j(X,\tau)\Psi_j(Y)\,; \qquad \text{(Transform, Inverse)} \tag{8a,b}$$

$$\tilde{V}_i(X,\tau)=\int_0^{L_y}\Phi_i(Y)V(X,Y,\tau)dY\,; \qquad V(X,Y,\tau)=\sum_{j=1}^{\infty}\tilde{V}_j(X,\tau)\Phi_j(Y)\,. \qquad \text{(Transform, Inverse)} \tag{9a,b}$$

Once obtained the transform-inverse pairs, equations (1a) and (1b) are multiplied by its respective eigenfunctions $\Psi_i(Y)$ and $\Phi_i(Y)$, corresponding to velocities $U(X,Y,\tau)$ and $V(X,Y,\tau)$, and in the sequel equations (4) and (5) are multiplied by $U(X,Y,\tau)/Re$ and $V(X,Y,\tau)/Re$, respectively. Resulting equations are subtracted each other and then integrated over the closed interval $[0,L_y]$. The final resulting equations are given by

$$\frac{\partial\tilde{U}_i(X,\tau)}{\partial\tau}+\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}A_{ijk}\tilde{U}_j(X,\tau)\frac{\partial\tilde{U}_k(X,\tau)}{\partial X}+\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}B_{ijk}\tilde{V}_j(X,\tau)\tilde{U}_k(X,\tau)=\frac{1}{Re}\left[\frac{\partial^2\tilde{U}_i(X,\tau)}{\partial X^2}-(\mu_i^u)^2\tilde{U}_i(X,\tau)\right] \tag{10}$$

and

$$\frac{\partial\tilde{V}_i(X,\tau)}{\partial\tau}+\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}C_{ijk}\tilde{U}_j(X,\tau)\frac{\partial\tilde{V}_k(X,\tau)}{\partial X}+\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}D_{ijk}\tilde{V}_j(X,\tau)\tilde{V}_k(X,\tau)=\frac{1}{Re}\left[\frac{\partial^2\tilde{V}_i(X,\tau)}{\partial X^2}-(\mu_i^v)^2\tilde{V}_i(X,\tau)\right] \tag{11}$$

in which $A_{ijk}=\int_0^{L_y}\Psi_i(Y)\Psi_j(Y)\Psi_k(Y)dY\,;$ $\quad B_{ijk}=\int_0^{L_y}\Psi_i(Y)\Phi_j(Y)\frac{\partial\Psi_k(Y)}{\partial Y}dY\,;$

$C_{ijk}=\int_0^{L_y}\Phi_i(Y)\Psi_j(Y)\Phi_k(Y)dY\,;$ $\quad D_{ijk}=\int_0^{L_y}\Phi_i(Y)\Phi_j(Y)\frac{\partial\Phi_k(Y)}{\partial Y}dY\,,$ are determined analytically.

Equations (10) and (11) define a coupled system of infinite non-linear partial differential equations for the transformed velocities $\tilde{U}_i(X,\tau)$ and $\tilde{V}_i(X,\tau)$.

The initial conditions (2a,b**)** and the boundary conditions (3a,b,e,f), are either transformed, doing the multiplication of equations (2a) and (3a,b) by the normalized eigenfunctions $\Psi_i(Y)$ and of equations (2b) and (3e,f) by the normalized eigenfunctions $\Phi_i(Y)$, respectively, and then integrating them regarding the axis-y, over the closed interval $[0,L_y]$, obtaining the following formula:

$$U_i(X,\tau_0)=0\,; \qquad \widetilde{U}_i(X,\tau)\Big|_{X=0}=0\,; \qquad \widetilde{U}_i(X,\tau)\Big|_{X=L_x}=0\,; \qquad \text{(12a,b,c)}$$

$$\widetilde{V}_i(X,\tau_0)=0\,; \qquad \widetilde{V}_i(X,\tau)\Big|_{X=0}=\widetilde{g}_i\,; \qquad \widetilde{v}_i(x,t)\Big|_{x=L_x}=0\,. \qquad \text{(13a,b,c)}$$

The boundary condition (3g) is not homogeneous. Therefore, to improve the solution convergence rate it is adequate to do the following variable change

$\widetilde{V}_i(X,\tau)=\widetilde{V}_i^{+}(X,\tau)+\widetilde{g}_i(1-X/L_x)$, where $\widetilde{g}_i$ can be understood as a boundary condition source term.

Thus, equations (10) and (11), become

$$\frac{\partial \widetilde{U}_i(X,\tau)}{\partial t}+\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}A_{ijk}\widetilde{U}_j(X,t)\frac{\partial \widetilde{U}_k(X,\tau)}{\partial X}+\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}B_{ijk}\left[\widetilde{V}_j^{+}(X,\tau)+\widetilde{g}_j\left(1-\frac{X}{L_x}\right)\right]\widetilde{U}_k(X,\tau)=\frac{1}{Re}\left[\frac{\partial^2\widetilde{U}_i(X,\tau)}{\partial x^2}-(\mu_i^u)^2\widetilde{U}_i(X,\tau)\right] \quad (14)$$

and

$$\frac{\partial \widetilde{V}_i^{+}(X,\tau)}{\partial t}+\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}C_{ijk}\widetilde{U}_j(X,\tau)\left[\frac{\partial \widetilde{V}_k^{+}(X,\tau)}{\partial X}-\frac{\widetilde{g}_k}{L_x}\right]+\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}D_{ijk}\left[\widetilde{V}_j^{+}(X,\tau)+\widetilde{g}_j\left(1-\frac{X}{L_x}\right)\right]\left[\widetilde{V}_k^{+}(X,\tau)+\widetilde{g}_k\left(1-\frac{X}{L_x}\right)\right]=$$

$$=\frac{1}{Re}\left[\frac{\partial^2\widetilde{V}_i(X,\tau)}{\partial x^2}-(\mu_i^v)^2\widetilde{V}_i(X,\tau)\right] \quad (15)$$

4. Transforming the Problem in the X Direction

Analogously, to the auxiliary eigenvalue problems regarding the Y axis, it is considered in this section the following eigenvalue problems, Eqs. (16) and (17), and its respective boundary conditions associated to the X axis, for the transformed velocities $\widetilde{U}_i(X,\tau)$ and $\widetilde{V}_i^{+}(X,\tau)$:

$$\frac{d^2\Psi_m(X)}{dX^2}+(\gamma_m^u)^2\Psi_m(X)=0\,; \quad 0<x<L_x\,; \qquad \Psi_m(0)=0 \text{ e } \Psi_m(L_x)=0\,; \qquad \text{(16a,b)}$$

$$\frac{d^2\Phi_m(X)}{dX^2}+(\gamma_m^v)^2\Phi_m(X)=0\,; \qquad \Phi_m(0)=0 \text{ e } \Phi_m(L_x)=0\,. \qquad \text{(17a,b)}$$

The eigenfunctions $\Psi_m(X)$ and $\Phi_m(X)$, the normalization integrals A_m^u and A_m^v, and the eigenvalues λ_m^u and λ_m^v, for each eigenvalue problem, are given by:

$$\Psi_m(X)=A_m^u sen(\lambda_m^u X), \quad A_m^u=\sqrt{\frac{2}{L_x}}, \quad \lambda_m^u=\frac{m\pi}{L_x}, \quad m=1,2,3,\ldots,\infty\,; \qquad (18)$$

$$\Phi_m(X)=A_m^v sen(\lambda_m^v X), \quad A_m^v=\sqrt{\frac{2}{L_x}}, \quad \lambda_m^v=\frac{m\pi}{L_x}, \quad m=1,2,3,\ldots,\infty\,. \qquad (19)$$

In according with the eigenvalue problems (16) and (17), it is determined the transform-inverse pair associated to the X axis, as:

$$\overline{\widetilde{U}}_{im}(\tau)=\int_0^{L_x}\Psi_m(X)\widetilde{U}_i(X,\tau)dX\,; \qquad \widetilde{U}_i(X,\tau)=\sum_{n=1}^{\infty}\overline{\widetilde{U}}_{in}(\tau)\Psi_n(X)\,; \qquad \text{(Transform, Inverse)} \quad (20)$$

$$\overline{\widetilde{V}}^{+}_{im}(\tau)=\int_0^{L_x}\Phi_m(X)\widetilde{V}_i(X,\tau)\,dX\,;\qquad \widetilde{V}_i(X,\tau)=\sum_{n=1}^{\infty}\overline{\widetilde{V}}_{in}(\tau)\Phi_n(X)\,.\qquad \text{(Transform, Inverse)}\quad (21)$$

After, obtained the transform-inverse pairs, equations (20) and (21) are multiplied by the respective eigenfunctions $\Psi_m(X)$ and $\Phi_m(X)$, in sequence equations (16) and (17) are multiplied by $\widetilde{U}_i(X,\tau)/Re$ and $\widetilde{V}_i^{+}(X,\tau)/Re$, respectively. The resulting equations are subtracted each other and then integrated over the interval $[0,L_x]$. The final formulae obtained are:

$$\frac{d\overline{\widetilde{U}}_{im}(\tau)}{dt}+\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}A_{ijk}E_{mnp}\overline{\widetilde{U}}_{jn}(\tau)\overline{\widetilde{U}}_{kp}(\tau)+\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}B_{ijk}F_{mnp}\overline{\widetilde{V}}^{+}_{jn}(\tau)\overline{\widetilde{U}}_{kp}(\tau)+$$
$$+\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}B_{ijk}\widetilde{g}_iG_{mn}\overline{\widetilde{U}}_{kn}(\tau)-\frac{1}{L_x}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}B_{ijk}\widetilde{g}_iH_{mn}\overline{\widetilde{U}}_{kn}(\tau)=\frac{1}{Re}\left\{\left[(\mu_i^u)^2+(\lambda_i^u)^2\right]\overline{\widetilde{U}}_{im}(\tau)\right\}\quad (22)$$

and

$$\frac{d\overline{\widetilde{V}}_{im}(\tau)}{dt}+\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}C_{ijk}I_{mnp}\overline{\widetilde{\hat{u}}}_{jn}(\tau)\,\overline{\widetilde{V}}^{+}_{kp}(\tau)-\frac{1}{L_x}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}C_{ijk}\widetilde{g}_kJ_{mn}\overline{\widetilde{U}}_{jn}(\tau)+$$
$$+\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}\sum_{p=1}^{\infty}D_{ijk}K_{mnp}\overline{\widetilde{V}}^{+}_{jn}(\tau)\overline{\widetilde{V}}^{+}_{kp}(\tau)+\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}D_{ijk}\widetilde{g}_kL_{mn}\overline{\widetilde{V}}^{+}_{jn}(\tau)-$$
$$-\frac{1}{L_x}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}D_{ijk}\widetilde{g}_kM_{mn}\overline{\widetilde{V}}^{+}_{jn}(\tau)+\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}D_{ijk}\widetilde{g}_jL_{mn}\overline{\widetilde{V}}^{+}_{kn}(\tau)-\frac{1}{L_x}\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\sum_{n=1}^{\infty}D_{ijk}\widetilde{g}_jM_{mn}\overline{\widetilde{V}}^{+}_{kn}(\tau)=$$
$$=\frac{1}{Re}\left\{-\left[(\mu_i^v)^2+(\lambda_m^v)^2\right]\overline{\widetilde{V}}^{+}_{im}(\tau)+\left[\frac{1}{L_x}O_m-N_m\right](\mu_i^v)^2\widetilde{g}_i\right\}+\sum_{j=1}^{\infty}\sum_{k=1}^{\infty}\left[\frac{2}{L_x}O_m-\left(\frac{1}{L_x}\right)^2P_m-N_m\right]D_{ijk}\widetilde{g}_j\widetilde{g}_k\quad (23)$$

where $E_{mnp}=\int_0^{L_x}\Psi_m(X)\Psi_n(X)\frac{d\Psi_p(X)}{dX}dX$; $F_{mnp}=\int_0^{L_x}\Psi_m(x)\Phi_n(X)\Psi_p(X)\,dX$; $G_{mn}=\int_0^{L_x}\Psi_m(X)\Psi_n(X)\,dX$;

$H_{mn}=\int_0^{L_x}X\;\Xi_m(X)\Xi_n(X)dX$; $I_{mnp}=\int_0^{L_x}\Phi_m(X)\Psi_n(X)\frac{d\Phi_p(X)}{dX}dX$; $J_{mn}=\int_0^{L_x}\Phi_m(X)\Psi_n(X)dX$;

$K_{mnp}=\int_0^{L_x}\Phi_m(X)\Phi_n(X)\Phi_p(X)dX$; $L_{mn}=\int_0^{L_x}\Theta_m(X)\Theta_n(X)dX$; $M_{mn}=\int_0^{L_x}\Phi_m(X)\Phi_n(X)\,X\,dX$;

$N_m=\int_0^{L_x}\Theta_m(x)\,dx$; $O_m=\int_0^{L_x}\Phi_m(X)\,X\,dX$; $P_m=\int_0^{L_x}X^2\,\Phi_m(X)dX$.

Equations (22) and (23) define an infinite and coupled system of non-linear first order ordinary differential equations for the, two times, transformed velocities $\overline{\widetilde{U}}_{im}(\tau)$ and $\overline{\widetilde{V}}^{+}_{im}(\tau)$, subjected to some initial conditions that are gotten by transforming the problem initial conditions. Doing the transforms on the initial conditions one obtain

$$\overline{\widetilde{U}}_{im}(\tau_0)=0\qquad \text{e}\qquad \overline{\widetilde{V}}^{+}_{im}(\tau_0)=\widetilde{g}_i\left(\frac{O_m}{L_x}-N_m\right).\qquad (24a,b)$$

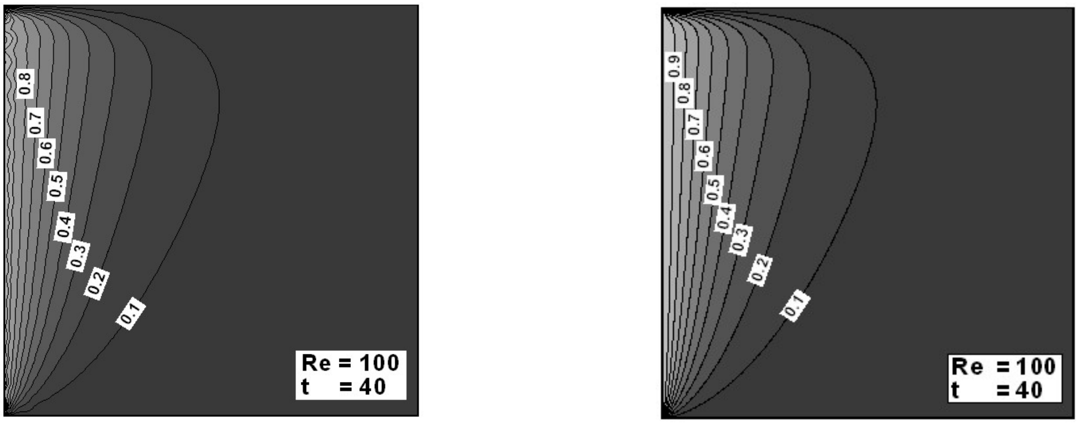
In order to obtain numerical results for the infinite size system described by equations (22) and (23), subjected to the transformed initial conditions, given by equations (24a,b), the series must be truncated to and order, big enough to guarantee convergence.

Once the system is truncated, it becomes finite and can be solved numerically. Therefore, the transformed velocities, $\tilde{\bar{U}}_{im}(\tau)$ and $\tilde{\bar{V}}^{+}_{im}(\tau)$, are obtained. Utilizing the inverse formulae, it is obtained the velocity distributions $U(X,Y,\tau)$ and $V(X,Y,\tau)$, building a solution to the original problem.

5. Results

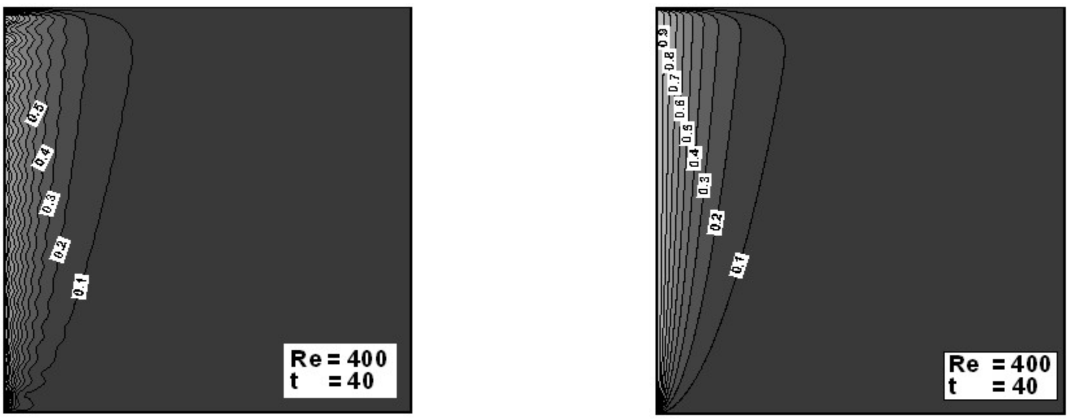
The velocities series convergence rate is quite slow, being necessary truncating orders, relatively, high to obtain more precise results. Otherwise, the strong non-linearity and the complexity in the system of differential equations require a great amount of computer processing time. Therefore, it was considered truncating orders in the series of up to 35 terms for each direction, or 1225 equations associated with each velocity component. For the two components there was a global system with 2450 first order non-linear ordinary differential equations. Aiming to compare results, the same problem was solved by using the finite difference method (FDM), in its implicit scheme. To discretize the domain it was used meshes with 401 nodes in each direction and for the time axis discretization it was used time intervals equal to $\Delta t = 10^{-4}$.

Since Burgers´ equation, for this application, has a trivial solution for the velocity component, $U(X,Y,\tau) = 0$, the diffusive effect is transmitted only through the viscosity property. As the Reynolds number increases, the velocity gradient increases near the moving wall. This fact can be seen in Figures (2)-(4), where are shown the iso-velocity curves. As can be seen, the results obtained by using GITT presented a wavy behavior mostly near the moving wall. This fact is due to a relatively small number o terms considered in the truncated series. Such wavy effect could be decreased by increasing the number of terms in the truncated series, but this overcame our present computer power. Broadly, the numerical results obtained by using GITT show good agreement with those ones obtained by using FDM.



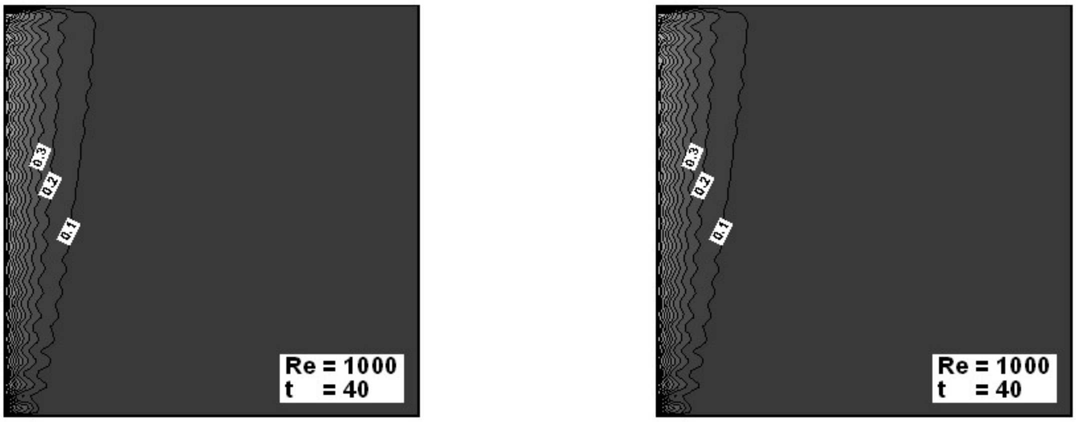
(a) GITT (b) FDM

Fig. 2: Velocity component field $V(X,Y,\tau)$, for Re = 100 and $\tau = 40$.



(a) GITT (b) FDM

Fig. 3: Velocity component field $V(X,Y,\tau)$, for Re = 400 and $\tau = 40$.



(a) GITT (b) FDM

Fig. 4: Velocity component field $V(X,Y,\tau)$, for Re = 1000 and $\tau = 40$.

In Figures 5, 6 and 7, are presented a comparison among the results obtained by using GITT and FDM for the component V of the velocity profile along the line for y = 0.5 for Reynolds numbers, 100, 400 and 1000, respectively. The agreement is fairly good. For a given dimensionless time and position in the domain, one can observe that as the Reynolds number increases the velocity decreases.

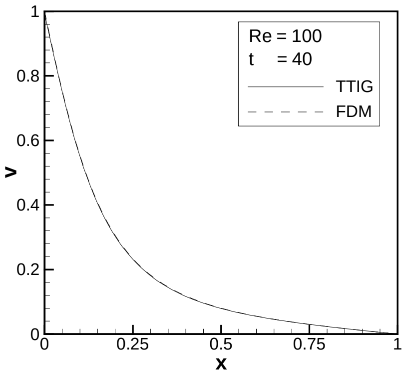


Fig. 5: Velocity component $V(X,Y,\tau)$, for Re = 100 and y = 0.5.

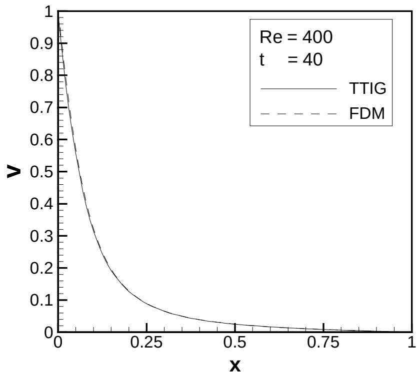


Fig. 6: Velocity component $V(X,Y,\tau)$, for Re = 400 and y = 0.5.

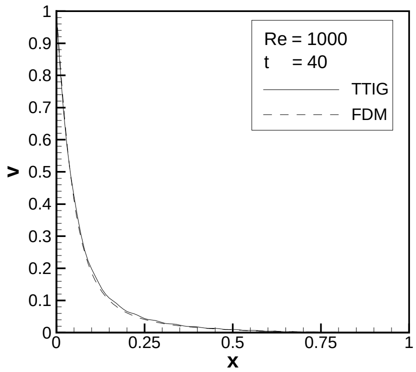


Fig. 7: Velocity component $V(X,Y,\tau)$, for Re = 1000 and y = 0.5.

In Figures 8, 9 and 10 are presented the velocity component V(x,y,t), at some given positions, as function of time, for flows with Reynolds numbers, 100, 400 and 1000. The chosen domain position are along the medium line, y = 0.5, and for x = 0.05, 0.1, 0.2 and 0.5. One can observe that the steady flow regime is achieved more quickly for the region near the moving wall for cases analyzed.

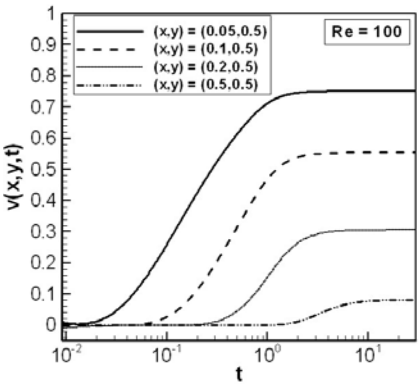


Fig. 8: Velocity × Time, for Re = 100 and y = 0.5.

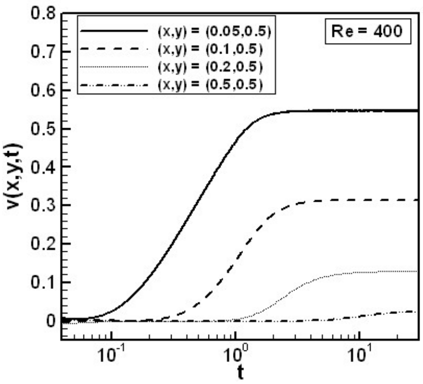


Fig. 9: Velocity × Time, for Re = 400 and y = 0.5.

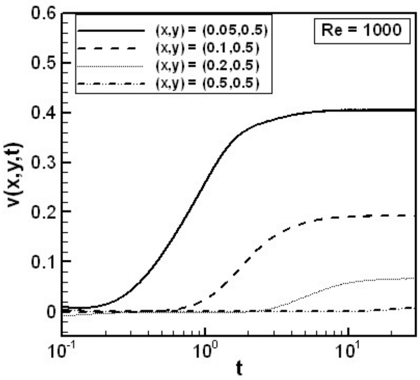


Fig. 10: Velocity × Time, for Re = 1000 and y = 0.5.

6. Conclusion

For Burgers´ equation study the two-dimensional cavity problem is characterized by a perturbation, caused by the displacement of a moving wall, which propagates through diffusion along the cavity. This kind of “imaginary” flow is interesting because it retain the most of difficulties encountered when solving “real” flow problems, modeled by

Navier-Stokes equations. So, the numerical methods used to solve it, and the obtained results are useful to understand and improve the solution of industrial flows.

It is noticed also that to solve the Burgers´ equation problem, for a given geometry and constraining conditions, was used the Generalized Integral Transform Technique (GITT). It was observed that, due to the equations non-linearity, the series solution shown a relatively slow convergence ratio, mainly for the beginning of the flow and also for positions near the moving wall. However, series expansion with 35 terms for each direction allowed obtaining fairly good results for qualitative analysis.

Finally, in order to validate the whole process and compare results, the problems was also solved by using the Method of Finite Difference, the details are not shown here due to lack o space. The agreement among the results obtained using GITT and FDM are quite good.

7. Acknowledgement

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9. Responsibility notice

The authors are the only responsible for the printed material included in this paper.